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On Groups and Their Weight

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Abstract

In this paper we compute weights of various groups like linear groups, finite p groups and groups of order upto order 50. The weights are expected to play a very vital role.

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Let G be a group and let $X \subseteq G$. We denote the normal closure of X in G by $\langle X\rangle_N$. Thus $\langle X\rangle_N = \langle x^g = g^{-1}xg | x \in X, g \in G\rangle$.The weight, $w(G)$, of a group G is the least cardinality of a subset X of G such that $G = \langle X\rangle_N$. A group is said to be of finite weight if X is finite. We define weight of a trivial group to be 1. For a normal subgroup H of G, we define G-weight of H, $w_G(H)$, as the least cardinality of a subset Y of H such that $H = Y^G = \langle y^g \mid y \in Y,\ g \in G\rangle$. Clearly $w_G(H) \leq w(H)$.

The concept of weight of groups and especially the relation between weight of a group and its abelianisation arise naturally in the study of knot theory. Kutzko [4] studied groups of finite weight and proved that
Theorem [Kutzko]. *If G is a group of finite weight and the lattice of normal subgroups of G contained in the commutator subgroup G' satisfies the minimum condition then $w(G) = w(G/G')$.*

Rhemtulla [5] carried the study further and proved the following theorem.
Theorem [Rhemtulla]. *If N is a normal subgroup of a group G and G acts on N by conjugation, then $d_G(N) = d_G(N/N')$ provided $d_G(N)$ is finite and N has the following property:*
There does not exist an infinite descending series of $G-$subgroups $N' = C_0 >$

$C_1 > \ldots$ *with each* C_i/C_{i+1} *perfect.*
Note that $d_G(N) = w_G(N)$ and $d_G(G) = w(G)$.

In this connection, Gupta, Sharma, Srivastava [2] proved the following theorem.
Theorem [Gupta, Sharma, Srivastava]. *Let G be a nilpotent group. Then weight of G is finite if and only if G is finitely generated.*

They also gave a counter example to show that it is not the case in solvable groups.

In this paper we compute weight of various classes of groups. In particular, we compute weight of linear groups like $GL_n(K)$, $UT_n(K)$ and $T_n(K)$ for any field K. We also compute the weight of finite p groups of order upto p^n, $n \leq 5$, p odd prime, and certain groups of small order upto 50.

1 Weight of linear groups

We first prove a lemma regarding weight of $K^* = K \setminus \{0\}$, where K is a field. This lemma is very helpful in computing weights of certain groups.

Lemma 1. *For any field K, $w(K^*)$ is finite if and only if K is finite. In this case $w(K^*) = 1$.*

Proof. Clearly for a finite field K, $w(K^*) = 1$. If K is a field of characteristic 0 then $w(K^*) < \infty$ will give that K^* is finitely generated abelian, which is not possible as $\mathbb{Q}^*$ is not finitely generated. Now let us consider the case when K is an infinite field of characteristic $p > 0$. If K is algebraic over $\mathbb{Z}_p$, then K^* is locally finite. Hence finite weight will give that K^* is finitely generated and hence finite which is a contradiction. Now in the case when K is not algebraic over $\mathbb{Z}_p$, let $\xi \in K$ be transcendental over $\mathbb{Z}_p$. Then $w(K^*) < \infty$ will give K^* is finitely generated abelian but $\mathbb{Z}_p(\xi)$ can not be finitely generated. Hence $w(K^*) = \infty$ if K is infinite. □

We begin by computing the weight of general linear group $GL_n(K)$, for any field K.

Theorem 1. *Let $GL_n(K)$ be the group of all $n \times n$ invertible matrices with entries from the field K. Then*

(*i*) $w(GL_n(K)) = 1$, *for any finite field K.*

(*ii*) $w(GL_n(K)) = \infty$, *when K is a infinite field of char $p > 0$.*

(*iii*) $w(GL_n(K)) = \infty$, *when K is a infinite field of char* 0.

Proof. We know that $GL_n(K)/SL_n(K) \cong K^*$, where $K^* = K\backslash\{0\}$. In case K is a finite field, K^* is cyclic. Further, $GL_n(K)' = SL_n(K)$ unless $n = 2$ and $K = \mathbb{Z}_2$. In case $n = 2$ and $K = \mathbb{Z}_2$, we know $GL_2(\mathbb{Z}_2) \cong S_3$. Hence in case of finite field K, $w(GL_n(K)) = 1$. For an infinite field K, using the Lemma 1 we get $w(GL_n(K)/SL_n(K)) = \infty$ and hence $w(GL_n(K)) = \infty$. □

Another important group is $UT_n(K)$, the group of all $n \times n$ unitriangular matrices over K. These groups provide a good source of nilpotent groups. We now obtain $w(UT_n(K))$, for any field K:

Theorem 2. *Let $UT_n(K)$ be the group of all $n \times n$ upper triangular matrices with* 1 *on the diagonal and entries from the field K. Then*

(*i*) $w(UT_n(K)) = m(n-1)$, *when K is a finite field and $|K| = p^m$.*

(*ii*) $w(UT_n(K)) = \infty$, *when K is a infinite field of char $p > 0$.*

(*iii*) $w(UT_n(K)) = \infty$, *when K is a infinite field of char* 0.

Proof. (*i*) As $(UT_n(K))' = \langle I + V |\ V$ is strictly upper triangular with first super diagonal zero $\rangle$, we have $\left|\frac{UT_n(K)}{(UT_n(K))'}\right| = p^{m(n-1)}$. Also as $char K = p$, every element of $\frac{UT_n(K)}{(UT_n(K))'}$ has order p. Hence $\frac{UT_n(K)}{(UT_n(K))'} \cong C_p \times C_p \times ... \times C_p$, $m(n-1)$ times. Thus $w(UT_n(K)) = w\left(\frac{UT_n(K)}{(UT_n(K))'}\right) = m(n-1)$.

(*ii*) For any $A \in UT_n(K)$, we have $o(A)$ is power of prime p. So $UT_n(K)$ is a torsion nilpotent group. Let if possible $w(UT_n(K)) < \infty$, then by [2] we get $UT_n(K)$ is finitely generated, hence finite. This is a contradiction as K is an infinite field. Hence $w(UT_n(K)) = \infty$ if F is an infinite field of *char* $p > 0$.

(*iii*) Let if possible $w(UT_n(K))$ be finite. Then by [2], $UT_n(K)$ is a finitely generated nilpotent group and hence a polycyclic group. Thus $UT_n(K)$ satisfies maximal condition on subgroups, hence every subgroup of $UT_n(K)$ is finitely generated. But

$$\left\{ \begin{bmatrix} 1 & 0 & 0 & \dots & \alpha \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & & \ddots & & \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix} \middle|\ \alpha \in K \right\} \leq UT_n(K)$$

which is not finitely generated. Hence $w(UT_n(K)) = \infty$.

□

We next compute weight of $T_n(K)$, for any field K. These groups provide a good source of solvable groups.

Theorem 3. *Let $T_n(K)$ be the group of all $n \times n$ upper triangular matrices with units on the diagonal and entries from the field K. Then*

(*i*) *For a finite field K,*

$$w(T_n(K)) = \begin{cases} n & , \quad K \neq GF(2) \\ n-1 & , \quad K = GF(2). \end{cases}$$

(*ii*) $w(T_n(K)) = \infty$, *when K is a infinite field of char $p > 0$.*

(*iii*) $w(T_n(K)) = \infty$, *when K is a infinite field of char* 0.

Proof. We know that $T_n(K)/UT_n(K) \cong (K^*)^n$. Further, $T_n(K)' = UT_n(K)$ unless $K = GF(2)$. In case $K = GF(2)$, we know $T_n(GF(2)) = UT_n(GF(2))$. So $w(T_n(GF(2))) = w(UT_n(GF(2))) = n-1$, using Theorem 2. In case K is a finite field and $K \neq GF(2)$, we have $w(T_n(K)) = w(T_n(K)/(T_n(K))') = w(T_n(K)/UT_n(K)) = w((K^*)^n) = n\, w(K^*) = n$. For an infinite field K, using Lemma 1 we get $w(T_n(K)/UT_n(K)) = \infty$ and hence $w(T_n(K)) = \infty$.

□

2 Finite p - groups

In this section, we compute the weights of all finite p-groups of order p^n, $n \leq 5$, p an odd prime. Due to space, weight of these groups is given in Table 1. For better comprehension of the table, the following be taken into consideration:

1. For the economy of space, all relations of the form $[a, b] = 1$(with a, b generators) have been omitted and should be assumed when reading the presentation.

2. Throughout, ν denotes the smallest positive integer which is a non-quadratic residue (mod p).

3. Throughout, g denotes the smallest positive integer which is a primitive root (mod p).

For notations and other details, we refer to [6]. Further, we note that for a non abelian group of order p^3, $|Z(G)| = p$ and $|G/Z(G)| = p^2$. Hence $G' = Z(G)$ and $G/Z(G) \cong C_p \times C_p$. Hence weight of all non abelian groups of order p^3, p an odd prime, is 2. We use the result for nilpotent groups that $w(G) = w(G/G')$.

3 Weights of Finite Groups of order up to 50

In this section, we compute weights of groups of small order, to be precise of order up to 50. We note that if G is a group of order $p_1 p_2 \cdots p_n$, where p_i's are distinct primes, then G/G' is abelian of order $m = p_{i_1} p_{i_2} \cdots p_{i_r}$ $(i_r \leq n)$, where p_i's are distinct primes and hence G/G' is cyclic of order m. Thus $w(G) = w(G/G') = w(C_m) = 1$. Hence groups of order $1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19, 21, 22, 23, 26, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 46, 47$ each have weight 1. Also one can see that for dihedral group D_{2n}, $w(D_{2n}) = 1$ if n is odd; $w(D_{2n}) = 2$ if n is even. Clearly if G is any non abelian simple group (finite or infinite) then $w(G) = 1$. The weight of small groups of order upto 50 are given in Table 2.

TABLE 1

Table of weight of groups of order p^n, $n \leq 5$, p an odd prime. Groups in the following table refer to those in [6].

Presentation of Group	$w(G)$
C_p	1
C_{p^2}	1
$C_p \times C_p$	2
C_{p^3}	1
$C_{p^2} \times C_p$	2
$C_p \times C_p \times C_p$	3
$< a, b, c \mid [b, a] = c, a^p = c, b^p = c^p = 1 >$	2
$< a, b, c \mid [b, a] = c, a^p = b^p = c^p = 1$	2
C_{p^4}	1
$C_{p^3} \times C_p$	2
$C_{p^2} \times C_{p^2}$	2
$C_{p^2} \times C_p \times C_p$	3
$C_p \times C_p \times C_p \times C_p$	4
$< a, b, c \mid [b, a] = c, a^p = c, b^p = c^p = 1 > \times C_p$	3
$< a, b, c \mid [b, a] = c, a^p = b^p = c^p = 1 > \times C_p$	3
$< a, b, c \mid [b, a] = a^{p^2} = c, b^p = c^p = 1 >$	2
$< a, b, c \mid [b, a] = a^p = c, b^{p^2} = c^p = 1 >$	2
$< a, b, c, d \mid [b, a] = d^p = c, a^p = b^p = c^p = 1 >$	3
$< a, b, c \mid [b, a] = c, a^{p^2} = b^p = c^p = 1 >$	2
$< a, b, c, d \mid [b, a] = c, [c, a] = a^p = d, b^p = c^p = d^p = 1 >$	2
$< a, b, c, d \mid [b, a] = c, [c, a] = b^p = d, a^p = c^p = d^p = 1 >$	2
$< a, b, c, d \mid [b, a] = c, [c, a]^\nu = b^p = d^\nu, a^p = c^p = d^p = 1 >$	2
$< a, b, c, d \mid [b, a] = c, [c, a] = d, a^p = b^p = c^p = d^p = 1 >$	2

Presentation of Group	$w(G)$
C_{p^5}	1
$C_{p^4} \times C_p$	2
$C_{p^3} \times C_{p^2}$	2
$C_{p^3} \times C_p \times C_p$	3
$C_{p^2} \times C_{p^2} \times C_p$	3
$C_{p^2} \times C_p \times C_p \times C_p$	4
$C_p \times C_p \times C_p \times C_p \times C_p$	5
$< a, b, c \mid [b, a] = a^{p^2} = c, b^p = c^p = 1 > \times C_p$	3
$< a, b, c \mid [b, a] = a^p = c, b^{p^2} = c^p = 1 > \times C_p$	3
$< a, b, c \mid [b, a] = c = a^p, b^p = c^p = 1 > \times C_{p^2}$	3
$< a, b, c \mid [b, a] = c = a^p, b^p = c^p = 1 > \times C_p \times C_p$	4
$< a, b, c, d \mid [b, a] = d^p = c, a^p = b^p = c^p = 1 > \times C_p$	4
$< a, b, c \mid [b, a] = c, a^{p^2} = b^p = c^p = 1 > \times C_p$	3
$< a, b, c \mid [b, a] = c, a^p = b^p = c^p = 1 > \times C_{p^2}$	3
$< a, b, c \mid [b, a] = c, a^p = b^p = c^p = 1 > \times C_p \times C_p$	4
$< a, b, c \mid [b, a] = a^{p^3} = c, b^p = c^p = 1 >$	2
$< a, b, c \mid [b, a] = a^{p^2} = c, b^{p^2} = c^p = 1 >$	2
$< a, b, c \mid [b, a] = b^p = c, a^{p^3} = c^p = 1 >$	2
$< a, b, c, d \mid [b, a] = d^{p^2} = c, a^p = b^p = c^p = 1 >$	3
$< a, b, c \mid [b, a] = c, a^{p^3} = b^p = c^p = 1 >$	2
$< a, b, c, d \mid [b, a] = d^p = c, a^{p^2} = b^p = c^p = 1 >$	3
$< a, b, c \mid [b, a] = c, a^{p^2} = b^{p^2} = c^p = 1 >$	2
$< a, b, c, d \mid [b, a] = c, [c, a] = a^p = d, b^p = c^p = d^p = 1 > \times C_p$	3
$< a, b, c, d \mid [b, a] = c, [c, a] = b^p = d, a^p = c^p = d^p = 1 > \times C_p$	3
$< a, b, c, d \mid [b, a] = c, [c, a]^\nu = b^p = d^\nu, a^p = c^p = d^p = 1 > \times C_p$	3
$< a, b, c, d \mid [b, a] = c, [c, a] = d, a^p = b^p = c^p = d^p = 1 > \times C_p$	3
$< a, b, c, d \mid [b, a] = c, [c, a] = a^{p^2} = d, b^p = c^p = d^p = 1 >$	2
$< a, b, c, d \mid [b, a] = c, [c, a] = b^{p^2} = d, a^p = c^p = d^p = 1 >$	2
$< a, b, c, d \mid [b, a] = c, [c, a]^\nu = b^{p^2} = d, a^p = c^p = d^p = 1 >$	2
$< a, b, c, d \mid [b, a] = c, [c, a] = a^p = d, b^{p^2} = c^p = d^p = 1 >$	2
$< a, b, c, d \mid [b, a] = c, [c, a] = b^p = d, a^{p^2} = c^p = d^p = 1 >$	2
$< a, b, c, d \mid [b, a] = c, [c, a]^\nu = b^p = d^\nu, a^{p^2} = c^p = d^p = 1 >$	2
$< a, b, c, d, f \mid [b, a] = c, [c, a] = f^p = d, a^p = b^p = c^p = d^p = 1 >$	3
$< a, b, c, d \mid [b, a] = c, [c, a] = d, a^{p^2} = b^p = c^p = d^p = 1 >$	2
$< a, b, c, d \mid [b, a] = c, [c, a] = d, a^p = b^{p^2} = c^p = d^p = 1 >$	2
$< a, b, c, d, f \mid [b, a] = d = b^p, [c, a] = f = a^p, c^p = d^p = f^p = 1 >$	3
$< a, b, c, d, f \mid [b, a] = d = c^p, [c, a] = f = a^p, b^p = d^p = f^p = 1 >$	3
$< a, b, c, d, f \mid [b, a] = d = b^p, [c, a] = f = c^p, a^p = d^p = f^p = 1 >$	3
$< a, b, c, d, f \mid [b, a] = d, [c, a] = f = c^p, b^p = d^k, a^p = d^p = f^p = 1 >$	3
where $k = g^r$ for $r = 1, 2, ..., (p-1)/2$	

Presentation of Group	$w(G)$
$< a,b,c,d,f \mid [b,a]=d,[c,a]=f,b^p=f^{-1/4},c^p=df,a^p=d^p=f^p=1 >$	3
$< a,b,c,d,f \mid [b,a]=d,[c,a]=f=b^p,c^p=d^\nu,a^p=d^p=f^p=1 >$	3
$< a,b,c,d,f \mid [b,a]=d,[c,a]=f,b^p=f^k,c^p=df,a^p=d^p=f^p=1 >$	3
where $4k=g^{2r+1}-1$ for $r=1,2,...,(p-1)/2$	
$< a,b,c,d,f \mid [b,a]=d,[c,a]=f=a^p,b^p=c^p=d^p=f^p=1 >$	3
$< a,b,c,d,f \mid [b,a]=d=b^p,[c,a]=f,a^p=c^p=d^p=f^p=1 >$	3
$< a,b,c,d,f \mid [b,a]=d=c^p,[c,a]=f,a^p=b^p=d^p=f^p=1 >$	3
$< a,b,c,d,f \mid [b,a]=d,[c,a]=f,a^p=b^p=c^p=d^p=f^p=1 >$	3
$< a,b,c,d,f \mid [a,b]=[c,d]=a^p=f,b^p=c^p=d^p=f^p=1 >$	4
$< a,b,c,d,f \mid [a,b]=[c,d]=f,a^p=b^p=c^p=d^p=f^p=1 >$	4
$< a,b,c,d,f \mid [a,b]=c,[c,a]=d=a^p,[c,b]=f=b^p,c^p=d^p=f^p=1 >$	2
$< a,b,c,d,f \mid [a,b]=c,[c,a]=d,[c,b]=f,a^p=d^k,b^p=f,c^p=d^p=f^p=1 >$	2
where $k=g^r$ for $r=1,2,...,(p-1)/2$	
$< a,b,c,d,f \mid [a,b]=c,[c,a]=d,[c,b]=f,a^p=f^{-r/4},b^p=d^rf^r,c^p=d^p=f^p=1 >$	2
where $r=1$ or ν	
$< a,b,c,d,f \mid [a,b]=c,[c,a]=d,[c,b]=f,a^p=f,b^p=d^\nu,c^p=d^p=f^p=1 >$	2
$< a,b,c,d,f \mid [a,b]=c,[c,a]=d,[c,b]=f,a^p=f^k,b^p=df,c^p=d^p=f^p=1 >$	2
where $4k=g^{2r+1}-1$ for $r=1,2,...,(p-1)/2$	
$< a,b,c,d,f \mid [a,b]=c,[c,a]=d,[c,b]=f,a^p=d,b^p=c^p=d^p=f^p=1 >$	2
$< a,b,c,d,f \mid [a,b]=c,[c,a]=d,[c,b]=f,b^p=d^r,a^p=c^p=d^p=f^p=1 >$	2
where $r=1$ or ν and $p>3$	
$< a,b,c,d,f \mid [a,b]=c,[c,a]=d,[c,b]=f,a^p=b^p=c^p=d^p=f^p=1 >$	2
$< a,b,c,d,f \mid [b,a]=c,[c,a]=d,[b,f]=a^p=d,b^p=c^p=d^p=f^p=1 >$	3
$< a,b,c,d,f \mid [b,a]=c,[c,a]=d,[b,f]^r=b^p=d^r,a^p=c^p=d^p=f^p=1 >$	3
where $r=1$ or ν	
$< a,b,c,d,f \mid [b,a]=c,[c,a]=d,[b,f]=f^p=d,a^p=b^p=c^p=d^p=1 >$	3
$< a,b,c,d,f \mid [b,a]=c,[c,a]=d,[b,f]=d,a^p=b^p=c^p=d^p=f^p=1 >$	3
$< a,b,c \mid [a,b]=c=a^p,c^{p^2}=b^{p^2}=1 >$	2
$< a,b,c,d,f \mid [b,a]=c,[c,a]=d,[d,a]=a^p=f,b^p=c^p=d^p=f^p=1 >$	2
$< a,b,c,d,f \mid [b,a]=c,[c,a]=d,[d,a]=f,b^p=f^k,a^p=c^p=d^p=f^p=1 >$	2
where $k=g^r$ for $r+1=1,2,...,(p-1,3)$	
$< a,b,c,d,f \mid [b,a]=c,[c,a]=d,[d,a]=f,a^p=b^p=c^p=d^p=f^p=1 >$	2
$< a,b,c,d,f \mid [b,a]=c,[c,a]=d,[d,a]=f,[b,c]^k=f^k=a^p,b^p=c^p=d^p=f^p=1 >$	2
where $k=g^r$ for $r+1=1,2,...,(p-1,4)$	
$< a,b,c,d,f \mid [b,a]=c,[c,a]=d,[d,a]=f,[b,c]^k=f^k=b^p,a^p=c^p=d^p=f^p=1 >$	2
where $k=g^r$ for $r+1=1,2,...,(p-1,3)$ and $p>3$	
$< a,b,c,d,f \mid [b,a]=c,[c,a]=d,[d,a]=f,[b,c]=f,a^p=b^p=c^p=d^p=f^p=1 >$	2

TABLE 2

Table of weights of groups of order upto 50 except 24, 32, 36, 40, 48.

Order	Groups	$w(G)$
1	C_1	1
2	C_2	1
3	C_3	1
4	C_4	1
	$C_2 \times C_2$	2
5	C_5	1
6	C_6	1
	$D_6 =< a, b \mid a^3 = b^2 = 1, b^{-1}ab = a^{-1} >$	1
7	C_7	1
8	C_8	1
	$C_4 \times C_2$	2
	$C_2 \times C_2 \times C_2$	3
	$D_8 =< a, b \mid a^4 = b^2 = 1, b^{-1}ab = a^{-1} >$	2
	$Q_8 =< a, b \mid a^4 = b^4 = 1, b^{-1}ab = a^{-1}, b^2 = a^2 >$	2
9	C_9	1
	$C_3 \times C_3$	2
10	C_{10}	1
	$D_{10} =< a, b \mid a^5 = b^2 = 1, b^{-1}ab = a^{-1} >$	1
11	C_{11}	1
12	C_{12}	1
	$C_6 \times C_2$	2
	$D_{12} =< a, b \mid a^6 = b^2 = 1, b^{-1}ab = a^{-1} >$	2
	$A_4 =< a, b \mid a^2 = b^3 = 1, a = babab >$	1
	$< a, b \mid a^4 = b^3 = 1, a^{-1}ba = b^{-1} >$	1
13	C_{13}	1
14	C_{14}	1
	$D_{14} =< a, b \mid a^7 = b^2 = 1, b^{-1}ab = a^{-1} >$	1
15	C_{15}	1
16	C_{16}	1
	$C_8 \times C_2$	2
	$C_4 \times C_4$	2
	$C_4 \times C_2 \times C_2$	3
	$C_2 \times C_2 \times C_2 \times C_2$	4

Order	Groups	$w(G)$
	$< a, b \mid a^8 = b^2 = 1, b^{-1}ab = a^5 >$	2
	$< a, b \mid a^4 = b^4 = 1, b^{-1}ab = a^3 >$	2
	$< a, b, c \mid a^4 = b^2 = c^2 = 1, c^{-1}bc = ba^2, b^{-1}ab = a, c^{-1}ac = a >$	3
	$< a, b, c \mid a^4 = b^2 = c^2 = 1, c^{-1}ac = a^3, a^{-1}ba = b, c^{-1}bc = b >$	3
	$< a, b, c \mid a^4 = b^2 = c^2 = 1, c^{-1}ac = ab, b^{-1}ab = a, c^{-1}bc = b >$	2
	$< a, b, c \mid a^4 = b^4 = c^2 = 1, b^{-1}ab = a^3, a^2 = b^2, c^{-1}bc = b, c^{-1}ac = a >$	3
	$< a, b \mid a^8 = b^2 = 1, b^{-1}ab = a^3 >$	2
	$D_{16} =< a, b \mid a^8 = b^2 = 1, b^{-1}ab = a^{-1} >$	2
	$Q_{16} =< a, b \mid a^8 = b^4 = 1, b^{-1}ab = a^{-1}, b^2 = a^4 >$	2
17	C_{17}	1
18	C_{18}	1
	$C_6 \times C_3$	2
	$D_{18} =< a, b \mid a^9 = b^2 = 1, b^{-1}ab = a^{-1} >$	1
	$< a, b, c \mid a^2 = b^3 = c^3 = 1, a^{-1}ba = b^2, a^{-1}ca = c^2, c^{-1}bc = b >$	1
	$< a, b, c \mid a^3 = b^3 = c^2 = 1, c^{-1}bc = b^2, c^{-1}ac = a, b^{-1}ab = a >$	1
19	C_{19}	1
20	C_{20}	1
	$C_{10} \times C_2$	2
	$D_{20} =< a, b \mid a^{10} = b^2 = 1, b^{-1}ab = a^{-1} >$	2
	$< a, b \mid a^4 = b^5 = 1, a^{-1}ba = b^2 >$	1
	$< a, b \mid a^4 = b^5 = 1, a^{-1}ba = b^4 >$	1
21	C_{21}	1
	$< a, b \mid a^3 = b^7 = 1, a^{-1}ba = b^2 >$	1
22	C_{22}	1
	$D_{22} =< a, b \mid a^{11} = b^2 = 1, b^{-1}ab = a^{-1} >$	1
23	C_{23}	1
25	C_{25}	1
	$C_5 \times C_5$	2
26	C_{26}	1
	$D_{26} =< a, b \mid a^{13} = b^2 = 1, b^{-1}ab = a^{-1} >$	1
27	C_{27}	1
	$C_9 \times C_3$	2
	$C_3 \times C_3 \times C_3$	3
	$< a, b \mid a^9 = b^3 = 1, b^{-1}ab = a^4 >$	2
	$< a, b, c \mid a^3 = b^3 = c^3 = 1, c^{-1}bc = ba, c^{-1}ac = a, b^{-1}ab = a >$	2
28	C_{28}	1
	$C_{14} \times C_2$	2
	$D_{28} =< a, b \mid a^{14} = b^2 = 1, b^{-1}ab = a^{-1} >$	2
	$< a, b \mid a^4 = b^7 = 1, a^{-1}ba = b^{-1} >$	1
29	C_{29}	1

Order	Groups	$w(G)$
30	C_{30}	1
	$D_{30} =< a, b \mid a^{15} = b^2 = 1, b^{-1}ab = a^{-1} >$	1
	$< a, b \mid a^5 = b^6 = 1, b^{-1}ab = a^{-1} >$	1
	$< a, b \mid a^3 = b^{10} = 1, b^{-1}ab = a^{-1} >$	1
31	C_{31}	1
33	C_{33}	1
34	C_{34}	1
	$D_{34} =< a, b \mid a^{17} = b^2 = 1, b^{-1}ab = a^{-1} >$	1
35	C_{35}	1
37	C_{37}	1
38	C_{38}	1
	$D_{38} =< a, b \mid a^{19} = b^2 = 1, b^{-1}ab = a^{-1} >$	1
39	C_{39}	1
	$< a, b \mid a^{13} = b^3 = 1, b^{-1}ab = a^{-1} >$	1
41	C_{41}	1
42	C_{42}	1
	$D_{42} =< a, b \mid a^{21} = b^2 = 1, b^{-1}ab = a^{-1} >$	1
	$< a, b, c \mid a^7 = b^3 = c^2 = 1, c^{-1}bc = b^{-1}, b^{-1}ab = a, c^{-1}ac = a >$	1
	$< a, b, c \mid a^7 = b^3 = c^2 = 1, c^{-1}ac = a^{-1}, b^{-1}ab = a, c^{-1}bc = b >$	1
	$< a, b, c \mid a^7 = b^3 = c^2 = 1, b^{-1}ab = a^2, c^{-1}bc = b, c^{-1}ac = a >$	1
	$< a, b, c \mid a^7 = b^3 = c^2 = 1, b^{-1}ab = a^2, c^{-1}ac = a^6, c^{-1}bc = b >$	1
43	C_{43}	1
44	C_{44}	1
	$C_{22} \times C_2$	2
	$D_{44} =< a, b \mid a^{22} = b^2 = 1, b^{-1}ab = a^{-1} >$	2
	$< a, b, c \mid a^{11} = b^2 = c^2 = 1, b^{-1}ab = a^{-1}, c^{-1}ac = a, c^{-1}bc = b >$	2
45	C_{45}	1
	$C_{15} \times C_3$	2
46	C_{46}	1
	$D_{46} =< a, b \mid a^{23} = b^2 = 1, b^{-1}ab = a^{-1} >$	1
47	C_{47}	1
49	C_{49}	1
	$C_7 \times C_7$	2
50	C_{50}	1
	$C_{10} \times C_5$	2
	$D_{50} =< a, b \mid a^{25} = b^2 = 1, b^{-1}ab = a^{-1} >$	1
	$< a, b, c \mid a^2 = b^5 = c^5 = 1, a^{-1}ba = b^4, a^{-1}ca = c^4, c^{-1}bc = b >$	1
	$< a, b, c \mid a^5 = b^5 = c^2 = 1, c^{-1}bc = b^4, c^{-1}ac = a, b^{-1}ab = a >$	1

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