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Intuitionistic Fuzzy Ideals of Subtraction Algebras

R. Ezhilarasi

Department of Mathematics
Annamalai University
Annamalainagar - 608 002, Tamilnadu, India
rearasi@gmail.com

S. Sriram

Mathematics Section
Faculty of Engineering and Technology
Annamalai University
Annamalainagar - 608 002, Tamilnadu, India
sriramcdm@gmail.com

Abstract

In this paper, we introduce the concept of intuitionistic fuzzy ideals of subtraction algebras. We obtain the characterizations of intuitionistic fuzzy ideals in subtraction algebras. Using a collection of ideals with additional conditions, we construct an intuitionistic fuzzy ideal and we obtain some related properties of it.

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1 Introduction

The concept of fuzzy set was introduced by Zadeh in 1965 [8]. The idea of intuitionistic fuzzy set was first introduced by Atanassov [2, 3] as a generalization of the notion of fuzzy set. Schein [7] considered systems of the form $(\Phi, \circ, \backslash)$, where Φ is a set of functions closed under the composition "$\circ$"of functions (and hence $(\Phi, \circ)$ is a function semigroup) and the set theoretic subtraction "$\backslash$"(and hence $(\Phi, \backslash)$ is a subtraction algebra in the sense of [1]). Jun et al.

[4] introduced the notion of ideals in subtraction algebras and discussed characterization of ideals. In [5], Jun and Kim established the ideal generated by a set and discussed related results. Lee and Park [6] introduced the notion of a fuzzy ideal in subtraction algebras.

In this paper, we introduce the concept of intuitionistic fuzzy ideals of subtraction algebras. We obtain the characterizations of intuitionistic fuzzy ideals in subtraction algebras. Using a collection of ideals with additional conditions, we construct an intuitionistic fuzzy ideal and we obtain some related properties of it.

2 Preliminaries

In this section, we shall recall some basic definitions and results that are required in the sequel.

A subtraction algebra is defined as an algebra $(X, -)$ with a single binary operation "–" that satisfies the following identities: for any $x, y, z \in X$,
(S1) $x - (y - x) = x$,
(S2) $x - (x - y) = y - (y - x)$,
(S3) $(x - y) - z = (x - z) - y$.
The last identity permits us to omit parentheses in expressions of the form $(x - y) - z$. The subtraction determines an order relation on $X : a \leq b \Leftrightarrow a - b = 0$, where $0 = a - a$ is an element that does not depend on the choice of $a \in X$. The ordered set $(X, \leq)$ is a semi-Boolean algebra in the sense of [1], that is, it is a meet semilattice with zero 0 in which every interval $[0, a]$ is a Boolean algebra with respect to the induced order. Here $a \wedge b = a - (a - b)$; the complement of an element $b \in [0, a]$ is $a - b$; and if $b, c \in [0, b]$, then

$$\begin{aligned} b \vee c &= (b' \wedge c')' = a - ((a - b) \wedge (a - c)) \\ &= a - ((a - b) - ((a - b) - (a - c))). \end{aligned}$$

In a subtraction algebra X, the following are true: [4]
(P1) $(x - y) - y = x - y$,
(P2) $x - 0 = x$ and $0 - x = 0$,
(P3) $(x - y) - x = 0$,
(P4) $x - (x - y) \leq y$,
(P5) $(x - y) - (y - x) = x - y$,
(P6) $x - (x - (x - y)) = x - y$,
(P7) $(x - y) - (z - y) \leq x - z$,
(P8) $x \leq y$ if and only if $x = y - w$ for some $w \in X$,
(P9) $x \leq y$ implies $x - z \leq y - z$ and $z - y \leq z - x$ for all $z \in X$,
(P10) $x, y \leq z$ implies $x - y = x \wedge (z - y)$.

Definition 2.1 [*4*] A non-empty subset A of a subtraction algebra X is called an ideal of X if it satisfies:

(I1) $a - x \in A$ for all $a \in A$ and $x \in X$,
(I2) for all $a, b \in A$, whenever $a \vee b$ exists in X then $a \vee b \in A$.

Proposition 2.2 [4] *Let X be a subtraction algebra and let $x, y \in X$. If $w \in X$ is an upper bound for x and y, then the element*

$$x \vee y = w - ((w - y) - x)$$

is a least upper bound for x and y.

Definition 2.3 A fuzzy set μ in a non-empty set S, we mean a function $\mu : S \to [0, 1]$, and the complement of μ, denoted by $\overline{\mu}$, is the fuzzy set in S given by $\overline{\mu}(x) = 1 - \mu(x)$ for all $x \in S$. For any $t \in [0, 1]$, and a fuzzy set μ in a non-empty set S, the set $U(\mu, t) = \{x \in S | \mu(x) \geq t\}$ is called an upper t-level cut of μ and the set $L(\mu, t) = \{x \in S | \mu(x) \leq t\}$ is called a lower t-level cut of μ.

An intuitionitic fuzzy set (briefly, IFS) A in a non-empty set S is an object having the form IFS $A = \{(x, \alpha_A(x), \beta_A(x)) | x \in S\}$, where the functions $\alpha_A : S \to [0, 1]$ and $\beta_A : S \to [0, 1]$ denote the degree of membership and degree of non-membership, respectively and $0 \leq \alpha_A(x) + \beta_A(x) \leq 1, x \in S$.

An IFS $A = \{(x, \alpha_A(x), \beta_A(x)) | x \in S\}$ in S can be identified to an order pair (α_A, β_A) in $I^S \times I^S$. For the sake of simplicity, we shall use the symbol IFS $A = (\alpha_A, \beta_A)$ for the IFS $A = \{(x, \alpha_A(x), \beta_A(x)) | x \in S\}$.

Definition 2.4 [6] A fuzzy set μ in a subtraction algebra X is called a fuzzy ideal of X if it satisfies:
$(F1)$ $\mu(x - y) \geq \mu(x)$ for all $x, y \in X$,
$(F2)$ $\exists\, x \vee y \Rightarrow \mu(x \vee y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in X$.

3 Main Results

Throughout this section, X denotes a subtraction algebra unless otherwise specified.

Definition 3.1 An IFS $A = (\alpha_A, \beta_A)$ in X is called an intuitionistic fuzzy ideal of X if it satisfies:
$(IF1)$ $\alpha_A(x - y) \geq \alpha_A(x)$ and $\beta_A(x - y) \leq \beta_A(x)$ for all $x, y \in X$,
$(IF2)$ $\exists\, x \vee y \Rightarrow \alpha_A(x \vee y) \geq \min\{\alpha_A(x), \alpha_A(y)\}$ and
$\beta_A(x \vee y) \leq \max\{\beta_A(x), \beta_A(y)\}$ for all $x, y \in X$.

Example 3.2 Consider a subtraction algebra $X = \{0, a, b\}$ with the following Cayley table:

$-$	0	a	b
0	0	0	0
a	a	0	a
b	b	b	0

Let an IFS $A = (\alpha_A, \beta_A)$ in X defined by $\alpha_A(0) = 0.9, \alpha_A(a) = 0.3$ and $\alpha_A(b) = 0.6\,;\, \beta_A(0) = 0.2, \beta_A(a) = 0.6$ and $\beta_A(b) = 0.4$. It is easy to check that IFS $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X.

Proposition 3.3 *Every intuitionistic fuzzy ideal $A = (\alpha_A, \beta_A)$ of X satisfies the following inequality:*

$(IF3)$ $\alpha_A(0) \geq \alpha_A(x)$ *and* $\beta_A(0) \leq \beta_A(x)$ *for all* $x \in X$.

Proof. If we take $y = x$ in (IF1), then $\alpha_A(0) = \alpha_A(x - x) \geq \alpha_A(x)$ and $\beta_A(0) = \beta_A(x - x) \leq \beta_A(x)$ for all $x \in X$.

Lemma 3.4 *An IFS $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy set in X such that*

$(IF4)$ $\alpha_A(0) \geq \alpha_A(x)$ *and* $\beta_A(0) \leq \beta_A(x)$ *for all* $x \in X$,

$(IF5)$ $\alpha_A(x - z) \geq \min\{\alpha_A((x - y) - z), \alpha_A(y)\}$ *and* $\beta_A(x - z) \leq \max\{\beta_A((x - y) - z), \beta_A(y)\}$ *for all* $x, y, z \in X$.

Then we have the following fact that

$$x \leq a \Rightarrow \alpha_A(x) \geq \alpha_A(a) \text{ and } \beta_A(x) \leq \beta_A(a) \text{ for all } a, x \in X.$$

Proof. Let $a, x \in X$ be such that $x \leq a$. Then

$$\begin{aligned}\alpha_A(x) = \alpha_A(x - 0) &\geq \min\{\alpha_A((x - a) - 0), \alpha_A(a)\} \\ &= \min\{\alpha_A(0), \alpha_A(a)\} = \alpha_A(a) \text{ and} \\ \beta_A(x) = \beta_A(x - 0) &\leq \max\{\beta_A((x - a) - 0), \beta_A(a)\} \\ &= \max\{\beta_A(0), \beta_A(a)\} = \beta_A(a),\end{aligned}$$

by applying (P2), (IF4) and (IF5). This completes the proof.

Theorem 3.5 *An IFS $A = (\alpha_A, \beta_A)$ in X satisfies $(IF4)$ and $(IF5)$ if and only if $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X.*

Proof. Let $A = (\alpha_A, \beta_A)$ be an IFS in X satisfying (IF4) and (IF5) and let $x, y \in X$. Then $x - y \leq x$ by using (P3). By Lemma 3.4, we have $\alpha_A(x - y) \geq \alpha_A(x)$ and $\beta_A(x - y) \leq \beta_A(x)$. Thus (IF1) is valid. Also we have $\alpha_A(x \vee y) \geq \alpha_A(x)$ and $\beta_A(x \vee y) \leq \beta_A(x)$ whenever $x \vee y$ exists in X by using Lemma 3.4 and so $\alpha_A(x \vee y) \geq \min\{\alpha_A(x), \alpha_A(y)\}$ and $\beta_A(x \vee y) \leq \max\{\beta_A(x), \beta_A(y)\}$. Thus (IF2) is valid. Hence A is an intuitionistic fuzzy ideal of X.

Conversely, assume that $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X. By Proposition 3.3, (IF4) is valid. Let $x, y, z \in X$. Putting $x = y$ and $y = x - z$ in (P3) and (P4), we have (P3) $\Rightarrow (x - z) - y \leq x - z$ and (P4) $\Rightarrow y - (y - (x - z)) \leq x - z$. It follows that $(x - z)$ is an upper bound for $(x - z) - y$ and $y - (y - (x - z))$. Then by Proposition 2.2, we have

$$\begin{aligned}&((x - z) - y) \vee (y - (y - (x - z))) \\ &\qquad = (x - z) - (((x - z) - ((x - z) - y)) - (y - (y - (x - z)))) \\ &\qquad = x - z. \qquad \text{[By using (S2) and (P2)]}\end{aligned}$$

Therefore $\alpha_A(x - z) = \alpha_A(((x - z) - y) \vee (y - (y - (x - z))))$

$\geq \min\{\alpha_A((x-z)-y), \alpha_A(y-(y-(x-z)))\}$ [Since A is an intuitionistic fuzzy ideal of X]

and $\beta_A(x-z) = \beta_A(((x-z)-y) \vee (y-(y-(x-z))))$

$\leq \max\{\beta_A((x-z)-y), \beta_A(y-(y-(x-z)))\}$. [Since A is an intuitionistic fuzzy ideal of X]

By using (S3), (P2) and putting $x-z=y$, then $\min\{\alpha_A((x-z)-y), \alpha_A(y-(y-(x-z)))\}$ reduces to $\min\{\alpha_A((x-y)-z)), \alpha_A(y)\}$ and $\max\{\beta_A((x-z)-y), \beta_A(y-(y-(x-z)))\}$ reduces to $\max\{\beta_A((x-y)-z)), \beta_A(y)\}$. Hence $\alpha_A(x-z) \geq \min\{\alpha_A((x-y)-z), \alpha_A(y)\}$ and $\beta_A(x-z) \leq \max\{\beta_A((x-y)-z), \beta_A(y)\}$. Thus (IF5) is valid.

Theorem 3.6 *An IFS $A = (\alpha_A, \beta_A)$ in X is an intuitionistic fuzzy ideal of X if and only if it satisfies*

$(IF6)$ $\alpha_A(x-((x-a)-b)) \geq \min\{\alpha_A(a), \alpha_A(b)\}$ *and*
$\beta_A(x-((x-a)-b)) \leq \max\{\beta_A(a), \beta_A(b)\}$ *for all* $x, a, b \in X$.

Proof. Let $A = (\alpha_A, \beta_A)$ be an IFS in X satisfying (IF6). Then

$\alpha_A(x-y) = \alpha_A((x-y)-(((x-y)-x)-x))$
$\geq \min\{\alpha_A(x), \alpha_A(x)\} = \alpha_A(x)$ and
$\beta_A(x-y) = \beta_A((x-y)-(((x-y)-x)-x))$
$\leq \max\{\beta_A(x), \beta_A(x)\} = \beta_A(x)$

by applying (P2), (P3) and (IF6). Therefore (IF1) is valid.

Now suppose that $x \vee y$ exists for $x, y \in X$. Putting $w = x \vee y$, we get $x \vee y = w-((w-x)-y)$ by Proposition 2.2. Then from (IF6) it follows that $\alpha_A(x \vee y) = \alpha_A(w-((w-x)-y)) \geq \min\{\alpha_A(x), \alpha_A(y)\}$ and $\beta_A(x \vee y) = \beta_A(w-((w-x)-y)) \leq \max\{\beta_A(x), \beta_A(y)\}$ and therefore (IF2) is also valid. Hence $A = (\alpha_A, \beta_A)$ in X is an intuitionistic fuzzy ideal of X.

Conversely, let an IFS $A = (\alpha_A, \beta_A)$ be an intuitionistic fuzzy ideal of X. By Theorem 3.5, $\alpha_A(x-z) \geq \min\{\alpha_A((x-y)-z), \alpha_A(y)\}$ and $\beta_A(x-z) \leq \max\{\beta_A((x-y)-z), \beta_A(y)\}$.

Taking $z = (x-a)-b$ and $y = b$, we have

$\alpha_A(x-((x-a)-b)) \geq \min\{\alpha_A((x-b)-((x-a)-b)), \alpha_A(b)\}$
$= \min\{\alpha_A((x-b)-((x-b)-a)), \alpha_A(b)\}$ [By (S3)] (1)

and

$\beta_A(x-((x-a)-b)) \leq \max\{\beta_A((x-b)-((x-a)-b)), \beta_A(b)\}$
$= \max\{\beta_A((x-b)-((x-b)-a)), \beta_A(b)\}$. [By (S3)] (2)

Putting $x = x-b$ and $y = a$ in (P4), we have $(x-b)-((x-b)-a) \leq a$. Then by Lemma 3.4, we have $\alpha_A((x-b)-((x-b)-a)) \geq \alpha_A(a)$ and $\beta_A((x-b)-((x-b)-a)) \leq \beta_A(a)$. Now (1) and (2) becomes $\alpha_A(x-((x-a)-b)) \geq \min\{\alpha_A(a), \alpha_A(b)\}$ and $\beta_A(x-((x-a)-b)) \leq \max\{\beta_A(a), \beta_A(b)\}$. Hence an IFS $A = (\alpha_A, \beta_A)$ in X satisfies (IF6).

Lemma 3.7 *An IFS $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X if and only if the fuzzy sets α_A and $\overline{\beta}_A$ are fuzzy ideals of X.*

Proof. Let $A = (\alpha_A, \beta_A)$ be an intuitionistic fuzzy ideal of X. Clearly, α_A is a fuzzy ideal of X. For every $x, y \in X$, we have $\overline{\beta}_A(x - y) = 1 - \beta_A(x - y) \geq 1 - \beta_A(x) = \overline{\beta}_A(x)$.
Suppose $x \vee y$ exists in X, we have

$$\begin{aligned}\overline{\beta}_A(x \vee y) &= 1 - \beta_A(x \vee y) \geq 1 - \max\{\beta_A(x), \beta_A(y)\} \\ &= \min\{1 - \beta_A(x), 1 - \beta_A(y)\} \\ &= \min\{\overline{\beta}_A(x), \overline{\beta}_A(y)\}.\end{aligned}$$

Hence $\overline{\beta}_A$ is a fuzzy ideal of X.

Conversely, assume that α_A and $\overline{\beta}_A$ are fuzzy ideals of X. For every $x, y \in X$, we get $\alpha_A(x-y) \geq \alpha_A(x), 1-\beta_A(x-y) = \overline{\beta}_A(x-y) \geq \overline{\beta}_A(x) = 1-\beta_A(x)$, that is $\beta_A(x - y) \leq \beta_A(x)$. Suppose $x \vee y$ exists in X, we get $\alpha_A(x \vee y) \geq \min\{\alpha_A(x), \alpha_A(y)\}$,

$$\begin{aligned}1 - \beta_A(x \vee y) = \overline{\beta}_A(x \vee y) &\geq \min\{\overline{\beta}_A(x), \overline{\beta}_A(y)\} \\ &= \min\{1 - \beta_A(x), 1 - \beta_A(y)\} \\ &= 1 - \max\{\beta_A(x), \beta_A(y)\},\end{aligned}$$

that is, $\beta_A(x \vee y) \leq \max\{\beta_A(x), \beta_A(y)\}$. Hence $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X.

Theorem 3.8 *Let $A = (\alpha_A, \beta_A)$ be an IFS in X. Then $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X if and only if $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\Diamond A = (\overline{\beta}_A, \beta_A)$ are intuitionistic fuzzy ideals of X.*

Proof. If $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X, then $\alpha_A = \overline{\overline{\alpha}_A}$ and $\overline{\beta}_A$ are fuzzy ideals of X from Lemma 3.7, hence $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\Diamond A = (\overline{\beta}_A, \beta_A)$ are intuitionistic fuzzy ideals of X.

Conversely, if $\Box A = (\alpha_A, \overline{\alpha}_A)$ and $\Diamond A = (\overline{\beta}_A, \beta_A)$ are intuitionistic fuzzy ideals of X, then the fuzzy sets α_A and $\overline{\beta}_A$ are fuzzy ideals of X. Hence $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X.

Theorem 3.9 *An IFS $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X if and only if for all $s, t \in [0, 1]$, the non-empty sets $U(\alpha_A, t)$ and $L(\beta_A, s)$ are ideals of X.*

Proof. Let an IFS $A = (\alpha_A, \beta_A)$ is an intuitionistic fuzzy ideal of X and let $s, t \in [0, 1]$ be such that $U(\alpha_A, t)$ and $L(\beta_A, s)$ are non-empty sets of X. For $y \in X$ and $x \in U(\alpha_A, t)$, we have $\alpha_A(x) \geq t$. Then $\alpha_A(x - y) \geq \alpha_A(x) \geq t$ by (IF1). Hence $x - y \in U(\alpha_A, t)$. Next for $y \in X$ and $x \in L(\beta_A, s)$, we have $\beta_A(x) \leq s$. Then $\beta_A(x - y) \leq \beta_A(x) \leq s$ by (IF1). Hence $x - y \in L(\beta_A, s)$.

Second, we assume that $a \vee b$ exists in X for all $a, b \in U(\alpha_A, t)$. By (IF2), we have $\alpha_A(a \vee b) \geq \min\{\alpha_A(a), \alpha_A(b)\} \geq t$, which implies that $a \vee b \in U(\alpha_A, t)$. Next we assume that $a \vee b$ exists in X for all $a, b \in L(\beta_A, s)$. By (IF2), we have $\beta_A(a \vee b) \leq \max\{\beta_A(a), \beta_A(b)\} \leq s$, which implies that $a \vee b \in L(\beta_A, s)$. Hence $U(\alpha_A, t)$ and $L(\beta_A, s)$ are ideals of X.

Conversely, assume that for all $s, t \in [0,1]$, the non-empty sets $U(\alpha_A, t)$ and $L(\beta_A, s)$ are ideals of X. If there exist $y_0 \in X$ and $x_0 \in U(\alpha_A, t_0)$ such that $\alpha_A(x_0 - y_0) < \alpha_A(x_0)$. Taking $t_0 = (\alpha_A(x_0 - y_0) + \alpha_A(x_0))/2$, we have $0 \leq \alpha_A(x_0 - y_0) < t_0 < \alpha_A(x_0) \leq 1$. It follows that $x_0 - y_0 \notin U(\alpha_A, t_0)$, but $x_0 \in U(\alpha_A, t_0)$, that is, $U(\alpha_A, t_0)$ is not an ideal of X. This is a contradiction.

If there exist $y_0 \in X$ and $x_0 \in L(\beta_A, s_0)$ such that $\beta_A(x_0 - y_0) > \beta_A(x_0)$. Taking $s_0 = (\beta_A(x_0 - y_0) + \beta_A(x_0))/2$, we have $0 \leq \beta_A(x_0) < s_0 < \beta_A(x_0 - y_0) \leq 1$. It follows that $x_0 \in L(\beta_A, s_0)$, but $x_0 - y_0 \notin L(\beta_A, s_0)$, which is a contradiction.

Secondly, suppose that there exist $x_0, y_0 \in U(\alpha_A, t_0)$ and for all $x \vee y$ in X such that $\alpha_A(x_0 \vee y_0) < \min\{\alpha_A(x_0), \alpha_A(y_0)\}$. Taking $t_0 = (\alpha_A(x_0 \vee y_0) + \min\{\alpha_A(x_0), \alpha_A(y_0)\})/2$, we have $0 \leq \alpha_A(x_0 \vee y_0) < t_0 < \min\{\alpha_A(x_0), \alpha_A(y_0)\} \leq 1$. Therefore $x_0 \vee y_0 \notin U(\alpha_A, t_0)$, but $x_0, y_0 \in U(\alpha_A, t_0)$. This is a contradiction.

Next if there exist $x_0, y_0 \in L(\beta_A, s_0)$ and for all $x \vee y$ in X such that $\beta_A(x_0 \vee y_0) > \max\{\beta_A(x_0), \beta_A(y_0)\}$. Taking $s_0 = (\beta_A(x_0 \vee y_0) + \max\{\beta_A(x_0), \beta_A(y_0)\})/2$, we have $0 \leq \max\{\beta_A(x_0), \beta_A(y_0)\} < s_0 < \beta_A(x_0 \vee y_0) \leq 1$. Therefore $x_0, y_0 \in L(\beta_A, s_0)$, but $x_0 \vee y_0 \notin L(\beta_A, s_0)$. This is also a contradiction. This completes the proof.

Theorem 3.10 *Let* $\{I_t | t \in \bigwedge\}$ *be a collection of ideals of* X *such that*
(i) $X = \bigcup_{t \in \bigwedge} I_t$,
(ii) $s > t$ *if and only if* $I_s \subset I_t$ *for all* $s, t \in \bigwedge$.
Then an IFS $A = (\alpha_A, \beta_A)$ *in* X *defined by* $\alpha_A(x) = \sup\{t \in \bigwedge | x \in I_t\}$ *and* $\beta_A(x) = \inf\{t \in \bigwedge | x \in I_t\}$ *for all* $x \in X$ *is an intuitionistic fuzzy ideal of* X.

Proof. According to Theorem 3.9, it is sufficient to show that the non-empty sets $U(\alpha_A, t)$ and $L(\beta_A, t)$ are ideals of X for every $t \in [0, \alpha_A(0)]$ and $s \in [\beta_A(0), 1]$. In order to prove that $U(\alpha_A, t)$ is an ideal of X, we divide the proof into the following two cases:
(i) $t = \sup\{q \in \bigwedge | q < t\}$,
(ii)$t \neq \sup\{q \in \bigwedge | q < t\}$.

The case (i) implies that $x \in U(\alpha_A, t) \Leftrightarrow x \in I_q, \forall\, q < t \Leftrightarrow x \in \bigcap_{q<t} I_q$, so that $U(\alpha_A, t) = \bigcap_{q<t} I_q$, which is an ideal of X.

For the case(ii), we claim that $U(\alpha_A, t) = \bigcup_{q \geq t} I_q$. If $x \in \bigcup_{q \geq t} I_q$, then $x \in I_q$ for some $q \geq t$. It follows that $\alpha_A(x) \geq q \geq t$, so that $x \in U(\alpha_A, t)$. This shows that $\bigcup_{q \geq t} I_q \subseteq U(\alpha_A, t)$. Now assume that $x \notin \bigcup_{q \geq t} I_q$. Then $x \notin I_q$ for all $q \geq t$. Since $t \neq \sup\{q \in \bigwedge | q < t\}$, there exists $\epsilon > 0$ such that $(t - \epsilon, t) \cap \bigwedge = \phi$. Hence $x \notin I_q$ for all $q > t - \epsilon$, which means that $x \in I_q$, then $q \leq t - \epsilon$. Thus $\alpha_A(x) \leq t - \epsilon < t$ and so $x \notin U(\alpha_A, t)$. Therefore $U(\alpha_A, t) \subseteq \bigcup_{q \geq t} I_q$ and thus $U(\alpha_A, t) = \bigcup_{q \geq t} I_q$, which is an ideal of X.

Next we prove that $L(\beta_A, t)$ is an ideal of X. We consider the following two cases:

(iii)$s = \inf\{r \in \bigwedge | s < r\}$,
(iv)$s \neq \inf\{r \in \bigwedge | s < r\}$.

For the case(iii), we have $x \in L(\beta_A, s) \Leftrightarrow x \in I_r, \forall\, s < r \Leftrightarrow x \in \bigcap_{s<r} I_r$ and hence $L(\beta_A, s) = \bigcap_{s<r} I_r$, which is an ideal of X.

For the case(iv), there exists $\epsilon > 0$ such that $(s, s+\epsilon) \cap \bigwedge = \phi$. We will show that $L(\beta_A, s) = \bigcup_{s \geq r} I_r$. If $x \in \bigcup_{s \geq r} I_r$, then $x \in I_r$ for some $r \leq s$. It follows that $\beta_A(x) \leq r \leq s$, so that $x \in L(\beta_A, s)$. Hence $\bigcup_{s \geq r} I_r \subseteq L(\beta_A, s)$. Conversely, if $x \notin \bigcup_{s \geq r} I_r$, then $x \notin I_r$ for all $r \leq s$, which implies that $x \notin I_r$ for all $r < s + \epsilon$, that is, if $x \in I_r$, then $r \geq s + \epsilon$. Thus $\beta_A(x) \geq s + \epsilon > s$, that is, $x \notin L(\beta_A, s)$. Therefore $L(\beta_A, s) \subseteq \bigcup_{s \geq r} I_r$ and consequently $L(\beta_A, s) = \bigcup_{s \geq r} I_r$, which is an ideal of X. This completes the proof.

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