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Restrict Edge-Connectivity of Circular Graphs

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Abstract

An edge cut X of G is restrict if $G - X$ is disconnected and has no trivial component. The minimum k such that G has a restrict k-edge cut is called the restrict edge-connectivity of G. Let n and $i_1, i_2, \cdots, i_k$ are positive integers such that $n > i_1 > i_2 > \cdots > i_k$. A circular graph $G = C(n; i_1, i_2, \cdots, i_k)$ of order n is one graph spanned by a n-cycle $C_n = (0, 1, \cdots, n, 1)$ together with the chords $(i, j) \in E$ if and only if $i + i_k \equiv j (\text{mod } n)$. The cycle C_n is the principal cycle of $C(n; i_1, i_2, \cdots, i_k)$. In this paper, we prove that the restrict edge connectivity of $C(n, n/2, k)$ is 8.

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1 Intruduction

Throughout this paper we only consider simple graphs and notions mentioned without specification can be found in Bondy's book[1]. In particular, $\kappa(G)$ and $\kappa'(G)$ represent the connectivity and edge-connectivity of a graph G. An edge cut X of G is *restrict* if $G - X$ is disconnected and has no trivial component. For an integer $k > 0$, a graph G is *restrict* k*−edge-connected* if G does not have an restrict edge cut Y with $|Y| < k$. We call the minimum k such that G has a restrict k-edge cut the *restrict edge-connectivity* of G, denote by $\lambda'(G)$. We call a minimum restrict edge cut of G a *restrict set*. It can be easily seen that if H is a restrict set of G, then $\omega(G - H) = 2$. And we will call the component of $G - H$ with less vertex number the *restrict subgraph* of H.

Let n and $i_1, i_2, \cdots, i_k$ are positive integers such that $n > i_1 > i_2 > \cdots > i_k$. A circular graph $G = C(n; i_1, i_2, \cdots, i_k)$ of order n is one graph spanned by a n-cycle $C_n = (0, 1, \cdots, n, 1)$ together with the chords $(i, j) \in E$ if and only if $i + i_k \equiv j(modn)$. The cycle C_n is the principal cycle of $C(n; i_1, i_2, \cdots, i_k)$. It is clear that every nontrivia1 circular graph is a 3-connected, of course 3-edge-connected nonplanar graph. Many characteristics of circular graphs have been studied, for example, the Hamiltonian, connectivity, edge coloring and so on(See reference [2], [3]). The discussion in this paper is restrict edge-connectivity of $C(n; n/2, k, 1)$. The following is a known result that will be used in our proof.

Lemma 1.1 *[4]* $\kappa(C(n; n/2, k, 1)) = 5$.

2 Main result

For the sake of brevity, we have $C(n; n/2, k, 1)$ simplified as Cr.

Theorem 2.1 $\lambda^{'}(C(n; n/2, k, 1)) = 8$.

Proof: Firstly we will proof $\lambda^{'}(Cr) \leq 8$. Let $H = \{(0, n/2), (0, n-k), (0; n-1), (0, k), (1, n/2+1), (1, n-k+1), (1, 2), (1, 1+k)\}$, then we know $Cr - H$ is disconnected, and the two components of $Cr - H$ are respectively $G_1 = Cr[\{0, 1\}]$ and $G_2 = Cr[\{2, 3, \cdots n-1\}]$. So H is an restrict edge cut of Cr. Thus

$$\lambda^{'}(Cr) \leq 8. \tag{1}$$

On the other hand, we will proof $\lambda^{'}(Cr) \geq 8$. Let H_m is an restrict set of Cr with m elements, and G^* is the restrict subgraph of H_m.

Case 1: The elements of $V(G^*)$ is an arithmetic series with tolerance 1(in the sense modulo n). Without loss of generality, we will suppose that $V(G^*) = \{1, 2, \cdots, m\}$. By Lemma 1.1 and the definition, we know $C(n; n/2, k, 1) = 5$ and each vertex is incident with five edges.

Subcase 1: $m \leq k$. Then the restrict edge cut

$H_m = \{(0, 1), (m, m+1), (1, 1-k), \cdots, (m, m-k), (1, 1+k), \cdots, (m, m+k), (1, 1+n/2), \cdots, (m, m+n/2)\}$.

So

$$|H_m| = |H_2| + 3(m-2).$$

Thus $H_m \geq 8$.

Subcase 2: $m > k$. Then the restrict edge cut

$H_m = \{(0,1),(m,m+1),(1,1-k),\cdots,(k,k-k),(m-k+1,m+1),\cdots,(m,m+k),(1,1+n/2),\cdots,(m,m+n/2)\}$.

So

$$|H_m| = m + 2k + 2 \geq 3k + 3.$$

Because $k \geq 2$, we get $H_m \geq 8$.

Case 2: $V(G^*) = V_1 \bigcup V_2$, the elements of V_1, V_2 respectively are an arithmetic series with tolerance 1, but $V(G^*)$ is not(mode n).

Let $|V_1| = m_1, |V_2| = m_2$, then we can easily know $|N(V_1)| \geq m_1 + 4, |N(H_2)| \geq m_2 + 4$. So $\sum_{v \in N(V(G^*))} d(v) \geq 8$, thus $H_m \geq 8$.

Case 3: $V(G^* = V_1 \bigcup V_2 \bigcup V_3$, the elements of V_1, V_2, V_3 respectively are an arithmetic series with tolerance 1, but $V(G^*)$ and $V_i \bigcup V_j$ are not(mode n).

Let $|V_i| = m_i, i = 1,2,3$, then we can easily know $N(V_i) \geq m_i + 4$. So $\sum_{v \in N(H_m)} d(v) \geq 8$, thus $H_m \geq 8$.

Case 4: $V(G^*) = V_1 \bigcup V_2 \bigcup \cdots \bigcup V_t, t > 3$, the elements of $V_i, i = 1, \cdots t$ respectively are an arithmetic series with tolerance 1, but $V(G^*)$ or $V_i \bigcup V_j$ is not(mode n).

Let $V_1 = (i_1, i_1+1, \cdots, i_1+m_1), V_2 = (i_2, i_2+1, \cdots, i_2+m_2), \cdots, V_t = (i_t, i_t+1, \cdots, i_t+m_t)$, then $(i_j, i_j-1), (i_j+m_j, i_j+m_j+1) \in H_m, j = 1,2,\cdots,t$. Because $t > 3$, so $H_m \geq 8$.

In any case, we obtain:

$$\lambda^{'}(Cr) \geq 8. \tag{2}$$

Combine (1) and (2), we have $\lambda^{'}(Cr) = 8$. ■

References

[1] J. A. Bondy, U. S. R. Murty, Graph Theory with Applications. American Elsevier, New York, 1976.

[2] Esfahanina A HHakimi S L, On computing a edge-connectivity of a graph, Information Processing Letters, 27(1988): 195-199.

[3] Esfahanian A H, Generalized measures offault tolerance with application to n-cube networks, Trans. Comput, 198938(11): 1586-1591.

[4] Zhai Xiaohon, The connectivity of circular graph $C_n < 1, k, n/2 >$, Journal of Kunming Science and Technology, 24(1999): 83-87.

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