BiHom-Poisson Algebra and Its Application

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Abstract
In this paper, we introduce BiHom-Poisson algebra and give some ways to construct a BiHom-Poisson algebra.

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1 Introduction

A Poisson algebra \((A, \{-, -\}, \mu)\) consists of a commutative associative \((A, \mu)\) together with a Lie algebra structure \(\{-, -\}\), satisfying the Leibniz identity:

\[\{\mu(x, y), z\} = \mu(\{x, z\}, y) + \mu(x, \{y, z\})\]

Poisson algebras are used in many fields in mathematics and physics. In mathematics, Poisson algebras play a fundamental role in Poisson geometry [4], quantum groups [9, 10] and deformation of commutative associative algebras [6]. In physics, Poisson algebras are a major part of deformation quantization[7], Hamiltonian mechanics[1] and topological field theories[2].

Algebras of Hom-type appeared in the physics literature of 1990’s, in the context of quantum deformations of some algebras of vector fields. A generalization has been given in [8], where a construction of Hom-category including a group action led to concepts of BiHom-type algebras. Hence, BiHom-associative algebras and BiHom-Lie algebras, involving two linear maps(called
structure maps), were introduced. The main axioms for these types of algebras (BiHom-associativity, BiHom-skew-symmetry and BiHom-Jacobi condition) were dictated by categorical considerations.

The purpose of this paper is to study a twisted generalization of Poisson algebras, called BiHom-Poisson algebras. In a BiHom-Poisson algebra $A$, there are two linear self-maps $\alpha, \beta$ and two binary operations $\{-,-\}$ and $\mu$. In particular, $(A, \mu, \alpha, \beta)$ is a BiHom-associative algebra and $(A, \{-,-\}, \alpha, \beta)$ is a BiHom-Lie algebra. If $\alpha = \beta$, then BiHom-Poisson algebra reduces to a Hom-Poisson algebras. If both of twisting maps are the identity maps, then a BiHom-Poisson algebra reduces to a Poisson algebra.

The paper is organized as follows. In section 2, we propose a definition for the BiHom-Poisson algebra and prove the tensor product of BiHom-Poisson algebra is a BiHom-Poisson algebra. In section 3, we give two ways to construct the BiHom-Poisson algebra.

**Definition 1.1.** [3] Let $\mathbb{K}$ be a field. A BiHom-associative algebra over $\mathbb{K}$ is a 4-tuple $(A, \mu, \alpha, \beta)$, where $A$ is a $\mathbb{K}$-linear space, $\alpha : A \to A$, $\beta : A \to A$ and $\mu : A \otimes A \to A$ are linear maps, with notation $\mu(a_1 \otimes a_2) = a_1 a_2$, satisfying the following conditions, for all $a_1, a_2, a_3 \in A$:

1. $\alpha \circ \beta = \beta \circ \alpha$,
2. $\alpha(a_1 a_2) = \alpha(a_1) \alpha(a_2)$ and $\beta(a_1 a_2) = \beta(a_1) \beta(a_2)$,
3. (BiHom-associativity) $\alpha(a_1)(a_2 a_3) = (a_1 a_2) \beta(a_3)$.

**Definition 1.2.** [3] A BiHom-Lie algebra over a field $\mathbb{K}$ is a 4-tuple $(A, [-,-], \alpha, \beta)$, where $A$ is a $\mathbb{K}$-linear space, $\alpha : A \to A$, $\beta : A \to A$ and $[-, -] : A \otimes A \to L$ are linear maps, with notation $[-, -](a_1 \otimes a_2) = [a_1, a_2]$, satisfying the following conditions, for all $a_1, a_2, a_3 \in L$:

1. $\alpha \circ \beta = \beta \circ \alpha$,
2. $\alpha[a_1, a_2] = [\alpha(a_1), \alpha(a_2)]$ and $\beta[a_1, a_2] = [\beta(a_1), \beta(a_2)]$,
3. (skew-symmetry) $[\beta(a_1), \alpha(a_2)] = -[\beta(a_2), \alpha(a_1)]$,
4. (BiHom-Jacobi condition) $[\beta^2(a_1), [\beta(a_2), \alpha(a_3)]] + [\beta^2(a_2), [\beta(a_3), \alpha(a_1)]] + [\beta^2(a_3), [\beta(a_1), \alpha(a_2)]] = 0$. 

Definition 1.3. [8] Let $A$ be a linear space and $\mu : A \otimes A \to A$, $\mu(a_1 \otimes a_2) = a_1 a_2$, for all $a_1, a_2 \in A$, is a linear multiplication on $A$. A Rota-Baxter operator of weight zero for $(A, \mu)$ is a linear map $R : A \to A$ satisfying the so-called Rota-Baxter condition

$$R(a_1)R(a_2) = R(R(a_1)a_2 + a_1R(a_2)), \forall a_1, a_2 \in A. \quad (1.1)$$

In this case, if we define a new multiplication by $a_1 * a_2 = a_1 R(a_2) + R(a_1)a_2$, for all $a_1, a_2 \in A$ on $A$, then $R(a_1 * a_2) = R(a_1)R(a_2)$, for all $a_1, a_2 \in A$ and $R$ is a Rota-Baxter operator for $(A, \ast)$. If $(A, \mu)$ is associative, then $(A, \ast)$ is also associative.

The notion of a Baxter operator can be defined for algebras over any bi-nary operated, in the obvious manner. For instance, for the associative and commutative operated, Baxter operators are defined by condition (1.1), while for the Lie operated they are defined by

$$\{R(a_1), R(a_2)\} = R(\{R(a_1), a_2\} + \{a_1, R(a_2)\}).$$

Lemma 1.4. [5] We consider a 4-tuple $(L, [-, -], \alpha, \beta)$, where $L$ is a linear space, $\alpha, \beta : L \to L$ are linear maps and $[-, -] : L \times L \to L$ is a bilinear map. Let $R : L \to L$ be a linear map such that

$$R \circ \alpha = \alpha \circ R \text{ and } R \circ \beta = \beta \circ R.$$

Define a new multiplication on $L$ by

$$\{x, y\} = [R(x), y] + [x, R(y)], \forall x, y \in L.$$

Then:

1. If $\alpha$ and $\beta$ satisfy

$$\alpha([x, y]) = [\alpha(x), \alpha(y)] \text{ and } \beta([x, y]) = [\beta(x), \beta(y)], \forall x, y \in L,$$

then they also satisfy

$$\alpha(\{x, y\}) = \{\alpha(x), \alpha(y)\} \text{ and } \beta(\{x, y\}) = \{\beta(x), \beta(y)\}, \forall x, y \in L;$$

2. If $\alpha$ and $\beta$ satisfy

$$[\beta(x), \alpha(y)] = -[\beta(y), \alpha(x)], \forall x, y \in L,$$

then they also satisfy
\[\{\beta(x), \alpha(y)\} = -\{\beta(y), \alpha(x)\}, \forall x, y \in L;\]

3. If \(R\) satisfies
\[
[R(x), R(y)] = R([R(x), y] + [x, R(y)]), \forall x, y \in L,
\]
then
\[R(\{x, y\}) = [R(x), R(y)], \forall x, y \in L.\]

2 The Tensor Product Of BiHom-Poisson Algebras

**Definition 2.1.** A BiHom-Poisson algebra over a field \(\mathbb{K}\) is a 5-tuple \((A, \mu, \{-,-\}, \alpha, \beta)\), where \(A\) is a \(\mathbb{K}\)-linear space, \(\alpha : A \rightarrow A\), \(\beta : A \rightarrow A\), \(\mu : A \otimes A \rightarrow A\) and \(\{-,-\} : A \otimes A \rightarrow A\) are linear maps, with notation \(\mu(a_1 \otimes a_2) = a_1 a_2\), \(\{a_1 \otimes a_2\} = \{a_1, a_2\}\), satisfying the following conditions.

1. \((A, \mu, \alpha, \beta)\) is a BiHom-associative algebra,
2. \((A, \{-,-\}, \alpha, \beta)\) is a BiHom-Lie algebra,
3. (BiHom Leibniz Identity) \(\{\alpha \beta(a_1), a_2 a_3\} = \{\beta(a_1), a_2\} \beta(a_3) + \beta(a_2) \{\alpha(a_1), a_3\}\), for \(a_1, a_2, a_3 \in A\).

**Lemma 2.2.** If \((A, \mu, \{-,-\}, \alpha, \beta)\) is a BiHom-Poisson algebra. Then
\[
\{\beta(a_1), \beta(a_2) \alpha(a_3)\} = \beta^2(a_2)\{\beta(a_1), \alpha a_3\} + \beta^2(a_3)\{\beta(a_1), \alpha(a_2)\}, \tag{2.1}
\]
\[
\beta^2(a_3)\{\beta(a_1), \alpha(a_2)\} = \beta^2(a_2)\{\beta(a_3), \alpha(a_1)\} = \beta^2(a_1)\{\beta(a_2), \alpha(a_3)\}, \tag{2.2}
\]
\[
\alpha \beta(a_1)(a_2 a_3) = \beta(a_2)(\alpha(a_1)a_3) = (\beta(a_1)a_2)\beta(a_3). \tag{2.3}
\]

**Lemma 2.3.** Let \((A, \cdot, \{-,-\}_1, \alpha_1, \beta_1), (B, *, \{-,-\}_2, \alpha_2, \beta_2)\) are BiHom-Poisson algebras. \(\alpha, \beta : A \otimes B \rightarrow A \otimes B\) and \(\{-,-\} : (A \otimes B) \otimes (A \otimes B) \rightarrow A \otimes B\) are linear maps such that the following conditions hold (for all \(a_1, a_2 \in A, b_1, b_2 \in B\)):
\[
\alpha = \alpha_1 \otimes \alpha_2, \quad \beta = \beta_1 \otimes \beta_2,
\]
\[
\{a_1 \otimes b_1, a_2 \otimes b_2\} = \{a_1, a_2\}_A \otimes b_1 \ast b_2 + (a_1 \cdot a_2) \otimes \{b_1, b_2\}_B,
\]
then \((A \otimes B, *, \{-,-\}, \alpha, \beta)\) is a BiHom-Lie algebra.

**Proof.** It is obvious that
So we just need prove the following conditions:

\[
\{\beta(a_1 \otimes b_1), \alpha(a_2 \otimes b_2)\} = -\{\beta(a_2 \otimes b_2), \alpha(a_1 \otimes b_1)\},
\]

\[
\{\beta^2(a_1 \otimes b_1), \{\beta(a_2 \otimes b_2), \alpha(a_3 \otimes b_3)\}\} + \{\beta^2(a_2 \otimes b_2), \{\beta(a_3 \otimes b_3), \alpha(a_1 \otimes b_1)\}\} + \{\beta^2(a_3 \otimes b_3), \{\beta(a_1 \otimes b_1), \alpha(a_2 \otimes b_2)\}\} = 0.
\]

Now, we compute, for \(a_1, a_2 \in A, b_1, b_2 \in B\):

\[
\{\beta(a_1 \otimes b_1), \alpha(a_2 \otimes b_2)\}
\]

\[
= \{\beta_1(a_1) \otimes \beta_2(b_1), \alpha_1(a_2) \otimes \alpha_2(b_2)\}
\]

\[
= \{\beta_1(a_1), \alpha_1(a_2)\}_1 \otimes \beta_2(b_1) \ast \alpha_2(b_2) + \beta_1(a_1) \cdot \alpha_1(a_2) \otimes \{\beta_2(b_1), \alpha_2(b_2)\}_2
\]

\[
= -\{\beta_1(a_2), \alpha_1(a_1)\}_1 \otimes \beta_2(b_2) \ast \alpha_2(b_1) - \beta_1(a_2) \cdot \alpha_1(a_1) \otimes \{\beta_2(b_2), \alpha_2(b_1)\}_2
\]

\[
= -\{\beta(a_2 \otimes b_2), \alpha(a_1 \otimes b_1)\}
\]

\[
= -\{\beta(a_2 \otimes b_2), \alpha(a_1 \otimes b_1)\}.
\]

We denote by summation over the cyclic permutations of two category elements \(a_1, a_2, a_3\) and \(b_1, b_2, b_3\). For example,

\[
\bigcirc_{a_1, a_2, a_3}^{b_1, b_2, b_3} (a_1 + b_3)(a_2 + b_2)(a_3 + b_1)
\]

\[= (a_1 + b_3)(a_2 + b_2)(a_3 + b_1) + (a_2 + b_1)(a_3 + b_3)(a_1 + b_2) + (a_3 + b_2)(a_1 + b_1)(a_2 + b_3)
\]

We compute, for \(a_1, a_2, a_3 \in A, b_1, b_2, b_3 \in B\):

\[
\bigcirc_{a_1, a_2, a_3}^{b_1, b_2, b_3} \{\beta^2(a_1 \otimes b_1), \{\beta(a_2 \otimes b_2), \alpha(a_3 \otimes b_3)\}\}
\]

\[
= \bigcirc_{a_1, a_2, a_3}^{b_1, b_2, b_3} \{\beta^2_1(a_1), \{\beta_1(a_2), \alpha_1(a_3)\}_1 \otimes \beta_2^2(b_1) \ast \alpha_2(b_3)\}
\]

\[
= \bigcirc_{a_1, a_2, a_3}^{b_1, b_2, b_3} \{\beta^2_1(a_1), \{\beta_1(a_2), \alpha_1(a_3)\}_1 \otimes \beta_2^2(b_1) \ast \alpha_2(b_3)\}
\]

\[
\overset{(2.2)}{=} \bigcirc_{a_1, a_2, a_3}^{b_1, b_2, b_3} \{\beta^2_1(a_1), \{\beta_1(a_2), \alpha_1(a_3)\}_1 \otimes \beta_2^2(b_1) \ast \alpha_2(b_3)\}
\]

\[
\overset{(2.1)}{=} 0
\]
Theorem 2.4. Let \((A,*, \{-,-\}, \alpha, \beta), (B,*, \{-,-\}, \alpha, \beta)\) are BiHom-Poisson algebras. \(\alpha, \beta : A \otimes B \to A \otimes B\) and \(\{-,-\} : (A \otimes B) \otimes (A \otimes B) \to A \otimes B\) are linear maps such that the following conditions hold (for all \(a_1, a_2 \in A, b_1, b_2 \in B\)):

\[
\alpha = \alpha_1 \otimes \alpha_2, \quad \beta = \beta_1 \otimes \beta_2,
\]

\[
(a_1 \otimes b_1) * (a_2 \otimes b_2) = (a_1 : a_2) \otimes (b_1 * b_2),
\]

\[
\{a_1 \otimes b_1, a_2 \otimes b_2\} = \{a_1, a_2\}_A \otimes b_1 * b_2 + (a_1 \cdot a_2) \otimes \{b_1, b_2\}_B.
\]

Then \((A \otimes B, *, \{-,-\}, \alpha, \beta)\) is a BiHom-Poisson algebra.

Proof. It was proved in [8] that \((A \otimes B, *, \alpha, \beta)\) is a BiHom-association algebra, and by lemma 2.2 \((A \otimes B, \{-,-\}, \alpha, \beta)\) is a BiHom-Lie algebra. So we just need prove BiHom-Leibniz identity hold.

\[
\{\beta(a_1 \otimes b_1), a_2 \otimes b_2\} * \beta(a_3 \otimes b_3) + \beta(a_2 \otimes b_2) * \{\alpha(a_1 \otimes b_1), a_3 \otimes b_3\} = \{\alpha\beta(a_1 \otimes b_1), (a_2 \otimes b_2) * (a_3 \otimes b_3)\}
\]

We compute, for \(a_1, a_2, a_3 \in A, b_1, b_2, b_3 \in B\):

\[
\{\beta(a_1 \otimes b_1), a_2 \otimes b_2\} * \beta(a_3 \otimes b_3) + \beta(a_2 \otimes b_2) * \{\alpha(a_1 \otimes b_1), a_3 \otimes b_3\}
\]

\[
= \{(\beta(a_1), a_2)_1 \cdot \beta_1(a_3) \} \otimes ((\beta_2(b_1) * b_2) * \beta_2(b_3)) + \beta_1(a_2) \cdot \{(a_1(a_1), a_3) \otimes (\beta_2(b_2) * (a_2(b_1) * b_3))
\]

\[
+ \beta_1(a_2) \cdot (a_1(a_1) \cdot a_3) \otimes (\beta_2(b_2) * \{a_2(b_1), b_3\}_2)
\]

\[
+ (\beta_1(a_1) \cdot a_2) \cdot \beta_1(a_3) \otimes (\{\beta_2(b_1), b_2\}_2 * \beta_2(b_3))
\]

\[
(2.3) = \{\beta_1(a_1), a_2\}_1 \cdot \beta_1(a_3) \otimes (\alpha_2 \beta_2(b_1) * (b_2 * b_3)))
\]

\[
+ \beta_1(a_2) \cdot \{(a_1(a_1), a_3) \otimes (a_2 \beta_2(b_1) * (b_2 * b_3))
\]

\[
+ \alpha_1 \beta_1(a_1) \cdot (a_2 \cdot a_3) \otimes (\beta_2(b_2) * \{a_2(b_1), b_3\}_2)
\]

\[
+ \alpha_1 \beta_1(a_1) \cdot (a_2 \cdot a_3) \otimes (\{\beta_2(b_1), b_2\}_2 * \beta_2(b_3))
\]

\[
= \{\alpha_1 \beta_1(a_1), a_2 \cdot a_3\}_1 \otimes (\alpha_2 \beta_2(b_1) * (b_2 * b_3))
\]

\[
+ \alpha_1 \beta_1(a_1) \cdot (a_2 \cdot a_3) \otimes \{a_2 \beta_2(b_1), b_2 b_3\}_2
\]

\[
= \{\alpha_1 \beta_1(a_1) \otimes \alpha_2 \beta_2(b_1), a_2 \cdot a_3 \otimes b_2 * b_3\}
\]

\[
= \{\alpha \beta(a_1 \otimes b_1), (a_2 \otimes b_2) * (a_3 \otimes b_3)\}.
\]

\[\square\]

3 Constructions Of BiHom-Poisson Algebra

Theorem 3.1. Let \((A, \mu, \{-,-\}, \alpha, \beta)\) be a BiHom-Poisson algebra.
1. \((A, \{-, -\}^n = \alpha^n \{-, -\}, \mu^n = \alpha^n \mu, \alpha^{n+1}, \alpha^n \beta)\) is a BiHom-Poisson algebra.

2. Define two new linear maps \(*\) and \((-,-)\) with

\[
x * y = R(x)y + xR(y), \quad \langle x, y \rangle = \{R(x), y\} + \{x, R(y)\}.
\]

Then \((A, *, (-,-), \alpha, \beta)\) is a BiHom-Poisson algebra.

**Proof.** We just prove (2) and leave (1) to the reader. It is easy to prove that \((A, *, \alpha, \beta)\) is a BiHom-associative algebra. And by Lemma 1.4, we get

\[
\alpha \langle a_1, a_2 \rangle = \langle \alpha (a_1), \alpha (a_2) \rangle,
\]

\[
\beta \langle a_1, a_2 \rangle = \langle \beta (a_1), \beta (a_2) \rangle,
\]

\[
\langle \beta (x), \alpha (y) \rangle = -\langle \beta (y), \alpha (x) \rangle.
\]

So we just need prove following conditions hold:

\[
\langle R\beta^2(a_1), \langle \beta (a_2), \alpha (a_3) \rangle \rangle + \langle R\beta^2(a_2), \langle \beta (a_3), \alpha (a_1) \rangle \rangle + \langle R\beta^2(a_3), \langle \beta (a_1), \alpha (y) \rangle \rangle = 0;
\]

\[
\langle \alpha \beta(a_1), a_2 * a_3 \rangle = \langle \beta (a_1), a_2 \rangle * \beta (a_3) + \beta (a_2) * \langle \alpha (a_1), a_3 \rangle.
\]

We compute, for \(a_1, a_2, a_3 \in A:\)

\[
\langle R\beta^2(a_1), \langle \beta (a_2), \alpha (a_3) \rangle \rangle
\]

\[
= \{R\beta^2(a_1), \{R\beta(a_2), \alpha (a_3)\}\} + \{\beta (a_2), R\alpha (a_3)\}
\]

\[
+ \{\beta^2(a_1), R\{R\beta(a_2), \alpha (a_3)\}\} + R\{\beta (a_2), R\alpha (a_3)\}
\]

\[
= \{R\beta^2(a_1), \{R\beta(a_2), \alpha (a_3)\}\} + \{R\beta^2(a_1), \{\beta (a_2), R\alpha (a_3)\}\}
\]

\[
+ \{\beta^2(a_1), \{R\beta(a_2), R\alpha (a_3)\}\}.
\]

Similarly,

\[
\langle R\beta^2(a_2), \langle \beta (a_3), \alpha (a_1) \rangle \rangle
\]

\[
= \{R\beta^2(a_2), \{R\beta(a_3), \alpha (a_1)\}\} + \{R\beta^2(a_2), \{\beta (a_3), R\alpha (a_1)\}\} + \{\beta^2(a_2), \{R\beta(a_3), R\alpha (a_1)\}\},
\]

\[
\langle R\beta^2(a_3), \langle \beta (a_1), \alpha (y) \rangle \rangle
\]

\[
= \{R\beta^2(a_3), \{R\beta(a_1), \alpha (a_2)\}\} + \{R\beta^2(a_3), \{\beta (a_1), R\alpha (a_2)\}\} + \{\beta^2(a_3), \{R\beta(a_1), R\alpha (a_2)\}\}.
\]
Thus, we get
\[ \langle [R\beta^2(a_1), \{\beta(a_2), \alpha(a_3)\}] \rangle \]
\[ = \langle [R\beta^2(a_1), \{R\beta(a_2), \alpha(a_3)\}] \rangle + \langle [R\beta^2(a_1), \{\beta(a_2), R\alpha(a_3)\}] \rangle + \langle [\beta^2(a_1), [R\beta(a_2), R\alpha(a_3)]] \rangle \]
\[ = \langle [R\beta^2(a_1), \{R\beta(a_2), \alpha(a_3)\}] \rangle + \langle [R\beta^2(a_1), \{\beta(a_2), R\alpha(a_3)\}] \rangle - \langle \beta^2(a_2), \{R\beta(a_3), R\alpha(a_1)\} \rangle - \langle \beta^2(a_2), \{\beta(a_1), \alpha R(a_2)\} \rangle \]
\[ = 0. \]

Next, we compute
\[ \langle \alpha \beta(a_1), a_2 * a_3 \rangle \]
\[ = \langle \alpha \beta(a_1), R(a_2)a_3 + a_2 R(a_3) \rangle \]
\[ = \{R(\alpha \beta(a_1)), R(a_2)a_3 + a_2 R(a_3)\} + \{\alpha \beta(a_1), R(R(a_2)a_3 + a_2 R(a_3))\} \]
\[ = \{R(\alpha \beta(a_1)), R(a_2)a_3 + a_2 R(a_3)\} + \{\alpha \beta(a_1), R(a_2)R(a_3)\} \]
\[ = \{R\beta(a_1), R(a_2)\} \beta(a_3) + R\beta(a_2)\{R\alpha(a_1), a_3\} + \{R\beta(a_1), a_2\} R\beta(a_3) \]
\[ + \beta(a_2)\{R\alpha(a_1), R(a_3)\} + \{\beta(a_1), R(a_2)\} \beta(a_3) + R\beta(a_2)\{\alpha(a_1), R(a_3)\} \]
\[ = \langle \beta(a_1), a_2 * \beta(a_3) + \beta(a_2) * \langle \alpha(a_1), a_3 \rangle \rangle \]
\[ = \langle \{R\beta(a_1), a_2\} + \{\beta(a_1), R(a_2)\} \rangle \beta(a_3) + \beta(a_2) \{\{R\alpha(a_1), a_3\} + \{\alpha(a_1), R(a_3)\}\} + \langle \{R\beta(a_1), a_2\} + \{\beta(a_1), R(a_2)\} \rangle \beta(a_3) \]
\[ + R\beta(a_2)\{\{R\alpha(a_1), a_3\} + \{\alpha(a_1), R(a_3)\}\} + \beta(a_2) \{\{R\alpha(a_1), a_3\} + \{\alpha(a_1), R(a_3)\}\} \]
\[ = \langle R\beta(a_1), R(a_2) \rangle \beta(a_3) + \{\{R\beta(a_1), a_2\} + \{\beta(a_1), R(a_2)\} \rangle \beta(a_3) \]
\[ + R\beta(a_2)\{\{R\alpha(a_1), a_3\} + \{\alpha(a_1), R(a_3)\}\} + \beta(a_2) \{\{R\alpha(a_1), a_3\} + \{\alpha(a_1), R(a_3)\}\} \]

Thus, (3.2) can be obtained. \( \square \)

References

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