H-Transversals in H-Groups

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Abstract

In this paper, we have defined the concept of H-subgroup and H-transversal in an H-group and then we have shown that there is a canonical H-group structure on $\tilde{p}(G)$ with respect to which the inclusion $\tilde{p}(G) \to G$ is an H-subgroup of an H-group $(G, \mu)$ where map $\tilde{p}$ be an H-transversal.

Mathematics Subject Classification: 20J99

Keywords: H-transversal; H-subgroup; H-group

1 Introduction

Let $G$ be a group with identity $e$. A map $\tilde{p} : G \to G$ satisfying the following properties (i) $\tilde{p}(e) = e$ (ii) $\tilde{p}^2 = \tilde{p}$ (iii) $\tilde{p}(g_1 g_2) = \tilde{p}(\tilde{p}(g_1) g_2)$ is called a $\tilde{p}$-map. Let $H$ be a subgroup of $G$ and $S$ be a right transversal to $H$ in $G$. Then $G = HS$. Thus each element $g$ of $G$ can be uniquely written as
where \( h \in H \) and \( x \in S \). Suppose \( x, y \in S \) and \( h \in H \). Then a map \( \tilde{p} : G \to G \) defined by \( \tilde{p}(g) = x \) is a \( \tilde{p} \)-map. For a \( \tilde{p} \)-map on \( G \), the subset \( H = \{ g : \tilde{p}(g) = e \} \) of \( G \) is a subgroup of \( G \) and the subset \( S = \{ \tilde{p}(g) : g \in G \} \) is a right transversal (with identity) of \( H \) in \( G \) \cite{5}. Ramji Lal \cite{4} in his paper 'Transversals in Groups' have studied transversals in much more detail. Ungar and Foguel \cite{3} has also given a way of decomposition of a group through an involution of a group into a twisted subgroup and a subgroup.

In this paper, using homotopy theory and taking \( \tilde{p} \) to be continuous, we have defined the concept of \( H \)-subgroup and \( H \)-transversal. We have shown that there is a canonical \( H \)-group structure on \( \tilde{p}(G) \) with respect to which the inclusion \( \tilde{p}(G) \hookrightarrow G \) is an \( H \)-subgroup of an \( H \)-group \( (G, \mu) \) where map \( \tilde{p} \) be an \( H \)-transversal.

Note: Throughout the paper \( \approx \) represents homotopy between two maps.

\section{H-Space}

In the present section, we have defined topological group, \( H \)-space, \( H \)-group, \( H \)-map, \( H \)-subgroup etc \cite{1,2}.

\textbf{Definition 2.1.} A \textbf{topological group} \( G \) is a group that is also a topological space, satisfying the requirements that the map of \( G \times G \) into \( G \) sending \( x \times y \) into \( x.y \), and the map of \( G \) into \( G \) sending \( x \) into \( x^{-1} \), are continuous.

\textbf{Definition 2.2.} A nonempty topological space with a base point is called a pointed topological space.

\textbf{Definition 2.3.} A pointed topological space \( G \) with base point \( e_0 \) together with a continuous multiplication \( \mu : G \times G \to G \) for which the unique constant map \( c : G \to G \) defined by \( c(x) = e_0 \), is a homotopy identity, that is, each composite \( G \overset{(c \times 1)}{\to} G \times G \overset{\mu}{\to} G \) and \( G \overset{(1 \times c)}{\to} G \times G \overset{\mu}{\to} G \) is homotopic to identity map \((1_G : G \to G)\), is called an \textbf{H-space}.

\textbf{Definition 2.4.} Let \( G \) be an \( H \)-space. The continuous multiplication \( \mu : G \times G \to G \) is said to be \textbf{homotopy associative} if the following diagram

\[
\begin{array}{ccc}
G \times G \times G & \overset{(\mu \times 1)}{\to} & G \times G \\
(1 \times \mu) \downarrow & & \downarrow \mu \\
G \times G & \overset{\mu}{\to} & G
\end{array}
\]

is homotopy commutative i.e \( \mu \circ (\mu \times 1) \approx \mu \circ (1 \times \mu) \).
Definition 2.5. Let $G$ be an $H$-space. A continuous function $\phi : G \to G$ is called a homotopy inverse for $G$ and $\mu$ if each of the composites $G \xrightarrow{(\phi \times 1)} G \times G \xrightarrow{\mu} G$ and $G \xrightarrow{(1 \times \phi)} G \times G \xrightarrow{\mu} G$ is homotopic to homotopy identity $c : G \to G$.

Definition 2.6. A homotopy associative $H$-space with a homotopy inverse satisfies the group axioms upto homotopy. Such a pointed space is called an $H$-group.

Example 2.7. Any topological group is an $H$-group.

Definition 2.8. The continuous multiplication $\mu : G \times G \to G$ in an $H$-group $G$ is said to be homotopy abelian if the diagram

$$
\begin{array}{ccc}
G \times G & \xrightarrow{T} & G \times G \\
\mu \downarrow & & \mu \downarrow \\
G & \underset{\sim}{\longrightarrow} & G
\end{array}
$$

is commutative upto homotopy i.e $\mu \circ T \approx \mu$ where $T(p_1, p_2) = (p_2, p_1)$.

Definition 2.9. An $H$-group with homotopy abelian multiplication is called an abelian $H$-group.

Definition 2.10. If $G$ and $G'$ are $H$-groups with multiplication $\mu$ and $\mu'$ respectively. A continuous map $\alpha : G \to G'$ is called a homomorphism if the diagram

$$
\begin{array}{ccc}
G \times G & \xrightarrow{\mu} & G \\
(\alpha, \alpha) \downarrow & & \downarrow \alpha \\
G' \times G' & \xrightarrow{\mu'} & G'
\end{array}
$$

is commutative upto homotopy.

Definition 2.11. If $G$ and $G'$ are $H$-groups with multiplication $\mu$ and $\mu'$ respectively. A homomorphism $\alpha : G \to G'$ is called an $H$-map if the diagram

$$
\begin{array}{ccc}
G & \xrightarrow{c} & G \\
\alpha \downarrow & & \downarrow \alpha \\
G' & \xrightarrow{c'} & G'
\end{array}
$$
is commutative up to homotopy that is $\alpha \circ c \approx c' \circ \alpha$ where $c$ and $c'$ are homotopy identity for $G$ and $G'$ respectively.

**Definition 2.12.** An equivalence class of monomorphism in the category of $H$-groups is called an $H$-subgroup. More explicitly, let $(G, \mu)$ be an $H$-group. An $H$-subgroup is an $H$-group $(K, \mu)$ together with an $H$-map $\phi : K \to G$ if given any $H$-group $(L, \eta)$ and two $H$-maps $f_1, f_2 : L \to K$ such that $\phi \circ f_1 \approx \phi \circ f_2 \Rightarrow f_1 \approx f_2$. Thus $[\phi]$ is a class of monomorphisms in the category of $H$-groups (objects are $H$-groups and morphisms are equivalence class of $H$-maps). This described a subgroup as an equivalence class of $H$-maps.

**Definition 2.13.** Consider the set $S = \{ \phi : K \to G \text{ is an } H\text{-map} : (K, \nu) \}$ is an $H$-subgroup of an $H$-group $(G, \mu)$. Define two $H$-maps $\phi_1 : K_1 \to G$ and $\phi_2 : K_2 \to G$ equivalent if there exists $H$-maps $h_1 : K_1 \to K_2$ and $h_2 : K_2 \to K_1$ such that $h_2 \circ h_1 \approx I_{K_1}$, $h_1 \circ h_2 \approx I_{K_2}$, $\phi_2 \circ h_1 \approx \phi_1$ and $\phi_1 \circ h_2 \approx \phi_2$.

**Proposition 2.14.** Let $(X, x_0)$ and $(Y, y_0)$ be two pointed topological spaces. Then $\Omega X = \{ \omega : \omega : I \to X \text{ is a loop based at } x_0 \}$ is an $H$-group with continuous multiplication $\mu$. Similarly $\Omega Y = \{ \omega : \omega : I \to Y \text{ is a loop based at } y_0 \}$ is an $H$-group with continuous multiplication $\nu$. Let $f : (Y, y_0) \to (X, x_0)$ is a continuous map. Then $(\Omega Y, \nu)$ is an $H$-subgroup together with an $H$-map $(\Omega Y, \nu) \xrightarrow{\Omega f} (\Omega X, \mu)$.

**Proof:** Let $(Z, +)$ be any $H$-group and $h_1, h_2 : (Z, +) \to \Omega Y$ are two $H$-maps given by $h_1(n) = \sigma^n, h_2(n) = \tau^n$.

Let $\sigma, \tau \in \Omega Y$ and $f \circ \sigma, f \circ \tau$ are in same path component of $\Omega X$ then we have $(Z, +) \xrightarrow{h_1} \Omega Y \xrightarrow{\Omega f} \Omega X$ and $(Z, +) \xrightarrow{h_2} \Omega Y \xrightarrow{\Omega f} \Omega X$ are path homotopic, that is, $\Omega f \circ h_1 \approx_p \Omega f \circ h_2$ because there is a path $\chi : I \to \Omega X$ such that $\chi(0) = f \circ \sigma$ and $\chi(1) = f \circ \tau$.

Now define $\chi : Z \times I \to \Omega X$ by $\chi(n, t) = (\chi(t))^n$.

Then $\chi(n, 0) = (\chi(0))^n = (f \circ \sigma)^n = (f \circ \sigma^n) = (\Omega f \circ h_1)(n)$

and $\chi(n, 1) = (\chi(1))^n = (f \circ \tau)^n = (f \circ \tau^n) = (\Omega f \circ h_2)(n)$

If $f \circ \sigma$ and $f \circ \tau$ lies in the same path component then $\sigma$ and $\tau$ also lies in the same path component.

Now $\Omega f \circ h_1 \approx_p \Omega f \circ h_2 \Rightarrow f \circ \sigma^n \approx_p f \circ \tau^n \Rightarrow \sigma^n \approx_p \tau^n \Rightarrow h_1(n) \approx_p h_2(n) \Rightarrow h_1 \approx_p h_2$.

Thus $\Omega Y$ is an $H$-subgroup with $H$-map $\Omega f : \Omega Y \to \Omega X$. 

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3 H-Transversal

In this section, by defining H-transversal, we have shown that there is a canonical H-group structure on \( \tilde{p}(G) \) with respect to which the inclusion \( \tilde{p}(G) \hookrightarrow G \) is an H-subgroup of \((G, \mu)\). (theorem 3.2)

**Definition 3.1.** An H-transversal in an H-group \((G, \mu)\) is a continuous identity preserving map \( \tilde{p} : G \to G \) such that

(i) \( \tilde{p}^2 \approx \tilde{p} \)

(ii) \( \mu \circ \tilde{p} \times \tilde{p} \approx \tilde{p} \circ \mu \circ (\tilde{p} \times I_G) \)

**Theorem 3.2.** Let \((G, \mu)\) be an H-group with base point identity element \(e\) of the group \(G\). Let \( \tilde{p} \) be an H-transversal in an H-group \((G, \mu)\). Then there is a canonical H-group structure on \( \tilde{p}(G) \) with respect to which the inclusion \( \tilde{p}(G) \hookrightarrow G \) is an H-subgroup of \((G, \mu)\).

**Proof:** From the definition of H-transversal, we have \( \mu \circ \tilde{p} \times \tilde{p} \approx \tilde{p} \circ \mu \circ (\tilde{p} \times I_G) \)

Thus there is a homotopy \( H : G \times G \times I \to G \) such that

\[
H(\{g_1, g_2\}, 0) = \mu(\tilde{p}(g_1), \tilde{p}(g_2)) \\
H((g_1, g_2), 1) = \tilde{p}(\mu(\tilde{p}(g_1), \tilde{p}(g_2))) \quad \text{for all } g_1, g_2 \in G
\]

Define a product \( \nu : \tilde{p}(G) \times \tilde{p}(G) \to \tilde{p}(G) \) by

\[
\nu(\tilde{p}(g_1), \tilde{p}(g_2)) = H((\tilde{p}(g_1), \tilde{p}(g_2)), 1) \\
= \tilde{p}(\mu(\tilde{p}(g_1), \tilde{p}(g_2))) \\
= (\mu(\tilde{p}(g_1), \tilde{p}(g_2))) \\
= (\tilde{p} \circ \mu)(\tilde{p}(g_1), \tilde{p}(g_2))
\]

We show that \((\tilde{p}(G), \nu)\) is an H-group. Since \( \tilde{p} \) and \( \mu \) are continuous so is \( \nu \).

Now,

(i) Since \( G \) is an H-group so the constant map \( c_G : G \to G \) given by \( c_G(g) = e \)

is a homotopy identity that is, \( \mu \circ (c_G \times 1_G) \) is homotopic to identity map \( 1_G \)

and similarly \( \mu \circ (1_G \times c_G) \) is also homotopic to identity map \( 1_G \)

Now for \( \tilde{p}(g) \in \tilde{p}(G) \)

\( (\mu \circ (c_G \times 1_G))\tilde{p}(g) = \mu \circ (c_G(\tilde{p}(g)), \tilde{p}(g)) = \mu(e, \tilde{p}(g)) \)

Let \( c_{\tilde{p}(G)} : \tilde{p}(G) \to \tilde{p}(G) \) denote constant map on \( \tilde{p}(G) \) defined by \( c_{\tilde{p}(G)}(\tilde{p}(g)) = \tilde{p}(e) = e \). Replacing \( G \) above by \( \tilde{p}(G) \). We have

\[
(\nu \circ (c_{\tilde{p}(G)} \times 1_{\tilde{p}(G)}))(\tilde{p}(g)) = (\nu((c_{\tilde{p}(G)}(\tilde{p}(g)), \tilde{p}(g))))
\]

\[
= \nu(e, \tilde{p}(g)) \\
= \nu(\tilde{p}(e), \tilde{p}(g)) \\
= (\tilde{p} \circ \mu)(\tilde{p}(e), \tilde{p}(g)) \\
= \tilde{p}(\mu(\tilde{p}(e), \tilde{p}(g))) \\
= \tilde{p}(\mu(c_{\tilde{p}(G)} \times 1_{\tilde{p}(G)}))\tilde{p}(g) \\
= \tilde{p}(\mu(c_G \times 1_G))\tilde{p}(g) \\
\approx \tilde{p}(1_G(\tilde{p}(g))) \\
\approx \tilde{p}(\tilde{p}(g)) \\
\approx \tilde{p}(g)
\]
\[ \approx 1_{\tilde{p}(G)}(\tilde{p}(g)) \]
Thus \( \nu \circ (c_{\tilde{p}(G)} \times 1_{\tilde{p}(G)}) \approx 1_{\tilde{p}(G)} \)
Similarly, \( \nu \circ (1_{\tilde{p}(G)} \times c_{\tilde{p}(G)}) \approx 1_{\tilde{p}(G)} \)
Thus \( c_{\tilde{p}(G)} \) is homotopic identity for \( (\tilde{p}(G), \nu) \)

(ii) Let \( \phi : G \to G \) be homotopy inverse for \( (G, \mu) \). So \( \mu \circ (\phi \times 1_G) \) and \( \mu \circ (1_G \times \phi) \) are homotopic to homotopy identity \( c_G \) for \( G \). Now
\[ (\nu \circ (1_{\tilde{p}(G)} \times \phi_{\tilde{p}(G)}))(\tilde{p}(g)) = \nu(\tilde{p}(g), \phi_{\tilde{p}(G)}(\tilde{p}(g))) \]
\[ = \nu(\tilde{p}(g), \tilde{p}(g_1)) \text{ for some } g_1 \in G \]
\[ = (\tilde{p} \circ \mu)(\tilde{p}(g), \tilde{p}(g_1)) \]
\[ = \tilde{p}(\mu((\tilde{p}(g), \tilde{p}(g_1))) \]
\[ = \tilde{p}(\mu(\tilde{p}(g), \phi_{\tilde{p}(G)}(\tilde{p}(g)))) \]
\[ = \tilde{p}(\mu(1_{\tilde{p}(G)} \times \phi_{\tilde{p}(G)})(\tilde{p}(g))) \]
\[ \approx \tilde{p}(c_{\tilde{p}(G)}(\tilde{p}(g))) \]
\[ \approx \tilde{p}(e) \]
\[ \approx e \]
\[ \approx c_{\tilde{p}(G)}(\tilde{p}(g)) \]
Thus, \( \nu \circ (1_{\tilde{p}(G)} \times \phi_{\tilde{p}(G)}) \approx c_{\tilde{p}(G)} \)
Similarly we can show that \( \nu \circ (\phi_{\tilde{p}(G)} \times 1_{\tilde{p}(G)}) \approx c_{\tilde{p}(G)} \)
Hence \( \phi_{\tilde{p}(G)} \) is homotopy inverse for \( \tilde{p}(G) \).

(iii) Associativity
Since \( (G, \mu) \) is associative. So the following diagram is commutative upto homotopy.

\[
\begin{array}{ccc}
G \times G & \xrightarrow{\mu \times 1_G} & G \times G \\
1_G \times \mu & \downarrow & \downarrow \mu \\
G \times G & \xrightarrow{\mu} & G
\end{array}
\]

that is, \( \mu \circ (\mu \times 1_G) \approx \mu \circ (1_G \times \mu) \)
Now, replacing \( G \) by \( \tilde{p}(G) \), we have to show that \( \nu \circ (1_{\tilde{p}(G)} \times \nu) \) is homotopic to \( \nu \circ (\nu \times 1_{\tilde{p}(G)}) \), that is, the following diagram is commutative upto homotopy.

\[
\begin{array}{ccc}
\tilde{p}(G) \times \tilde{p}(G) & \xrightarrow{\nu \times 1_{\tilde{p}(G)}} & \tilde{p}(G) \times \tilde{p}(G) \\
1_{\tilde{p}(G)} \times \nu & \downarrow & \downarrow \nu \\
\tilde{p}(G) \times \tilde{p}(G) & \xrightarrow{\nu} & \tilde{p}(G)
\end{array}
\]

Then
\[ (\nu \circ (1_{\tilde{p}(G)} \times \nu))(\tilde{p}(g_1), \tilde{p}(g_2), \tilde{p}(g_3)) = \nu(\tilde{p}(g_1), \nu(\tilde{p}(g_2), \tilde{p}(g_3))) \]
\[ = \nu(\tilde{p}(g_1), (\tilde{p} \circ \mu)(\tilde{p}(g_2), \tilde{p}(g_3))) \]
\[ \approx (\tilde{p} \circ \mu)((\tilde{p}(g_1)), \mu((\tilde{p}(g_2), \tilde{p}(g_3)))) \text{ [Since } \tilde{p}^2 = \tilde{p} \text{ ]} \]
\[ \approx (\tilde{p} \circ \mu \circ \tilde{p}(g_1), \mu(\tilde{p}(g_2), \tilde{p}(g_3))) \]
\[ \approx (\tilde{p} \circ \mu \circ \tilde{p} \times 1_G)(\tilde{p}(g_1), \mu(\tilde{p}(g_2), \tilde{p}(g_3))) \]
\[ \approx (\tilde{p} \circ \tilde{p}) \circ \mu (\tilde{p}(g_1), \mu (\tilde{p}(g_2), \tilde{p}(g_3))) \]
\[ \approx \tilde{p} \circ \mu (\tilde{p}(g_1), \mu (\tilde{p}(g_2), \tilde{p}(g_3))) \]
\[ \approx \tilde{p} \circ (\mu \circ (1_G \times \mu)) (\tilde{p}(g_1), (\tilde{p}(g_2), \tilde{p}(g_3))) \]
\[ \approx \tilde{p} \circ (\mu \circ (\mu \times 1_G)) ((\tilde{p}(g_1), \tilde{p}(g_2)), \tilde{p}(g_3)) \]
\[ \approx \tilde{p} \circ \mu (\mu (\tilde{p}(g_1), \tilde{p}(g_2)), \tilde{p}(g_3)) \]
\[ \approx \tilde{p} \circ \mu (\mu (\tilde{p}(g_1)), \tilde{p}(g_2), \tilde{p}(g_3)) \]
\[ \approx \tilde{p} \circ \mu (\mu (\tilde{p}(g_1)), \tilde{p}(g_2), \tilde{p}(g_3)) \]
\[ \approx (\tilde{p} \circ \mu) ((\tilde{p} \circ \mu \times \tilde{p})(\tilde{p}(g_1), \tilde{p}(g_2), \tilde{p}(g_3))) \]
\[ \approx (\tilde{p} \circ \mu) ((\tilde{p} \circ \mu \times (\tilde{p} \times 1_G)) (\tilde{p}(g_1), \tilde{p}(g_2), \tilde{p}(g_3))) \]
\[ \approx (\tilde{p} \circ \mu) ((\tilde{p} \circ \mu)(\tilde{p}(g_1), \tilde{p}(g_2), \tilde{p}(g_3))) \]
\[ \approx (\tilde{p} \circ \mu)((\nu \circ \nu \times 1_{\tilde{p}(G)})) (\tilde{p}(g_1), \tilde{p}(g_2), \tilde{p}(g_3)) \]

Thus
\[ \nu \circ (1_{\tilde{p}(G)} \times \nu) \approx \nu \circ (\nu \times 1_{\tilde{p}(G)}) \]

Now, we show that inclusion map \( i : \tilde{p}(G) \to G \) is an H-map.
First, we show that the diagram

\[
\begin{array}{ccc}
\tilde{p}(G) \times \tilde{p}(G) & \xrightarrow{i \times i} & G \times G \\
\downarrow \nu & & \downarrow \mu \\
\tilde{p}(G) & \xrightarrow{i} & G 
\end{array}
\]

is commutative up to homotopy which proves that \( i \) is an homomorphism. We have
\[ (\mu \circ (i \times i))(\tilde{p}(g_1), \tilde{p}(g_2)) = \mu(\tilde{p}(g_1), \tilde{p}(g_2)) \]

Now, \( (i \circ \nu)(\tilde{p}(g_1), \tilde{p}(g_2)) = i(\nu(\tilde{p}(g_1), \tilde{p}(g_2))) \]
\[ = i((\tilde{p} \circ \mu)(\tilde{p}(g_1), \tilde{p}(g_2))) \]
\[ = i(\tilde{p}(\mu(\tilde{p}(g_1), \tilde{p}(g_2)))) \]
\[ = \tilde{p}(\mu(\tilde{p}(g_1), \tilde{p}(g_2))) \]
\[ \approx \tilde{p}(\mu(\tilde{p}(g_1), \tilde{p}(g_2))) \quad \text{[Since } \tilde{p}^2 = \tilde{p} \text{]} \]
\[ \approx \tilde{p}(\mu((\tilde{p} \times 1_G)(\tilde{p}(g_1), \tilde{p}(g_2)))) \]
\[ \approx (\tilde{p} \circ \mu \circ (\tilde{p} \times 1_G)(\tilde{p}(g_1), \tilde{p}(g_2)) \]
\[ \approx (\mu \circ \tilde{p} \circ \tilde{p})(\tilde{p}(g_1), \tilde{p}(g_2)) \]
\[ \approx \mu(\tilde{p}(\tilde{p}(g_1), \tilde{p}(g_2))) \]
\[ \approx \mu(\tilde{p}(g_1), \tilde{p}(g_2)) \]
\[ \approx (\mu \circ i \times i)(\tilde{p}(g_1), \tilde{p}(g_2)) \]

Thus \( i \circ \nu \approx \mu \circ i \times i \)

Now, to show that the homomorphism \( i \) is an H-map, we prove that the diagram

\[
\begin{array}{ccc}
\tilde{p}(G) & \xrightarrow{i} & G \\
\downarrow c_{p(G)} & & \downarrow c_G \\
\tilde{p}(G) & \xrightarrow{i} & G 
\end{array}
\]
is commutative up to homotopy where $c_{\tilde{p}(G)}$ and $c_G$ denote constants map on $\tilde{p}(G)$ and $G$ respectively.

\[(c_G \circ i)(\tilde{p}(g)) = c_G(i(\tilde{p}(g)))\]

\[= c_G(\tilde{p}(g))\]

\[= c_{\tilde{p}(G)}(\tilde{p}(g))\]

\[= i(c_{\tilde{p}(G)}(\tilde{p}(g)))\]

So, $c_G \circ i \approx i \circ c_{\tilde{p}(G)}$

Hence inclusion $i : \tilde{p}(G) \to G$ is an $H$-map. Now for any $H$-group $(L, \eta)$, we have

\[L \xrightarrow{f_1} \tilde{p}(G) \xrightarrow{i} G\]

where $f_1, f_2$ are $H$-maps, therefore $i \circ f_1 \approx i \circ f_2 \Rightarrow f_1 \approx f_2$. Thus $[i]$ is an $H$-subgroup in the category of $H$-groups (with objects as $H$-groups and morphisms as equivalence class of $H$-maps).

**References**


**Received:** September 3, 2014