

Study of Γ -Semigroups via its Operator Semigroups in Terms of Atanassov's Intuitionistic Fuzzy Ideals

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Abstract

In this paper some fundamental relationships of a Γ -semigroup and its operator semigroups in terms of intuitionistic fuzzy subsets, intuitionistic fuzzy ideals, intuitionistic fuzzy prime(semiprime) ideals, intuitionistic fuzzy ideal extensions are obtained. These are then used to obtain some important characterization theorems of Γ -semigroups in terms of intuitionistic fuzzy subsets so as to highlight the role of operator semigroups in the study of Γ -semigroups in terms of intuitionistic fuzzy subsets.

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1 Introduction

This is a continuation of our paper on Atanassov's Intuitionistic Fuzzy Ideals of Γ -semigroups[7] wherein we have investigated properties of Atanassov's intuitionistic fuzzy

(prime, semiprime) ideals and intuitionistic fuzzy ideal extension and obtained characterization of regular Γ -semigroup and of prime ideals of Γ -semigroups.

Dutta and Adhikari[2] have found operator semigroups of a Γ -semigroup to be a very effective tool in studying Γ -semigroups. The principal objective of this paper is to investigate as to whether the concept of operator semigroups can be made to work in the study of Γ -semigroups in terms of intuitionistic fuzzy subsets. In order to do this we deduce some fundamental relationships of a Γ -semigroup and its operator semigroups in terms of intuitionistic fuzzy subsets, intuitionistic fuzzy ideals, intuitionistic fuzzy prime(semiprime) ideals and intuitionistic fuzzy ideal extension. We then use these relationships to obtain some characterization theorems thereby establishing the effectiveness of operator semigroups in the study of Γ -semigroups in terms of intuitionistic fuzzy subsets.

For preliminaries we refer to [7].

2 Main Results

Many results of semigroups could be extended to Γ -semigroups directly and via operator semigroups[1](left, right) of a Γ -semigroup. In this section in order to make operator semigroups of a Γ -semigroup work in the context of IFS^2 as it worked in the study of Γ -semigroups[1, 2], we obtain various relationships between $IFI(S)$ and that of its operator semigroups. Here, among other results we obtain an inclusion preserving bijection between the set of all $IFI(S)$ and that of its operator semigroups. Among other applications of this bijection we apply it to give new proofs of its ideal analogue obtained in [2] by Dutta and Adhikari.

Definition 2.1. [2] Let S be a Γ -semigroup. Let us define a relation ρ on $S \times \Gamma$ as follows : $(x, \alpha)\rho(y, \beta)$ if and only if $x\alpha s = y\beta s$ for all $s \in S$ and $\gamma x\alpha = \gamma y\beta$ for all $\gamma \in \Gamma$. Then ρ is an equivalence relation. Let $[x, \alpha]$ denote the equivalence class containing (x, α) . Let $L = \{[x, \alpha] : x \in S, \alpha \in \Gamma\}$. Then L is a semigroup with respect to the multiplication defined by $[x, \alpha][y, \beta] = [x\alpha y, \beta]$. This semigroup L is called the left operator semigroup of the Γ -semigroup S .

² $IFS, IFS(L), IFS(R), IFS(S), IFI(S)$ respectively denote intuitionistic fuzzy subset(s), intuitionistic fuzzy subset(s) of L , intuitionistic fuzzy subset(s) of R , intuitionistic fuzzy subset(s) of S , intuitionistic fuzzy ideal(s) of S .

Dually the right operator semigroup R of Γ -semigroup S is defined where the multiplication is defined by $[\alpha, a][\beta, b] = [\alpha\beta, b]$.

If there exists an element $[e, \delta] \in L([\gamma, f] \in R)$ such that $e\delta s = s$ (resp. $s\gamma f = s$) for all $s \in S$ then $[e, \delta]$ (resp. $[\gamma, f]$) is called the left (resp. right) unity of S .

Definition 2.2. For an $IFS(R)$, $A = (\mu_A, \nu_A)$ we define an $IFS(S)$, $A^* = (\mu_A, \nu_A)^* = (\mu_A^*, \nu_A^*)$ by $\mu_A^*(a) = \inf_{\gamma \in \Gamma} \mu_A([\gamma, a])$ and $\nu_A^*(a) = \sup_{\gamma \in \Gamma} \nu_A([\gamma, a])$, where $a \in S$. For an $IFS(S)$, $B = (\mu_B, \nu_B)$ we define an $IFS(R)$, $B^{*'} = (\mu_B, \nu_B)^{*' } = (\mu_B^{*' }, \nu_B^{*' })$ by $\mu_B^{*' }([\alpha, a]) = \inf_{s \in S} \mu_B(s\alpha a)$ and $\nu_B^{*' }([\alpha, a]) = \sup_{s \in S} \nu_B(s\alpha a)$, where $[\alpha, a] \in R$. For an $IFS(L)$, $C = (\mu_C, \nu_C)$ we define an $IFS(S)$, $C^+ = (\mu_C, \nu_C)^+ = (\mu_C^+, \nu_C^+)$ by $\mu_C^+(a) = \inf_{\gamma \in \Gamma} \mu_C([a, \gamma])$ and $\nu_C^+(a) = \sup_{\gamma \in \Gamma} \nu_C([a, \gamma])$, where $a \in S$. For an $IFS(S)$, $D = (\mu_D, \nu_D)$ we define an $IFS(L)$, $D^{+' } = (\mu_D, \nu_D)^{+' } = (\mu_D^{+' }, \nu_D^{+' })$ by $\mu_D^{+' }([a, \alpha]) = \inf_{s \in S} \mu_D(a\alpha s)$ and $\nu_D^{+' }([a, \alpha]) = \sup_{s \in S} \nu_D(a\alpha s)$ where $[a, \alpha] \in L$.

Now we recall the following propositions from [2] which were proved therein for one sided ideals. But the results can be proved to be true for two sided ideals.

Proposition 2.3. [2] *Let S be a Γ -semigroup with unities and R be its right operator semigroup. If P is a $LI(R)(I(R))^3$ then P^* is a $LI(S)(I(S))$.*

Proposition 2.4. [2] *Let S be a Γ -semigroup with unities and R be its right operator semigroup. If Q is a $LI(S)(I(S))$ then $Q^{*' }$ is a $LI(R)(I(R))$.*

For convenience of the readers, we may note that for a Γ -semigroup S and its left, right operator semigroups L, R respectively four mappings namely $()^+, ()^{+' }, ()^*, ()^{*' }$ occur. They are defined as follows: For $I \subseteq R, I^* = \{s \in S, [\alpha, s] \in I \forall \alpha \in \Gamma\}$; for $P \subseteq S, P^{*' } = \{[\alpha, x] \in R : s\alpha x \in P \forall s \in S\}$; for $J \subseteq L, J^+ = \{s \in S, [s, \alpha] \in J \forall \alpha \in \Gamma\}$; for $Q \subseteq S, Q^{+' } = \{[x, \alpha] \in L : x\alpha s \in Q \forall s \in S\}$.

Proposition 2.5. *Let $A = (\mu_A, \nu_A)$ be an $IFS(R)$, then $[U(\mu_A; t)]^* = U((\mu_A)^*; t)$ and $[L(\nu_A; t)]^* = L((\nu_A)^*; t)$ for all $t \in [0, 1]$, provided the sets are non-empty.*

³ $LI(R), LI(S), I(R), I(S), IFLI(R), IFLI(S), IFI(R), IFI(S)$ respectively denote left ideal(s) of R , left ideal(s) of S , ideal(s) of R , ideal(s) of S , intuitionistic fuzzy left ideal(s) of R , intuitionistic fuzzy left ideal(s) of S , intuitionistic fuzzy ideal(s) of R , intuitionistic fuzzy ideal(s) of S .

Proof. Let $m \in S$. Then $m \in [U(\mu_A; t)]^* \Leftrightarrow [\gamma, m] \in U(\mu_A; t) \forall \gamma \in \Gamma \Leftrightarrow \mu_A([\gamma, m]) \geq t \forall \gamma \in \Gamma \Leftrightarrow \inf_{\gamma \in \Gamma} \mu_A([\gamma, m]) \geq t \Leftrightarrow (\mu_A)^*(m) \geq t \Leftrightarrow m \in U((\mu_A)^*; t)$. Again let $n \in S$. Then $n \in [L(\nu_A; t)]^* \Leftrightarrow [\gamma, n] \in L(\nu_A; t) \forall \gamma \in \Gamma \Leftrightarrow \nu_A([\gamma, n]) \leq t \forall \gamma \in \Gamma \Leftrightarrow \sup_{\gamma \in \Gamma} \nu_A([\gamma, n]) \leq t \Leftrightarrow (\nu_A)^*(n) \leq t \Leftrightarrow n \in L((\nu_A)^*; t)$. Hence $[U(\mu_A; t)]^* = U((\mu_A)^*; t)$ and $[L(\nu_A; t)]^* = L((\nu_A)^*; t)$. \square

Proposition 2.6. *Let $B = (\mu_B, \nu_B)$ be an $IFIS(S)$. Then $[U(\mu_B; t)]^{*'} = U((\mu_B)^{*}'; t)$ and $[L(\nu_B; t)]^{*'} = L((\nu_B)^{*}'; t)$ for all $t \in [0, 1]$, provided the sets under consideration are non-empty.*

Proof. Let $[\alpha, x] \in R$ and t is as mentioned in the statement. Then $[\alpha, x] \in [U(\mu_B; t)]^{*'} \Leftrightarrow m\alpha x \subseteq U(\mu_B; t) \forall m \in S \Leftrightarrow \mu_B(m\alpha x) \geq t \forall m \in S \Leftrightarrow \inf_{m \in S} \mu_B(m\alpha x) \geq t \Leftrightarrow (\mu_B)^{*}'([\alpha, x]) \geq t \Leftrightarrow [\alpha, x] \in U((\mu_B)^{*}'; t)$. Again let $[\beta, y] \in R$ and t is as mentioned in the statement. Then $[\beta, y] \in [L(\nu_B; t)]^{*'} \Leftrightarrow n\beta y \subseteq L(\nu_B; t) \forall n \in S \Leftrightarrow \nu_B(n\beta y) \leq t \forall n \in S \Leftrightarrow \sup_{n \in S} \nu_B(n\beta y) \leq t \Leftrightarrow (\nu_B)^{*}'([\beta, y]) \leq t \Leftrightarrow [\beta, y] \in L((\nu_B)^{*}'; t)$. Hence $[U(\mu_B; t)]^{*'} = U((\mu_B)^{*}'; t)$ and $[L(\nu_B; t)]^{*'} = L((\nu_B)^{*}'; t)$. \square

In what follows S denotes a Γ -semigroup with unities[2], L, R be its left and right operator semigroups respectively.

Proposition 2.7. *If $A = (\mu_A, \nu_A) \in IFI(R)(IFLI(R))$, then $A^* = (\mu_A, \nu_A)^* = (\mu_A^*, \nu_A^*) \in IFI(S)(respectively IFLI(S))$.*

Proof. Suppose $A = (\mu_A, \nu_A) \in IFI(R)$. Then $U(\mu_A; t)$ and $L(\nu_A; t)$ are $I(R)$, $\forall t \in [0, 1]$. Hence $[U(\mu_A; t)]^*$ and $[L(\nu_A; t)]^*$ are $I(S)$, $\forall t \in [0, 1]$ (cf. Proposition 2.3). Now since $A = (\mu_A, \nu_A)$ is an $IFI(R)$, $A = (\mu_A, \nu_A)$ is a non-empty $IFIS(R)$. Hence for some $[\alpha, m] \in R$, $0 < \mu_A([\alpha, m]) + \nu_A([\alpha, m]) \leq 1$. Then $U(\mu_A; t) \neq \phi$ and $L(\nu_A; t) \neq \phi$ where $t := \mu_A([\alpha, m]) = \nu_A([\alpha, m])$. So by the same argument applied above $[U(\mu_A; t)]^* \neq \phi$ and $[L(\nu_A; t)]^* \neq \phi$. Let $u \in [U(\mu_A; t)]^*$. Then $[\beta, u] \in U(\mu_A; t)$ for all $\beta \in \Gamma$. Hence $\mu_A([\beta, u]) \geq t$. This implies that $\inf_{\beta \in \Gamma} \mu_A([\beta, u]) \geq t$, i.e., $(\mu_A)^*(u) \geq t$. Hence $u \in U((\mu_A)^*; t)$. Hence $U((\mu_A)^*; t) \neq \phi$. By similar argument we can show that $L((\nu_A)^*; t) \neq \phi$. Consequently, $[U(\mu_A; t)]^* = U((\mu_A)^*; t)$ and $[L(\nu_A; t)]^* = L((\nu_A)^*; t)$ (cf. Proposition 2.5). It follows that $U((\mu_A)^*; t)$ and $L((\nu_A)^*; t)$ are $I(S)$ for all $t \in [0, 1]$. Hence $A^* = (\mu_A, \nu_A)^* = (\mu_A^*, \nu_A^*)$ is an $IFI(S)$ (cf. Theorem 3.10[7]). Similarly we can prove the other case also. \square

In a similar fashion by using Propositions 2.4, 2.6 and Theorems 3.9[7], 3.10[7] we deduce the following proposition.

Proposition 2.8. *If $B = (\mu_B, \nu_B) \in IFI(S)(IFLI(S))$, then $B^{*\prime} = (\mu_B, \nu_B)^{*\prime} = (\mu_B^{*\prime}, \nu_B^{*\prime}) \in IFI(R)$ (respectively $IFLI(R)$).*

REMARK 1. The left operator analogues of Propositions 2.3-2.8. are also true.

In view of Remark 1, we deduce the following theorem.

Theorem 2.9. *Let S be a Γ -semigroup with unities and L be its left operator semigroup. Then there exists an inclusion preserving bijection $A \mapsto A^{+\prime}$ between the set of all $IFI(S)(IFRI(S))^4$ and set of all $IFI(L)$ (resp. $IFRI(L)$), where $A = (\mu_A, \nu_A)$ is an $IFI(S)$ (resp. $IFRI(S)$).*

Proof. Let $A = (\mu_A, \nu_A) \in IFI(S)(IFRI(S))$ and $x \in S$. Then

$$(\mu_A^{+\prime})^+(x) = \inf_{\gamma \in \Gamma} \mu_A^{+\prime}([x, \gamma]) = \inf_{\gamma \in \Gamma} [\inf_{s \in S} \mu_A(x\gamma s)] \geq \mu_A(x).$$

Again

$$(\nu_A^{+\prime})^+(x) = \sup_{\gamma \in \Gamma} \nu_A^{+\prime}([x, \gamma]) = \sup_{\gamma \in \Gamma} [\sup_{s \in S} \nu_A(x\gamma s)] \leq \nu_A(x).$$

Hence $A \subseteq (A^{+\prime})^+$. Let $[\gamma, f]$ be the right unity of S . Then $x\gamma f = x$ for all $x \in S$. Then

$$\mu_A(x) = \mu_A(x\gamma f) \geq \inf_{\alpha \in \Gamma} [\inf_{s \in S} \mu_A(x\alpha s)] = \inf_{\alpha \in \Gamma} \mu_A^{+\prime}([x, \alpha]) = (\mu_A^{+\prime})^+(x).$$

Again

$$\nu_A(x) = \nu_A(x\gamma f) \leq \sup_{\alpha \in \Gamma} [\sup_{s \in S} \nu_A(x\alpha s)] = \sup_{\alpha \in \Gamma} \nu_A^{+\prime}([x, \alpha]) = (\nu_A^{+\prime})^+(x).$$

So $A \supseteq (A^{+\prime})^+$. Hence $(A^{+\prime})^+ = A$. Thus the said mapping is one-one. Now let $B = (\mu_B, \nu_B) \in IFI(L)(IFRI(S))$. Then

$$\begin{aligned} (\mu_B^+)^{\prime}([x, \alpha]) &= \inf_{s \in S} \mu_B^+(x\alpha s) = \inf_{s \in S} [\inf_{\gamma \in \Gamma} \mu_B([x\alpha s, \gamma])] \\ &= \inf_{s \in S} [\inf_{\gamma \in \Gamma} \mu_B([x, \alpha][s, \gamma])] \geq \mu_B([x, \alpha]). \end{aligned}$$

Again

$$\begin{aligned} (\nu_B^+)^{\prime}([x, \alpha]) &= \sup_{s \in S} \nu_B^+(x\alpha s) = \sup_{s \in S} [\sup_{\gamma \in \Gamma} \nu_B([x\alpha s, \gamma])] \\ &= \sup_{s \in S} [\sup_{\gamma \in \Gamma} \nu_B([x, \alpha][s, \gamma])] \geq \nu_B([x, \alpha]). \end{aligned}$$

⁴ $IFRI(L), IFRI(S)$ respectively denote intuitionistic fuzzy right ideal(s) of L and intuitionistic fuzzy right ideal(s) of S .

So $B \subseteq (B^+)^+$. Let $[e, \delta]$ be the left unity of L . Then

$$\begin{aligned}\mu_B([x, \alpha]) &= \mu_B([x, \alpha][e, \delta]) \geq \inf_{s \in S} [\inf_{\gamma \in \Gamma} \mu_B([x, \alpha][s, \gamma])] \\ &= (\mu_B^+)^+([x, \alpha]).\end{aligned}$$

Again

$$\begin{aligned}\nu_B([x, \alpha]) &= \nu_B([x, \alpha][e, \delta]) \leq \sup_{s \in S} [\sup_{\gamma \in \Gamma} \nu_B([x, \alpha][s, \gamma])] \\ &= (\nu_B^+)^+([x, \alpha])\end{aligned}$$

So $B \supseteq (B^+)^+$ and hence $B = (B^+)^+$. Consequently, the correspondence $A \mapsto A^+$ is a bijection. Now let $C = (\mu_C, \nu_C), D = (\mu_D, \nu_D) \in IFI(S)(IFLI(S))$ be such that $C \subseteq D$, i.e., $\mu_C \subseteq \mu_D$ and $\nu_C \supseteq \nu_D$. Then for all $[x, \alpha] \in L$,

$$\mu_C^+([x, \alpha]) = \inf_{s \in S} \mu_C(x\alpha s) \leq \inf_{s \in S} \mu_D(x\alpha s) = \mu_D^+([x, \alpha])$$

and

$$\nu_C^+([x, \alpha]) = \sup_{s \in S} \nu_C(x\alpha s) \geq \sup_{s \in S} \nu_D(x\alpha s) = \nu_D^+([x, \alpha]).$$

Thus $\mu_C^+ \subseteq \mu_D^+$ and $\nu_C^+ \supseteq \nu_D^+$. Consequently, $C^+ \subseteq D^+$. Hence $A \mapsto A^+$ is an inclusion preserving bijection. The rest of the proof follows from Remark 1. □

In a similar way by using Proposition 2.7 and Proposition 2.8 we can deduce the following theorem.

Theorem 2.10. *Let S be a Γ -semigroup with unities and R be its right operator semigroup. Then there exists an inclusion preserving bijection $B \mapsto B^*$ between the set of all $IFI(S)(IFLI(S))$ and set of all $IFI(R)$ (resp. $IFLI(R)$), where $B = (\mu_B, \nu_B)$ is an IFI (resp. $IFLI(S)$).*

Now to apply the above theorem for giving a new proof of Theorem 4.6[2] and its two sided ideal analogue we deduce the following lemmas.

Lemma 2.11. *Let I be a $LI(R)(I(R))$ of a Γ -semigroup S and $P = (\chi_I, \chi_I^c)$ where χ_I is the characteristic function of I . Then $P^* = (\chi_I, \chi_I^c)^* = ((\chi_I)^*, (\chi_I^c)^*) = (\chi_{I^*}, \chi_{I^*}^c)$.*

Proof. Suppose $s \in I^*$. Then $[\beta, s] \in I$ for all $\beta \in \Gamma$. This means $\inf_{\beta \in \Gamma} (\chi_I([\beta, s])) = 1$ and $\sup_{\beta \in \Gamma} (\chi_I^c([\beta, s])) = 0$. Also $\chi_{I^*}(s) = 1$ and $\chi_{I^*}^c(s) = 0$. Now suppose $s \notin I^*$. Then there exists $\delta \in \Gamma$ such that $[\delta, s] \notin I$. Hence $\chi_I([\delta, s]) = 0, \chi_I^c([\delta, s]) = 1$ and so $\inf_{\beta \in \Gamma} (\chi_I([\beta, s])) = 0, \sup_{\beta \in \Gamma} (\chi_I^c([\beta, s])) = 1$. Hence $(\chi_I)^*(s) = 0$ and $(\chi_I^c)^*(s) = 1$. Again $(\chi_{I^*})(s) = 0$ and $(\chi_{I^*}^c)(s) = 1$. Thus $P^* = (\chi_I, \chi_I^c)^* = ((\chi_I)^*, (\chi_I^c)^*) = (\chi_{I^*}, \chi_{I^*}^c)$.

□

The following lemma follows in a similar way.

Lemma 2.12. *Let I be a $RI(S)(I(S))$, $P = (\chi_I, \chi_I^c)$ and R be the right operator semigroup of S . Then $P^{*'} = (\chi_I, \chi_I^c)^{*'} = ((\chi_I)^{*'}, (\chi_I^c)^{*'}) = (\chi_{I^{*'}}, \chi_{I^{*'}}^c)$, where χ_I is the characteristic function of I .*

REMARK 2. By drawing an analogy we deduce results similar to the above lemmas for left operator semigroup L of the Γ -semigroup S , i.e., for the functions $+$ and $+'$.

Now we present a new proof of the following result which is originally due to Dutta and Adhikari[2].

Theorem 2.13. [2] *Let S be a Γ -semigroup with unities. Then there exists an inclusion preserving bijection between the set of all $I(S)(LI(S))$ and that of its right operator semigroup R via the mapping $I \rightarrow I^{*'}$.*

Proof. Let us denote the mapping $I \rightarrow I^{*'}$ by ϕ . This is actually a mapping follows from Proposition 2.8. Now let $\phi(I_1) = \phi(I_2)$. Then $I_1^{*' } = I_2^{*'}$. This implies that $(\chi_{I_1^{*' }}, \chi_{I_1^{*' }}^c) = (\chi_{I_2^{*' }}, \chi_{I_2^{*' }}^c)$ (where χ_I is the characteristic function I).

Hence by Lemma 2.12, $(\chi_{I_1}, \chi_{I_1}^c)^{*'} = (\chi_{I_2}, \chi_{I_2}^c)^{*'}$. This together with Theorem 2.10, gives $(\chi_{I_1}, \chi_{I_1}^c) = (\chi_{I_2}, \chi_{I_2}^c)$ whence $I_1 = I_2$. Consequently ϕ is one-one.

Let I be a $I(R)(LI(R))$. Then (χ_I, χ_I^c) is an $IFI(R)(IFLI(R))$. Hence by Theorem 2.10, $((\chi_I, \chi_I^c)^*)^{*' } = (\chi_I, \chi_I^c)$. This implies that $(\chi_{(I^*)^{*' }}, \chi_{(I^*)^{*' }}^c) =$

(χ_I, χ_I^c) (cf. Lemma 2.11 and Lemma 2.12). Hence $(I^*)^{*' } = I$, i.e., $\phi(I^*) = I$.

Now since I^* is a $I(S)(LI(S))$ (cf. Proposition 2.3), it follows that ϕ is onto.

Let I_1, I_2 be two $I(S)(LI(S))$ with $I_1 \subseteq I_2$. Then $\chi_{I_1} \subseteq \chi_{I_2}$ and $\chi_{I_1}^c \supseteq \chi_{I_2}^c$.

Hence by Theorem 2.10, we see that $(\chi_{I_1})^{*' } \subseteq (\chi_{I_2})^{*' }$ and $(\chi_{I_1}^c)^{*'} \supseteq (\chi_{I_2}^c)^{*'}$ i.e.,

$\chi_{I_1^{*' }} \subseteq \chi_{I_2^{*' }}$ and $\chi_{I_1^{*' }}^c \supseteq \chi_{I_2^{*' }}^c$ (cf. Lemma 2.12) which gives $I_1^{*' } \subseteq I_2^{*'}$.

□

REMARK 3. Now by using a similar argument as above and with the help of lemmas dual to the lemmas 2.11, 2.12 (cf. Remark 2) and Theorem 2.9 we

deduce that the mapping $()^+$ is an inclusion preserving bijection (with $()^+$ as the inverse) between the set of all $I(S)(RI(S))$ and that of its left operator semigroup L .

In what follows S denotes a Γ -semigroup not necessarily with unities, L, R be its left and right operator semigroups respectively.

Proposition 2.14. [2, 5] *Let S be a Γ -semigroup and R be its right operator semigroup. If P is $PI(R)(SPI(R))$ ⁵ then P^* is $PI(S)(SPI(S))$.*

Proposition 2.15. [2, 5] *Let S be a Γ -semigroup and R be its right operator semigroup. If Q is $PI(S)(SPI(S))$ then $Q^{* \prime}$ is $PI(R)(SPI(R))$.*

Proposition 2.16. *If $A = (\mu_A, \nu_A) \in IFPI(R)(IFSPI(R))$, then $A^* = (\mu_A^*, \nu_A^*) \in IFPI(S)$ (resp. $IFSPI(S)$).*

Proof. Let $A = (\mu_A, \nu_A) \in IFPI(R)$. Then it is in $IFI(R)$. Hence by Proposition 2.7, $A^* = (\mu_A^*, \nu_A^*) \in IFI(S)$. Since $A = (\mu_A, \nu_A) \in IFPI(R)$, so $U(\mu_A; t)$ and $L(\nu_A; t)$ are $PI(R)$. Now by Proposition 2.14, for all $t \in [0, 1]$, $[U(\mu_A; t)]^*$ and $[L(\nu_A; t)]^*$ are $PI(S)$. By Proposition 2.5, $[U(\mu_A; t)]^* = U((\mu_A)^*; t)$ and $[L(\nu_A; t)]^* = L((\nu_A)^*; t)$. So $U((\mu_A)^*; t)$ and $L((\nu_A)^*; t)$ are $PI(S)$. Hence $A^* = (\mu_A^*, \nu_A^*) \in IFPI(S)$. Similarly we can prove the other case also. \square

In a similar fashion by using Propositions 2.6, 2.8 and Theorem 3.9[7], 3.10[7] we deduce the following proposition.

Proposition 2.17. *If $B = (\mu_B, \nu_B) \in IFPI(S)(IFSPI(S))$, then $B^{* \prime} = (\mu_B^{* \prime}, \nu_B^{* \prime}) \in IFPI(R)$.*

REMARK 4. We can also deduce the following left operator analogues of Propositions 2.14-2.17.

The following theorem is on the inclusion preserving bijection between the set of all $IFPI(S)$ and the set of all $IFPI(R)$. It may be noted that S need not have unities here which was the case for the set of all IFI (cf. Theorems 2.9, 2.10).

Theorem 2.18. *Let S be a Γ -semigroup and R be its right operator semigroup. Then there exist an inclusion preserving bijection $B = (\mu_B, \nu_B) \mapsto B^{* \prime} = (\mu_B^{* \prime}, \nu_B^{* \prime})$ between the set of all $IFPI(S)(IFSPI(S))$ and set of all $IFPI(R)$ (resp. $IFSPI(R)$).*

⁵ $PI(R), PI(S), SPI(R), SPI(S), IFPI(R), IFPI(S), IFSPI(R), IFSPI(S)$ respectively denote prime ideal(s) of R , prime ideal(s) of S , semiprime ideal(s) of R , semiprime ideal(s) of S , intuitionistic fuzzy prime ideal(s) of R , intuitionistic fuzzy prime ideal(s) of S , intuitionistic fuzzy semiprime ideal(s) of R , intuitionistic fuzzy semiprime ideal(s) of S .

Proof. Let $B = (\mu_B, \nu_B) \in IFPI(R)$ and $x \in S$. Then

$$(\mu_B^*)'(x) = \inf_{\gamma \in \Gamma} \mu_B^*([\gamma, x]) = \inf_{\gamma \in \Gamma} \inf_{s \in S} \mu_B(s\gamma x) \geq \mu_B(x) \text{ (since } B \in IFI(S)\text{)}.$$

$$\begin{aligned} \text{Again for } x \in S, (\mu_B^*)'(x) &= \inf_{\gamma \in \Gamma} \inf_{s \in S} \mu_B(s\gamma x) = \inf_{s \in S} \inf_{\gamma \in \Gamma} \mu_B(s\gamma x) \\ &= \inf_{s \in S} \max\{\mu_B(s), \mu_B(x)\} \text{ (since } B \in IFPI(S)\text{)} \\ &\leq \max\{\mu_B(x), \mu_B(x)\} = \mu_B(x). \end{aligned}$$

Hence $(\mu_B^*)'(x) = \mu_B(x)$. Also

$$(\nu_B^*)'(x) = \sup_{\gamma \in \Gamma} \nu_B^*([\gamma, x]) = \sup_{\gamma \in \Gamma} \sup_{s \in S} \nu_B(s\gamma x) \leq \nu_B(x) \text{ (since } B \in IFI(S)\text{)}.$$

$$\begin{aligned} \text{Again for } x \in S, (\nu_B^*)'(x) &= \sup_{\gamma \in \Gamma} \sup_{s \in S} \nu_B(s\gamma x) = \sup_{s \in S} \sup_{\gamma \in \Gamma} \nu_B(s\gamma x) \\ &= \sup_{s \in S} \min\{\nu_B(s), \nu_B(x)\} \text{ (since } B \in IFPI(S)\text{)} \\ &\geq \min\{\nu_B(x), \nu_B(x)\} = \nu_B(x). \end{aligned}$$

Hence $(\nu_B^*)'(x) = \nu_B(x)$. Consequently, $(B^*)' = B$. Hence the mapping is one-one. Now let $[\alpha, x] \in R$. Then

$$\begin{aligned} (\mu_B^*)'([\alpha, x]) &= \inf_{s \in S} \mu_B^*(s\alpha x) = \inf_{s \in S} \inf_{\beta \in \Gamma} \mu_B([\beta, s\alpha x]) \\ &= \inf_{s \in S} \inf_{\beta \in \Gamma} \mu_B([\beta, s][\alpha, x]) \geq \mu_B([\alpha, x]) \quad \dots\dots\dots (*_1). \end{aligned}$$

$$\begin{aligned} \text{Also, } (\nu_B^*)'([\alpha, x]) &= \sup_{s \in S} \nu_B^*(s\alpha x) = \sup_{s \in S} \sup_{\beta \in \Gamma} \nu_B([\beta, s\alpha x]) \\ &= \sup_{s \in S} \sup_{\beta \in \Gamma} \nu_B([\beta, s][\alpha, x]) \leq \nu_B([\alpha, x]) \quad \dots\dots\dots (**_1). \end{aligned}$$

Since $B = (\mu_B, \nu_B) \in IFPI(R)$, for all $\alpha, \beta \in \Gamma$, for all $x, s \in S$. $\mu_B([\alpha, x][\beta, s]) = \max\{\mu_B([\alpha, x]), \mu_B([\beta, s])\}$ and $\nu_B([\alpha, x][\beta, s]) = \min\{\nu_B([\alpha, x]), \nu_B([\beta, s])\} \forall s \in S, \forall \beta$

$\in \Gamma$. Hence for $s = x$ and $\beta = \alpha$ we obtain $\mu_B([\alpha, x][\beta, s]) = \mu_B([\alpha, x])$ and $\nu_B([\alpha, x][\beta, s]) = \nu_B([\alpha, x])$. This together with the relations $(\mu_B^*)'([\alpha, x]) = \inf_{s \in S} \inf_{\beta \in \Gamma} \mu_B([\alpha, x][\beta, s])$

and $(\nu_B^*)'([\alpha, x]) = \sup_{s \in S} \sup_{\beta \in \Gamma} \nu_B([\alpha, x][\beta, s])$ give $(\mu_B^*)'([\alpha, x]) \leq \mu_B([\alpha, x])$(*₂)

and $(\nu_B^*)'([\alpha, x]) \geq \nu_B([\alpha, x])$(**₂). By (*₁) and (*₂) we obtain $(\mu_B^*)'([\alpha, x]) = \mu_B([\alpha, x])$ and by (**₁) and (**₂) we have $(\nu_B^*)'([\alpha, x]) = \nu_B([\alpha, x])$. Consequently, $(B^*)' = B$. Hence the mapping is onto. Inclusion preserving property is similar as in Theorem 2.9. Hence $B \mapsto B'$ is an inclusion preserving bijection. □

REMARK 5. (i) Similar results hold for $IFSPI(S)$. (ii) Similar result holds for the Γ -semigroup S and the left operator semigroup L of S .

Corollary 2.19. Let S be a Γ -semigroup and R, L be respectively its right and left operator semigroups. Then there exists an inclusion preserving bijection between the set of all $IFPI(R)(IFSPI(R))$ and the set of all $IFPI(L)(IFSPI(L))$.

REMARK 6. In view of Theorem 2.18, we see that in a Γ -semigroup S with unities the above result also holds for IFI .

Now we revisit the following theorem which is originally due to Dutta and Adhikari[2] via intuitionistic fuzzy ideals by using Theorem 2.18 and applying similar argument as applied in Theorem 2.13.

Theorem 2.20. Let S be a Γ -semigroup. Then there exists an inclusion preserving bijection between the set of all $PI(S)(SPI(S))$ and that of its right operator semigroup R via the mapping $I \rightarrow I^*$.

The definition of an IFE ,⁶ $\langle x, A \rangle$ of an IFS , $A = (\mu_A, \nu_A)$ is given in [7]. Now by routine verification we obtain the following two propositions.

Proposition 2.21. Let S be a commutative Γ -semigroup and $L(R)$ the left(respectively the right) operator semigroups of S . Let $A = (\mu_A, \nu_A)$ be an $IFLI(S)(IFRI(S), IFI(S))$ then $\langle x, A^{+'} \rangle$ (respectively $\langle x, A^{*'} \rangle$) is an $IFLI(L(R))(IFRI(L(R)), IFI(L(R)))$ for all $x \in L(R)$.

Proposition 2.22. (With same notation as in the above proposition) If $B = (\mu_B, \nu_B)$ is an $IFLI(L(R))(IFRI(L(R)), IFI(L(R)))$ then $\langle x, B^+ \rangle$ (respectively $\langle x, B^* \rangle$) is an $IFLI(S)(IFRI(S), IFI(S))$ for all $x \in S$.

Now we deduce the following two lemmas on the relationships between a Γ -semigroup and its operator semigroups in terms of IFE .

Lemma 2.23. Let $A = (\mu_A, \nu_A)$ be an $IFS(S)$ where S is commutative. Then for all $x \in S$,

- (1) $\langle x, A \rangle^{*'} \subseteq \langle [\alpha, x], A^{*'} \rangle \forall \alpha \in \Gamma$.
- (2) $\langle x, A \rangle^{+'} = (\langle x, \mu_A \rangle^{*'}, \langle x, \nu_A \rangle^{*'})$
 $= (\inf_{\alpha \in \Gamma} \langle [\alpha, x], \mu_A^{*'} \rangle, \sup_{\alpha \in \Gamma} \langle [\alpha, x], \nu_A^{*'} \rangle)$.

Proof. (1) Let $[\beta, y] \in R$. Then $\langle x, \mu_A \rangle^{*'}([\beta, y]) = \inf_{s \in S} \langle x, \mu_A \rangle(s\beta y) = \inf_{s \in S} \inf_{\gamma \in \Gamma} \mu_A(x\gamma s\beta y) = \inf_{\gamma \in \Gamma} \inf_{s \in S} \mu_A(x\gamma s\beta y)$. Again $\langle [\alpha, x], \mu_A^{*'} \rangle([\beta, y]) = \mu_A^{*'}([\alpha, x][\beta, y]) =$

⁶ IFE denote an intuitionsitic fuzzy extension.

$$\mu_A^*([\alpha, x\beta y]) = \inf_{s \in S} \mu_A^*(s\alpha x\beta y) = \inf_{s \in S} \mu_A^*(x\alpha s\beta y) \text{ (using the commutativity of } S\text{)}.$$

Since $\inf_{\gamma \in \Gamma} \inf_{s \in S} \mu_A^*(x\gamma s\beta y) \leq \inf_{s \in S} \mu_A^*(x\alpha s\beta y)$, we obtain $\langle x, \mu_A \rangle^*([\beta, y]) \leq \langle [\alpha, x], \mu_A^* \rangle([\beta, y])$. By similar argument we can show that $\langle x, \nu_A \rangle^*([\beta, y]) \geq \langle [\alpha, x], \nu_A^* \rangle([\beta, y])$. Hence $\langle x, A \rangle^* \subseteq \langle [\alpha, x], A^* \rangle \forall \alpha \in \Gamma$.

(2) Let $[\beta, y] \in R$. Then $\inf_{\alpha \in \Gamma} \langle [\alpha, x], \mu_A^* \rangle([\beta, y]) = \inf_{\alpha \in \Gamma} \mu_A^*([\alpha, x][\beta, y]) = \inf_{\alpha \in \Gamma} \mu_A^*([\alpha, x\beta y]) = \inf_{\alpha \in \Gamma} \inf_{s \in S} \mu_A^*(s\alpha x\beta y) = \inf_{s \in S} \langle x, \mu_A \rangle(s\beta y) = \langle x, \mu_A \rangle^*([\beta, y])$. By applying similar argument we obtain $\sup_{\alpha \in \Gamma} \langle [\alpha, x], \nu_A^* \rangle([\beta, y]) = \langle x, \nu_A \rangle^*([\beta, y])$. Hence $\langle x, A \rangle^* = (\langle x, \mu_A \rangle^*, \langle x, \nu_A \rangle^*) = (\inf_{\alpha \in \Gamma} \langle [\alpha, x], \mu_A^* \rangle, \sup_{\alpha \in \Gamma} \langle [\alpha, x], \nu_A^* \rangle)$. \square

Lemma 2.24. *If $B = (\mu_B, \nu_B)$ is an IFS(R) then for all $x \in S$, $\langle [\beta, x], B \rangle^* \supseteq \langle x, B^* \rangle \forall \beta \in \Gamma$.*

Proof. Let $p \in S$. Then $\langle [\beta, x], \mu_B \rangle^*(p) = \inf_{\gamma \in \Gamma} \langle [\beta, x], \mu_B \rangle([\gamma, p]) = \inf_{\gamma \in \Gamma} \mu_B([\beta, x][\gamma, p]) = \inf_{\gamma \in \Gamma} \mu_B([\beta, x\gamma p])$. Again $\langle x, \mu_B^* \rangle(p) = \inf_{\gamma \in \Gamma} \mu_B^*(x\gamma p) = \inf_{\gamma \in \Gamma} \inf_{\beta \in \Gamma} \mu_B([\beta, x\gamma p]) = \inf_{\beta \in \Gamma} \inf_{\gamma \in \Gamma} \mu_B([\beta, x\gamma p])$. Since $\inf_{\gamma \in \Gamma} \mu_B([\beta, x\gamma p]) \geq \inf_{\beta \in \Gamma} \inf_{\gamma \in \Gamma} \mu_B([\beta, x\gamma p])$, we have $\langle [\beta, x], \mu_B \rangle^*(p) \geq \langle x, \mu_B^* \rangle(p)$. Similarly we can show that $\langle [\beta, x], \nu_B \rangle^*(p) \leq \langle x, \nu_B^* \rangle(p)$. Hence $\langle [\beta, x], B \rangle^* \supseteq \langle x, B^* \rangle \forall \beta \in \Gamma$. \square

Lemma 2.25. *Let I be an $I(S)$. Then $((\mu_I)^*, (\nu_I)^*) = (\mu_{I^*}, \nu_{I^*})$.*

Proof. Let $[\beta, y] \in R$. Then $(\mu_I)^*([\beta, y]) = \inf_{s \in S} \mu_I(s\beta y)$ and $(\nu_I)^*([\beta, y]) = \sup_{s \in S} \nu_I(s\beta y)$. Suppose $[\beta, y] \in I^*$. Then $s\beta y \in I$ for all $s \in S$. Hence $\mu_I(s\beta y) = 1$ and $\nu_I(s\beta y) = 0$ for all $s \in S$ which implies that $\inf_{s \in S} \mu_I(s\beta y) = 1$ and $\sup_{s \in S} \nu_I(s\beta y) = 0$ whence $(\mu_I)^*([\beta, y]) = 1$ and $(\nu_I)^*([\beta, y]) = 0$. Also $\mu_{I^*}([\beta, y]) = 1$ and $\nu_{I^*}([\beta, y]) = 0$. If $[\beta, y] \notin I^*$ then $(\mu_{I^*})([\beta, y]) = 0$, $(\nu_{I^*})([\beta, y]) = 1$ and there exists $s \in S$ such that $s\beta y \notin I$. Hence $\mu_I(s\beta y) = 0$ and $\nu_I(s\beta y) = 1$ whence $(\mu_I)^*([\beta, y]) = 0$ and $(\nu_I)^*([\beta, y]) = 1$. Consequently, $((\mu_I)^*, (\nu_I)^*) = (\mu_{I^*}, \nu_{I^*})$. \square

Lemma 2.26. [4] *Let $\{A_\alpha\}_{\alpha \in I}$ be a family of $I(S)$. Then $(\bigcap_{\alpha \in I} A_\alpha)^* = \bigcap_{\alpha \in I} (A_\alpha)^*$.*

Lemma 2.27. *Let S be a Γ -semigroup, R be its right operator semigroup and $\{A_i\}_{i \in I} = (\mu_{A_i}, \nu_{A_i})_{i \in I}$ be a family of IFS(S) such that $A = (\mu_A, \nu_A) := \inf_{i \in I}$*

$A_i = (\inf_{i \in I} \mu_{A_i}, \sup_{i \in I} \nu_{A_i})$. Then $A^{*'} := ((\mu_A)^{*'}, (\nu_A)^{*'}) = \inf_{i \in I} (A_i)^{*'}$ = $(\inf_{i \in I} (\mu_{A_i})^{*'}, \sup_{i \in I} (\nu_{A_i})^{*'})$.

Proof. Let $[\alpha, x] \in R$. Then $(\mu_A)^{*'([\alpha, x])} = (\inf_{i \in I} \mu_{A_i})^{*'([\alpha, x])} = \inf_{s \in S} \inf_{i \in I} \mu_{A_i}(s\alpha x) = \inf_{i \in I} \inf_{s \in S} \mu_{A_i}(s\alpha x) = \inf_{i \in I} (\mu_{A_i})^{*'([\alpha, x])}$. Similarly we can show that $(\nu_A)^{*'([\alpha, x])} = \sup_{i \in I} (\nu_{A_i})^{*'([\alpha, x])}$. Hence $A^{*'} = \inf_{i \in I} (A_i)^{*'}$. □

Proposition 2.28. *Let S be a commutative Γ -semigroup with unities and $A = (\mu_A, \nu_A)$ be an $IFI(S)$. Then $\langle x, A \rangle := (\langle x, \mu_A \rangle, \langle x, \nu_A \rangle)$ is an $IFI(S)$ for all $x \in S$.*

Proof. Let R be the right operator semigroup of a Γ -semigroup S . Since S is commutative, R is also commutative. By Proposition 2.8, $A^{*'} = (\mu_{A'}^{*'}, \nu_{A'}^{*'})$ is an $IFI(R)$. Let $x \in S$. Then for any $\alpha \in \Gamma$, $\langle [\alpha, x], A^{*'} \rangle = (\langle [\alpha, x], \mu_{A'}^{*' *} \rangle, \langle [\alpha, x], \nu_{A'}^{*' *} \rangle)$ is an $IFI(R)$ (cf. Proposition 3.2[6]) and hence $(\inf_{\alpha \in \Gamma} \langle [\alpha, x], \mu_{A'}^{*' *} \rangle, \sup_{\alpha \in \Gamma} \langle [\alpha, x], \nu_{A'}^{*' *} \rangle)$ is an $IFI(R)$. So by Lemma 2.23(2), $(\langle x, \mu_A \rangle^{*' *} , \langle x, \nu_A \rangle^{*' *}) = \langle x, A \rangle^{*' *}$ is an $IFI(R)$. Consequently, $(\langle x, A \rangle^{*' *})^*$ is an $IFI(S)$ (cf. Proposition 2.7). Hence $\langle x, A \rangle$ is an $IFI(S)$ (cf. Theorem 2.10). □

Now by applying Proposition 2.17, Remark 5(i) we obtain the following proposition.

Proposition 2.29. *Let S be a commutative Γ -semigroup and $A = (\mu_A, \nu_A)$ be an $IFSPI(S)$. Then $\langle x, A \rangle = (\langle x, \mu_A \rangle, \langle x, \nu_A \rangle)$ is an $IFSPI(S)$ for all $x \in S$.*

Proposition 2.30. *Let S be a Γ -semigroup, $\{A_i\}_{i \in I} = (\mu_{A_i}, \nu_{A_i})_{i \in I}$ be a non-empty family of $IFSPI(S)$ and let $A = (\mu_A, \nu_A) = \inf_{i \in I} A_i = (\inf_{i \in I} \mu_{A_i}, \sup_{i \in I} \nu_{A_i})$. Then for any $x \in S$, $\langle x, A \rangle$ is an $IFSPI(S)$.*

Proof. Let R be the right operator semigroup of S . Since S is commutative, R is also commutative. Now $\{A_i^{*'}\}_{i \in I} = (\mu_{A_i}^{*' *} , \nu_{A_i}^{*' *})_{i \in I}$ is a non-empty family of $IFSPI(R)$ (cf. Proposition 2.17). Hence $\inf_{i \in I} A_i^{*' *}$ is an $IFSPI(R)$. Thus by Lemma 2.27, $A^{*' *}$ is an $IFSPI(R)$. Then for any $[\alpha, x] \in R$, $\langle [\alpha, x], A^{*' *} \rangle = (\langle [\alpha, x], \mu_{A'}^{*' *} \rangle, \langle [\alpha, x], \nu_{A'}^{*' *} \rangle)$ is an $IFSPI(R)$ (cf. Proposition 2.11[6]) and hence $(\inf_{\alpha \in \Gamma} \langle [\alpha, x], \mu_{A'}^{*' *} \rangle, \sup_{\alpha \in \Gamma} \langle [\alpha, x], \nu_{A'}^{*' *} \rangle)$ is an $IFSPI(R)$. So by Lemma

2.23(2), $(\langle x, \mu_A \rangle^{*'}, \langle x, \nu_A \rangle^{*'}) = \langle x, A \rangle^{*'}$ is an $IFSPI(R)$. Consequently, $(\langle x, A \rangle^{*'})^*$ is an $IFSPI(S)$ (cf. Proposition 2.16). Hence $\langle x, A \rangle$ is an $IFSPI(S)$ (cf. Proposition 2.17 and Remark 5(i)) \square

Theorem 2.31. *Let S be a commutative Γ -semigroup, $\{S_i\}_i$ be a non-empty family of $SPI(S)$, $A := \bigcap_{i \in I} S_i \neq \phi$ and $M = (\mu_A, \mu_A^c)$. Then $\langle x, M \rangle$ is an $IFSPI(S)$ for all $x \in S$.*

Proof. Since $\forall i \in I, S_i$ is a $SPI(S)$, $S_i^{*'}$ is a $SPI(R)$, $\forall i \in I$. Now $A := \bigcap_{i \in I} S_i$. Then $A^{*'}$ $= (\bigcap_{i \in I} S_i)^{*'}$ $= \bigcap_{i \in I} S_i^{*'}$ (cf. Lemma 2.27) $\neq \phi$. So by Corollary 3.12[6], $\langle [\alpha, x], M \rangle = (\langle [\alpha, x], \mu_A \rangle, \langle [\alpha, x], \mu_A^c \rangle)$ is an $IFSPI(R)$, $\forall \alpha \in \Gamma$. This implies that $(\inf_{\alpha \in \Gamma} \langle [\alpha, x], \mu_A \rangle, \sup_{\alpha \in \Gamma} \langle [\alpha, x], \mu_A^c \rangle)$ is an $IFSPI(R)$, i.e., $(\langle [\alpha, x], \mu_A \rangle^{*'}, \langle [\alpha, x], \mu_A^c \rangle^{*'}) = \langle x, M \rangle^{*'}$ is an $IFSPI(R)$. Hence $(\langle x, M \rangle^{*'})^*$ is an $IFSPI(S)$. Consequently, by Theorem 2.18, $\langle x, M \rangle$ is an $IFSPI(S)$. \square

To conclude the paper we obtain the following characterization of a prime ideal of a Γ -semigroup S in terms of intuitionistic fuzzy ideal extension.

Theorem 2.32. *Let S be a Γ -semigroup, P be an $I(S)$ and $M = (\mu_P, \mu_P^c)$ where μ_P is the characteristic function of P . Then P is $PI(S)$ if and only if for $x \in S$ with $x \notin P$, $\langle x, M \rangle = M$.*

Proof. Let P be a $PI(S)$ and $x \notin P$. Then $P^{*'}$ is a $PI(R)$ (cf. Proposition 2.15). Also $[\alpha, x] \notin P^{*'}$ for some $\alpha \in \Gamma$. Then by Corollary 3.15[6], $\langle [\alpha, x], M^{*'} \rangle = M^{*'}$ i.e., $(\langle [\alpha, x], \mu_P^{*' \prime} \rangle, \langle [\alpha, x], (\mu_P^c)^{*' \prime} \rangle) = ((\mu_P)^{*' \prime}, (\mu_P^c)^{*' \prime}) = M^{*'}$. Now by Lemma 2.12, $(\langle [\alpha, x], \mu_P^{*' \prime} \rangle, \langle [\alpha, x], (\mu_P^c)^{*' \prime} \rangle) = (\mu_P, \mu_P^c)^{*'}$. Hence $(\langle [\alpha, x], M^{*' \prime} \rangle)^* = (\langle [\alpha, x], \mu_P^{*' \prime} \rangle, \langle [\alpha, x], (\mu_P^c)^{*' \prime} \rangle)^* = ((\mu_P, \mu_P^c)^{*' \prime})^* = (\mu_P, \mu_P^c) = M$. By Lemma 2.23(1), $\langle x, M \rangle^{*' \prime} \subseteq \langle [\alpha, x], M^{*' \prime} \rangle \forall \alpha \in \Gamma$. So $(\langle x, M \rangle^{*' \prime})^* \subseteq M$. Consequently by Theorem 2.10, $\langle x, M \rangle \subseteq M$. Again by Proposition 5.5(1)[7], we have $M \subseteq \langle x, M \rangle$. Hence $\langle x, M \rangle = M$.

Conversely, suppose $\langle z, M \rangle = M$ for all $z \in S$ with $z \notin P$. Let $x\Gamma y \subseteq P$ where $x, y \in S$. Then $\mu_P(x\gamma y) = 1$ and $\mu_P^c(x\gamma y) = 0 \forall \gamma \in \Gamma$. Let $x \notin P$. Then by hypothesis $\langle x, M \rangle = M$. This gives $\langle x, \mu_P \rangle(y) = \mu_P(y)$ and $\langle x, \mu_P^c \rangle(y) = \mu_P^c(y)$. Then $\inf_{\gamma \in \Gamma} \mu_P(x\gamma y) = \mu_P(y)$ and $\sup_{\gamma \in \Gamma} \mu_P^c(x\gamma y) = \mu_P^c(y)$ which implies that $\mu_P(y) = 1$ and $\mu_P^c(y) = 0$ whence $y \in P$. Hence P is a $PI(S)$. \square

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