International Journal of Algebra, Vol. 5, 2011, no. 6, 295 - 299

Solving Fuzzy Nonlinear Equation with Secand Method

Seied Mahmoud Khorasani

Department of Mathematics,Islamic Azad University
Shahr-E-Rey Branch, Tehran, Iran

M. H. Dogani Aghcheghloo

Pure math Department of the shahid Bahonar
University of Kerman, Kerman, Iran

Abstract

In this paper we introduce an algebraic fuzzy equation of degree n with fuzzy coefficients and crisp variable, and we present an iterative method to find the real roots of such equations, numerically. We present an algorithm to generate a sequence that can be converged to the root of an algebraic fuzzy equation.

Keywords: Fuzzy number, Triangular fuzzy number, Secant Method

1 Introduction

Since 1965 where Zadeh [3] presented fuzzy logic, up to now, this logic is applicable in many fields of sciences. Sometimes outcome of a system depends on the roots of a fuzzy equation. Some fuzzy equations were checked in [6].

There are some works on fuzzy equations in [5]. A recent work has been done on fuzzy linear systems by Muzzioli and Reynaerts [4]. All these methods compute the roots of an algebraic fuzzy equation analytically, but there are not any analytically solution for algebraic fuzzy equations with degree greater than 3.

In this paper we want to solve the algebraic fuzzy equation of degree n, with fuzzy coefficients and crisp variable, numerically.

The structure of this paper is as follows. In Section 2 we introduce an algebraic fuzzy polynomial of degree n, with fuzzy coefficients and crisp variables and some needed concepts to solve it. In Section 3 an algorithm is presented to solve an algebraic equation of degree n, numerically. In Section 4 we introduce an algebraic fuzzy equation and numerically algorithm.There is an examples in this Section .

2 Preliminary Notes

Definition 2.1

A fuzzy number is a fuzzy set like $u : R \to I = [0,1]$which satisfies:
1. $\tilde{u}$ is upper semi continuous.
2. $\tilde{u}(x) = 0$ outside some interval [c,d]
3. there are real number $a, b : c \leq a \leq b \leq d$ for which:
1.1 $\tilde{u}(x)$ is monotonic increasing on [c,a].
1.2 $\tilde{u}(x)$ is monotonic decreasing on [b,d].
1.3 $\tilde{u}(x) = 1, a \leq x \leq b$

All of the entire fuzzy number (as given definition 2.1) if denoted by$\mathbf{F}(R)$.Another definition of fuzzy number defined as follow,

Definition 2.2

A fuzzy number $\tilde{u}$ is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r), \overline{u}(r); 0 \leq r \leq 1$ which satisfy the following requirements:
1. $\underline{u}(r)$ is monotonically increasing left continuous function .
2. $\overline{u}(r)$ is monotonically decreasing left continuous function.
3. $\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

A popular fuzzy number is trapezoidal fuzzy number with tolerance interval [a,b], left width α and right width β if its membership function has the following form and we use the notation:

$$\tilde{u} = (a, b, \alpha, \beta)$$

Its parametric form :

$$\underline{u}(r) = a - (1-r)\alpha \quad , \quad \overline{u}(r) = b + (1-r)\beta$$

If $a = b$ then trapezoidal transform to triangular fuzzy number and we denote all of the triangular fuzzy number with$\mathbf{FT}(R)$.

Definition 2.3

Let $\tilde{v} = (\underline{v}(r), \overline{v}(r))$, $\tilde{u} = (\underline{u}(r), \overline{u}(r))$. Some results of applying fuzzy arithmetics on fuzzy numbers $\tilde{v}$, $\tilde{u}$ are as follows:[3]

- $x > 0 : x = (x\underline{v}(r), x\overline{v}(r));$
- $x < 0 : x = (x\overline{v}(r), x\underline{v}(r))$
- $\tilde{v} + \tilde{u} = (\underline{v}(r) + \underline{u}(r), \overline{v}(r) + \overline{u}(r))$

- $\tilde{v} - \tilde{u} = (\underline{v}(r) - \overline{u}(r), \overline{v}(r) - \underline{u}(r))$

Definition 2.4

$\tilde{P}_n(x)$ is a fuzzy polynomial of degree at most n , if there are some fuzzy numbers $\tilde{a}_0, ..., \tilde{a}_n$ such that

$$\tilde{P}_n(x) = \sum_{j=0}^{n} \tilde{a}_j x^j \tag{1}$$

3 functional iteration

Let $f(x) = 0$ be a continuous real-valued function with as many derivatives. Our approach here is to derive iterative methods for the solution of $f(x) = 0$ by using inverse interpolation.

3.1 The linear secant method

Our approach in this section will be to use linear inverse interpolation to derive method. Suppose we can find two points x_1 , x_2 such that $f(x_1)f(x_2) < 0$ then we have:

$$x_{i+1} = \frac{y_i}{y_i - y_{i-1}} x_{i-1} + \frac{y_{i-1}}{y_{i-1} - y_i} x_i \quad , i = 2, 3, ... \tag{2}$$

where y_i 's is value of $f(x)$ in the x_i.

4 fuzzy equation

In this section we definition fuzzy equation that we want solving with Secant Method.

Definition 4.1

Fuzzy equation defined as follow:

$$\tilde{P}_n(x) = \tilde{a} \tag{3}$$

that $\tilde{a}$ have same LR as $\tilde{a}_j$ in $\tilde{P}_n(x)$

If we denote $\tilde{a}_j = (\underline{a}_j \ , \ \overline{a}_j)$ and $\tilde{a} = (\underline{a} \ , \ \overline{a})$ where

$$\underline{a}_j = \sum_{i=0}^{n} b_{ij} r^i \ , \ \overline{a}_j = \sum_{i=0}^{n} c_{ij} r^i \ , \underline{a} = \sum_{i=0}^{n} h_i r^i \ , \ \overline{a} = \sum_{i=0}^{n} k_i r^i \tag{4}$$

and x is positive number then we will have:

$$(\sum_{j=0}^{n} \underline{a}_j x^j \ , \ \sum_{j=0}^{n} \overline{a}_j x^j) = (\underline{a} \ , \ \overline{a}) \tag{5}$$

then:

$$\sum_{j=0}^{n} \underline{a}_j x^j = \underline{a} \quad , \quad \sum_{j=0}^{n} \overline{a}_j x^j = \overline{a} \tag{6}$$

where by replacing (4) into (6) then:

$$\sum_{j=0}^{n} b_{ij} x^j = h_i \quad , \quad \sum_{j=0}^{n} c_{ij} x^j = k_i \quad i = 0, 1, 2, ..., n \tag{7}$$

now we solving this equation with Secant method.

Theorem 4.2

If solutions of the equations (7) were be equal together then the equation (3) have solution.
Proof:It is obvious.

Example 4.3

In the follow fuzzy equation the exact solution is $x^* = 1$ then with Secant Method will have:[2]

$(r, 3 - 2r)x^3 + 2(r, 2 - r)x^2 + (r + 4, 7 - 2r)x + (6r - 14, -4 - 14r) = (10r - 10, 10 - 10r)$

with $x_0 = 0.9$, $x_1 = 1.05$ then we have:

$x_2 = 0.996599$, $x_3 = 0.999888$, $x_4 = 1.00000$

References

[1] .J. Zimmermann,Fuzzy Set Theory and its Applications, third ed., Kluwer Academic, Norwell, MA, 1996

[2] M.Amirfakhrian, Numerical solution of algebraic fuzzy equations with crisp variable by Gauss-Newton method ,App.Math.Modelling,32(2008)1859-1868

[3] L.A. Zadeh, Fuzzy sets, Inform. and Control 8 (1965) 338353.

[4] S. Muzzioli, H. Reynaerts, Fuzzy linear systems of the form $A_1 x + b_1 = A_2 x + b_2$, Fuzzy Sets and Systems 157 (7) (2006) 939951.

[5] J.J. Buckley, E. Eslami, T. Feuring, Fuzzy mathematics in economics and engineering, Studies in fuzziness and soft computing, Phisica-Verlag, 2002.

[6] J.J. Buckley, Yunxia Qu, Solving linear and quadratic fuzzy equations, Fuzzy Sets and Systems 38 (1990) 4359.

Received: November, 2010