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αgs Closed Sets in BiČech Closure Spaces

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Abstract

In this paper, we introduce the concepts of α - generalised semi closed (resp. open) sets in biČech closure space and some characterizations and properties are investigated.

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1 Introduction

Čech closure spaces were introduced by Čech [1]. In Čech's approach the operator satisfies idempotent condition among Kuratowski axioms. This condition need not hold for every set A of X. When this condition is also true, the operator becomes topological closure operator. Thus the concept of closure space is the generalisation of a topological space. Closure functions that are more general than the topological ones have been studied already by Day [6]. A thorough discussion on closure functions is due to Hammer, see eg [10, 11] and more recently Gnilka[8, 9].

Definition 1.1 *[3] Two functions k_1 and k_2 from power set X to itself are called biČech closure operators (simply biclosure operator) for X if they satisfies the following properties:*

i. $k_1(\phi) = \phi$ *and* $k_2(\phi) = \phi$

ii. $A \subset k_1(A)$ *and* $A \subset k_2(A)$ *for any set* $A \subset X$

iii. $k_1(A \cup B) = k_1(A) \cup k_1(B)$ *and* $k_2(A \cup B) = k_2(A) \cup k_2(B)$ *for any* $A, B \subset X$

(X, k_1, k_2) *is called biČech closure space.*

Example 1.2 *Let* $X = \{a, b, c\}$ *and let* k_1 *and* k_2 *be defined as:*

$$
\begin{array}{lcl}
k_1(\{a\}) & = & \{a\} \\
k_1(\{b\}) & = & k_1(\{c\}) \\
 & = & k_1(\{b,c\}) \\
 & = & \{b,c\} \\
k_1(\{a,b\}) & = & k_1(\{a,c\}) \\
 & = & k_1(\{X\}) \\
 & = & X \\
k_1(\phi) & = & \phi
\end{array}
\qquad
\begin{array}{lcl}
k_2(\{a\}) & = & \{a\} \\
k_2(\{b\}) & = & \{b,c\} \\
k_2(\{c\}) & = & k_2(\{a,c\}) \\
 & = & \{a,c\} \\
k_2(\{a,b\}) & = & k_2(\{b,c\}) \\
 & = & k_2(\{X\}) \\
 & = & X \\
k_2(\phi) & = & \phi
\end{array}
$$

Now, (X, k_1, k_2) *is biČech closure space.*

Definition 1.3 *[4] A subset A in a biČech closure space* (X, k_1, k_2) *is said to be*

i. k_i *- regular open if* $A = int_{k_i}(k_i(A))$, $i = 1, 2$

ii. k_i *- regular closed if* $A = k_i\,[int_{k_i}(A)]$, $i = 1, 2$

iii. k_i *- semi open if* $A \subseteq k_i\,[int_{k_i}(A)]$

iv. k_i *- preopen if* $A \subseteq int_{k_i}(k_i(A))$, $i = 1, 2$

v. k_i *- preclosed if* $k_i[int_{k_i}(A)] \subseteq A$, $i = 1, 2$

vi. $k_i\alpha$ *- open if* $A \subseteq int_{k_i}(k_i[int_{k_i}(A)])$, $i = 1, 2$

vii. $k_i\alpha$ *- closed if* $k_i[int_{k_i}(k_i(A))] \subseteq A$, $i = 1, 2$

2 (k_1, k_2) - αgs closed sets

Definition 2.1 *A subset A in a biČech closure space (X, k_1, k_2) is said to be (k_1, k_2) - αgs closed if $k_{\alpha_2}(A) \subseteq U$, whenever $A \subseteq U$ and U is k_1 - semi open set in X.*

Example 2.2 *In example 1.2 when $U = \{b, c\}$, $A = \{b\}$ is (k_1, k_2) - αgs closed.*

Result 2.3 *If A and B are (k_1, k_2) - αgs closed sets and so is $A \cup B$.*

Proof. Let A and B be two (k_1, k_2) - αgs closed sets.
Let U be k_1 - semi open set in X. Let $(A \cup B) \subseteq U$.
Since $(A \cup B) \subseteq U$, we have $A \subseteq U$ and $B \subseteq U$.
Then $K_{\alpha_2}(A) \subseteq U$ and $k_{\alpha_2}(B) \subseteq U$ implies $k_{\alpha_2}(A) \cup k_{\alpha_2}(B) \subseteq U$.
Hence $k_{\alpha_2}(A \cup B) \subseteq U$.
Thus $A \cup B$ is (k_1, k_2) - αgs closed set. □

Result 2.4 *If A is (k_1, k_2) - αgs closed set, then $k_{\alpha_2}(A) - A$ contains no non-empty k_1 - semi closed sets.*

Proof. Let A be (k_1, k_2) - αgs closed. Let U be k_1 - semi closed contained in $k_{\alpha_2}(A) - A$. Now,

$$U \subseteq k_{\alpha_2}(A) \text{ and } U \subseteq A^c. \tag{2.1}$$

Now,

$$U \subseteq A^c \text{ then } A \subseteq U^c.$$

Since U is k_1 - semi closed, U^c is k_1 - semi open. Thus we have,

$$k_{\alpha_2}(A) \subseteq U^c.$$

Consequently,

$$U \subseteq [k_{\alpha_2}(A)]^c. \tag{2.2}$$

From (2.1) and (2.2),

$$U \subseteq k_{\alpha_2}(A) \cap [k_{\alpha_2}(A)]^c = \phi.$$

Therefore, $U = \phi$. Hence $k_{\alpha_2}(A) - A$ contains no non-empty k_1 - semi closed sets. □

Result 2.5 *If A is (k_1, k_2) - αgs closed set, then $k_{\alpha_1}(x) \cap A \neq \phi$ holds for each $x \in k_{\alpha_2}(A)$.*

Proof. Let A be (k_1, k_2) - αgs closed set. Suppose $k_{\alpha_1}(x) \cap A = \phi$, for some $x \in k_{\alpha_2}(A)$, we have

$$A \subseteq [k_{\alpha_1}(x)]^c .$$

Now, $k_{\alpha_1}(x)$ is k_1 - α closed. Therefore $[k_{\alpha_1}(x)]^c$ is k_1 - α open.
Thus $[k_{\alpha_1}(x)]^c$ is k_1 - semi open.
Since A is (k_1, k_2) - αgs closed set, we have

$$k_{\alpha_2}(A) \subseteq [k_{\alpha_1}(x)]^c$$

implies $k_{\alpha_2}(A) \cap k_{\alpha_1}(x) = \phi$.

Then $x \notin k_{\alpha_2}(A)$ is a contradiction.

Hence $k_{\alpha_1}(x) \cap A \neq \phi$ holds for each $x \in k_{\alpha_2}(A)$. □

Result 2.6 *Let (X, k_1, k_2) be biČech closure space. For each x in X, $\{x\}$ is k_1 - semi closed or $\{x\}^c$ is (k_1, k_2) - αgs closed set.*

Proof. Let (X, k_1, k_2) be biČech closure space. Suppose that $\{x\}$ is not k_1 - semi closed set, $\{x\}^c$ is not k_1 - semi open.
Therefore, the only k_1 - semi open set containing $\{x\}^c$ is X.
Thus, $\{x^c\} \subset X$.
Now, $k_{\alpha_2}[(x)^c] \subseteq k_{\alpha_2}(X) = X$.
Hence $\{x\}^c$ is (k_1, k_2) - αgs closed set. □

Result 2.7 *Let A be (k_1, k_2) - αgs closed set and if A is k_1 - semi open then $A = k_{\alpha_2}(A)$.*

Proof. Let A be (k_1, k_2) - αgs closed subset of a biČech closure space (X, k_1, k_2) and let A be k_1 - semi open set. Then

$$k_{\alpha_2}(A) \subseteq U$$

whenever $A \subseteq U$ and U is k_1 - semi open set in X.

Since A is k_1 - semi open and $A \subseteq A$, we have

$$k_{\alpha_2}(A) \subseteq A.$$

But always,

$$A \subseteq k_{\alpha_2}(A).$$

Thus, $A = k_{\alpha_2}(A)$. □

Result 2.8 *Let $A \subseteq Y \subseteq X$ and suppose that A is (k_1, k_2) - αgs closed in (X, k_1, k_2). Then A is (k_1, k_2) - αgs closed relative to Y.*

Proof. Let S be any k_1 - semi open set in Y such that $A \subseteq S$.
Then $S = U \cap Y$ for some U is k_1 - semi open set in X.
Therefore $A \subset U \cap Y$ implies $A \subseteq U$.
Since A is (k_1, k_2) - αgs closed set in X, we have $k_{\alpha_2}(A) \subseteq U$.
Hence $Y \cap k_{\alpha_2}(A) \subseteq Y \cap U = S$.
Thus A is αgs closed set relative to Y. □

3 αgs open sets

Definition 3.1 *A subset A in biČech closure space (X, k_1, k_2) is called (k_1, k_2) - αgs open set if A^c is (k_1, k_2) - αgs closed in (X, k_1, k_2).*

Result 3.2 *A subset A of (X, k_1, k_2) is (k_1, k_2) - αgs open set if and only if $F \subset (int_{k_2}(A))$ whenever F is k_1 - semi closed set and $F \subseteq A$.*

Proof. Suppose A is (k_1, k_2) - αgs open in (X, k_1, k_2). Let F be k_1 - semi closed set and $F \subseteq A$. Then F^c is k_1 - semi open set and $A^c \subseteq F^c$. Since A^c is (k_1, k_2) - αgs closed set, we have

$$k_{\alpha_2}(A^c) \subseteq F^c.$$

Implies

$$F \subseteq [k_{\alpha_2}(A^c)]^c = int_{k_{\alpha_2}}(A).$$

That is $F \subseteq int_{k_{\alpha_2}}(A)$ whenever F is k_1 - semi closed set and $F \subseteq A$.
Let V be any k_1 - semi open set in X such that $A^c \subseteq V$.
Thus $V^c \subseteq A$ and V^c is k_1 - semi closed.

Therefore,

$$V^c \subseteq int_{k_{\alpha_2}}(A).$$

Then

$$\left[int_{k_{\alpha_2}}(A)\right]^c \subseteq V.$$

Implies

$$k_{\alpha_2}(A^c) \subseteq V$$

gives A^c is (k_1, k_2) - αgs closed set.

Thus A is (k_1, k_2) - αgs open set. □

Corollary 3.3 *A subset A of (X, k_1, k_2) is (k_1, k_2) - αgs closed set, then $k_{\alpha_2}(A) - A$ is (k_1, k_2) - αgs open set.*

Proof. Let F be k_1 - semi closed set such that $F \subseteq k_{\alpha_2}(A) - A$.

$F = \phi$ (by Result 2.7)

Therefore $F \subseteq int_{\alpha_2}\{k_{\alpha_2}(A) - A\}$

$k_{\alpha_2}(A) - A$ is (k_1, k_2) - αgs open set. □

Result 3.4 *If A and B be (k_1, k_2) - αgs open sets, then so is $A \cap B$.*

Proof. Let $A^c \cup B^c \subseteq U$ where U is k_1 - semi open. This implies $A^c \subseteq U$ and $B^c \subseteq U$, gives $k_{\alpha_2}(A^c) \subseteq U$ and $k_{\alpha_2}(B^c) \subseteq U$. Thus $k_{\alpha_2}(A^c) \cup k_{\alpha_2}(B^c) \subseteq U$.

Thus $k_{\alpha_2}(A^c \cup B^c) \subseteq U$.

Therefore $A \cap B$ is (k_1, k_2) - αgs open set. □

Result 3.5 *Let $A \subseteq Y \subseteq X$ and suppose that Y is k_2 - α closed in X and A is (k_1, k_2) - αgs open set in X, then A is (k_1, k_2) - αgs open set relative to Y.*

Proof. It is analogous to the proof of result (2.8). □

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