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Ranking Functions and its Application to Fuzzy DEA

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Abstract

Data envelopment analysis is widely applied approach for measuring the relative efficiencies of a set of decision making units(DMUs) using various inputs to produce various output are limited to crisp data.In this paper, the focus will be on the nonradial model because the nonradial model is best efficiency estimator for DEA model.However, in real-world problems inputs and outputs are often imprecise.This paper develops DEA models using imprecise data represented by fuzzy sets.By use of a ranking function we introduce the approach to solving the fuzzy nonradial model.

Mathematics Subject Classification: Operations Research, No. 90

Keywords: fuzzy linear programming problem, Data envelopment analysis DEA, fuzzy ranking functions

1 Introduction

One approach for measuring the relative efficiencies of decision making units (DMUs) which consume multiple inputs to produce multiple outputs is data envelopment analysis (DEA) developed by Charnes et al.(1978,1979).This approach is essentially the counterpart of the pare to optimality in economics which states that a DMU is said to reduce the consumption of one input, another input will necessarily be raised in producing the same amount of output, or when it tries to raise the production of one output, another output will necessarily be reduced in consuming the same amount of input (see, for example, Ferguson and Gould,1975). To deal quantitatively with impression in decision process, Bellman and Zadeh (1970) introduce the notion of fuzziness.In the conventional DEA approach a set of weighs which satisfies a set of constraints is selected to give the highest possible efficiency measure for each DMU. when some observations are fuzzy the goal and constraints in the decision process become fuzzy as well. since the DEA model is essentially a linear program, one straightforward idea is to apply the existing fuzzy linear programming (FLP) to the fuzzy DEA problems. In this paper we use a new method for fuzzy linear programming to treat fuzzy nonradial models.

2 preliminaries

We review the fundamental notions of fuzzy set theory, initiated by Bellman and Zadeh [1], to be used throughout this note. Below, we give definitions and notations taken from Bezdek [2].

Definition 2.1. Let X be the universal set $\tilde{A}$ is called a fuzzy set in X if $\tilde{A}$ is a set of ordered pairs

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x))|x \in X\},$$

where $\mu_{\tilde{A}}(x)$ is the membership value of x in $\tilde{A}$.

Definition 2.2. A fuzzy set $\tilde{A}$ is convex if

$$\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}, \quad \forall x, y \in X \text{ and } \lambda \in [0,1].$$

Definition 2.3. A convex fuzzy set $\tilde{A}$ on $\Re$ is a fuzzy number if the following conditions hold:

(a) Its membership function is piecewise continuous.
(b) There exist only x_0 that $\mu_A(x_0) = 1$.

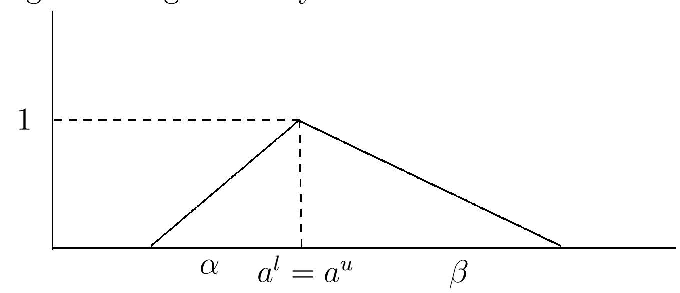
Definition 2.4. (L-R fuzzy number) A fuzzy number $\tilde{A}$ is of L-R type if there exit reference function L (L for left), (R for right) and scalers $\alpha > 0$, $\beta > 0$ with

$$\mu_{\tilde{A}}(x) = \begin{cases} L(\frac{m-x}{\alpha}) & x \leq m \\ R(\frac{x-m}{\beta}) & x > m \end{cases}$$

m, called the mien value of $\tilde{A}$, is real number, and α, β are called the left and right of $\tilde{A}$, is real number, and α, β are called the left and right expanse respectively, $\tilde{A}$ is denoted by $(m, \alpha, \beta)_{LR}$.

When $L(x) = R(x) = \begin{cases} 1-x & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$ then we have triangular fuzzy number.

Fig 1. Triangular fuzzy number.



compatibility

Definition 2.5. The extension principle: [13]. One of the most basic concepts of fuzzy set theory that can be used generalize crisp mathematical concepts to fuzzy set is the extension principle. In its elementary form, it was already implied in Zadeh's first contribution [14]. Zadeh and Dubois and Prade [4], we define the extension principle as follows:

Let X be a cartesian product of universes $X = X_1 \times \cdots \times X_r$, and $\tilde{A}_1, \ldots, \tilde{A}_r$ be r fuzzy sets in $X_1, \ldots, X_r$, respectively, f is mapping form X to universe Y, $Y = f(x_1, \ldots, x_r)$. Then the extension principle allows us to define a fuzzy set $\tilde{B}$ in Y by $\tilde{B} = \{(Y, M_{\tilde{B}}(Y)) | Y = f(x_1, \ldots, x_r), (x_1, \ldots, x_r) \in X\}$ where

$$\mu_{\tilde{B}}(y) = \begin{cases} \sup_{(x_1,\ldots,x_n)\in f^{-1}(y)} & \min\{\mu_{\tilde{A}_1}(x_1), \ldots, \mu_{\tilde{A}_r}(x_r) iff^{-1}(u) \neq \emptyset\} \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

where f^{-1} is the inverse of f. for $r = 1$, the extension principle, of course, reduces to

$$\tilde{B} = f(\tilde{A}) = \{(Y, \mu_{\tilde{B}}(Y)) | Y = f(x), x \in X\}$$

where

$$\mu_{\tilde{B}}(Y) = \begin{cases} \sup_{x\in f^{-1}(Y)} \mu_{\tilde{A}}(x), & iff^{-1}(Y) \neq \theta \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

3 Ranking Functions and Its Application to Fuzzy Number Linear Programming

3.1 Ranking Functions

Several methods for solving fuzzy linear programing problems can be Fang (1999),Lai and Hwang (1992),Maleki et al (2000).one of the most convenient of these methods is based on the concept of comparison of fuzzy numbers by use of ranking functions.In fact,an efficient approach for ordering the elements of $F(\Re)$ is to define a ranking function$\mathcal{M} : F(\Re) \rightarrow \Re$ which maps each fuzzy number into the line, where a natural order exists. We define orders on $F(\Re)$ by

$$\tilde{a} \succeq \tilde{b} \quad \text{if and only if} \quad \mathcal{M}(\tilde{a}) \geq \mathcal{M}(\tilde{b}), \tag{3.3}$$

$$\tilde{a} \succ \tilde{b} \quad \text{if and only if} \quad \mathcal{M}(\tilde{a}) > \mathcal{M}(\tilde{b}), \tag{3.4}$$

$$\tilde{a} \cong \tilde{b} \quad \text{if and only if} \quad \mathcal{M}(\tilde{a}) = \mathcal{M}(\tilde{b}), \tag{3.5}$$

where $\tilde{a}$ and $\tilde{b}$ are in $F(\Re)$. Also we write $\tilde{a} \preceq \tilde{b}$ if and only if $\tilde{b} \succeq \tilde{a}$.

Here, we introduce a linear ranking function that is similar to the ranking function adopted by Maleki(2000).For a triangular fuzzy number $\tilde{a} = (m, \alpha, \beta)$,we use ranking function as follows:

$$\mathcal{M}(\tilde{a}) = \frac{1}{2} \int_0^1 (\inf \tilde{a}_\alpha + \sup \tilde{a}_\alpha) d\alpha. \tag{3.6}$$

This reduces to

$$\mathcal{M}(\tilde{a}) = m + \frac{1}{4}(\beta - \alpha)) \tag{3.7}$$

Then, for triangular fuzzy numbers $\tilde{a} = (m, \alpha, \beta)$ and $\tilde{b} = (n, \gamma, \delta)$,we have

$$[\tilde{a} \succeq \tilde{b}] \Longleftrightarrow [(m + \frac{1}{4}(\beta - \alpha)) \geq (n + \frac{1}{4}(\delta - \gamma))]. \tag{3.8}$$

Authors who use ranking function for comparison of fuzzy linear programming problems usually define a crisp model which is equivalent to the Fuzzy linear programing problem and then use optimal solution of this model as the optimal solution of fuzzy linear programming problems. We now define fuzzy linear programming problems and the corresponding crisp model.

3.2 Fuzzy Number Linear Programming Problem

Definition 3.1. *A fuzzy number linear programing problem (FNLP) is defined as follows:*

$$\begin{aligned} Min\ \tilde{z} &\simeq \tilde{c}x \\ s.t.\quad \tilde{A}x &\succeq \tilde{b} \\ x &\geq 0, \end{aligned} \tag{3.9}$$

where "$\simeq$" and "$\preceq$" mean equality and inequality with respect to the ranking function τ, $\tilde{A} = [\tilde{a_{ij}}]_{m\times n}$, $\tilde{c} = (\tilde{c_1}, ..., \tilde{c_n})$,
$\tilde{b} = (\tilde{b_1}, ..., \tilde{b_m})^T$,$x = (x_1, ..., x_n)$, and $\tilde{a_{ij}}, \tilde{b_i}, \tilde{c_j} \in F(\Re)$ and $x_j \in \Re$ for $i = 1, ..., m; j = 1, ..., n$.

Definition 3.2 Any x which satisfies the set of constraints of FNLP is called a feasible solution. Let $\tilde{S}$ be the set of all crisp feasible solutions of FNLP . We say that $x^* \in \tilde{S}$ is an optimal feasible solution for FNLP if $\tilde{c}x^* \preceq \tilde{c}x$ for all $x \in \tilde{S}$.

Definition 3.3 We say that the real number a corresponds to the fuzzy number $\tilde{a}$, with respect to a given linear ranking function $\mathcal{M}$, if $a = \mathcal{M}(\tilde{a})$.

However The following theorem shows that any FNLP can be reduced to a linear programming problem (see Maleki [13] and Maleki et al. [14]).

Theorem 3.1. *The following linear programming problem (LP) and the FNLP in (4.16) are equivalent:*

$$\begin{aligned} \min\ z &= cx \\ s.t.\quad Ax &\geq b \\ x &\geq 0, \end{aligned} \tag{3.10}$$

where a_{ij}, b_i, c_j are real numbers corresponding to the fuzzy numbers $\tilde{a_{ij}}, \tilde{b_i}, \tilde{c_j}$ with respect to a given linear ranking function $\mathcal{M}$, respectively.

Remark 3.4 The above theorem shows that the set of all crisp feasible solutions of FNLP and all feasible solutions of LP are the same. Also if $\bar{x}$ is an optimal feasible solution for FNLP, then $\bar{x}$ is an optimal feasible solution for LP.

4 DEA and Fuzzy Non-radial DAE model

4.1 DEA

The most frequently used DEA model is the CCR model, name after Charnes, Cooper and Rhodes (1978).In this paper we considered a Non-radial. Suppose that there are n DMUs, each of which consumers the same type of inputs and produces the same type of outputs. Let m be the number of inputs and let r be the number of outputs.All inputs and outputs are assumed to be nonnegative, but at least one input and one output are positive. The following notation will be used throughout this paper.

Notation

- DMU_j is the jth DMU.
- DMU_o is the target DMU.
- $X_j \in R^{m\times 1}$ is the column vector of inputs consumed by DMU_j.
- $X_o \in R^{m\times 1}$ is the column vector of inputs consumed by DMU_o.
- $X \in R^{m\times n}$ is the matrix of inputs of all DMUs.
- $Y_j \in R^{s\times 1}$ is the column vector of outputs consumed by DMU_j.
- $Y_o \in R^{s\times 1}$ is the column vector of outputs consumed by DMU_o.
- $Y \in R^{s\times n}$ is the matrix of outputs of all DMUs.
- $\lambda = (\lambda_j)_{n\times 1}, \lambda \in R^n$ is the column vector of a linear combination of n DMUs.
- θ is the objective value (efficiency) of the nonradial model.
- $V \in R^{m\times 1}$ is the column vector of input weights.
- $U \in R^{s\times 1}$ is the column vector of output weights.
- α_i is the certian weights values based on inputs improve .

4.2 A Non-radial DAE model

In this Non-radial model, the multiple input and multiple output of each DMU are aggregated into a single virtual input and a single virtual output, respectively. The Non-radial model and its dual are formulated as the following linear programming models:
$(Non-radial)$

$$Min \quad \frac{1}{\sum_{i=1}^{m}\alpha_i}\sum_{i=1}^{m}\alpha_i\theta_i$$

$$s.t. \quad \sum_{j=1}^{n}\lambda_j x_{ij} \leq \theta_i x_{io} \qquad i=1,\ldots,m$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} \geq y_{ro} \qquad r=1,\ldots,s$$

$$\lambda_j \leq 0 \qquad j=1,\ldots,n$$

$$0 \leq \theta_i \leq 1 \qquad i=1,\ldots,m$$

Whereα_i $\quad i=1,\ldots,m$ are input weights. In optimal solution there isn‘t any nonzero input slacks.

4.3 Non-radial measures model with fuzzy data

In recent years, fuzzy set theory has been proposed as a way to quantify imprecise and vague data in DEA models. fuzzy DEA models take the form of fuzzy number linear programming model. The Non-radial model with fuzzy coefficients and its dual are formulated as the following linear programming model:

$$Min \quad \frac{1}{\sum_{i=1}^{m}\alpha_i}\sum_{i=1}^{m}\alpha_i\theta_i$$

$$s.t. \quad \sum_{j=1}^{n}\lambda_j \tilde{x_{ij}} \leq \theta_i \tilde{x_{io}} \qquad i=1,\ldots,m$$

$$\sum_{j=1}^{n} \lambda_j \tilde{y_{rj}} \geq \tilde{y_{ro}} \qquad r = 1, \ldots , s$$

$$\lambda_j \leq 0 \qquad j = 1, \ldots , n$$

$$0 \leq \theta_i \leq 1 \qquad i = 1, \ldots , m$$

The fuzzy Non-radial models cannot be solved by a standard LP solver like a crisp Non-radial model because coefficients in the fuzzy Non-radial model are fuzzy sets. With the fuzzy inputs and fuzzy outputs, the optimality conditions for the crisp DEA model need to be clarified and generalized. This approach by using the concept FLP can be used to where x_{ij}, y_{rj} are real numbers corresponding to the fuzzy numbers $\tilde{x}_{ij}, \tilde{y}_{rj}$ with respect to a given linear ranking function $\mathcal{M}$, respectively.

5 Methodology and examples

We evaluate thirty branches of Tehran Social Security Insurance Organization at this section.Each branch uses of four inputs in order to produce four outputs. The labels of inputs and outputs are presented in under table.

	Input	Output
1	The number of personals	The total number of insured persons
2	The total number of computers	The number of insured persons'agreements
3	The area of the branch	The total number of life-pension receivers
4	Administrative expenses	The receipt total sum (Incom)

Table1.The labels of inputs and outputs.

In this example we consider $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$. The total triangular Fuzzy data has been viewed in tables (3), and (4).We have input 3 as crisp data in table (3). It is considered that "M"as number middle, "α"left expanse. "β"as right expanse. For example if $\tilde{a} = (m, \alpha, \beta)$ denote a triangular fuzzy number then we use ranking function as follows:

$$\tau(\tilde{a}) = m + \frac{1}{4}(\beta - \alpha)$$

After using of ranking function τ , the dates are given as crisp and then with applying explained method on the essay the results are presented in the table (5).

	$Im1$	$I\alpha_1$	$I\beta_1$	$Im2$	$I\alpha_2$	$I\beta_2$	$I3$	$Im4$	$I\alpha_4$	$I\beta_4$
1	94.83	1.83	2.16	84.5	0.5	2.5	4000	78262041.33	25392261.33	69544898.67
2	77	2	2	93	2	2	2565	73241407	18700853	65837545
3	76.5	1.5	1.5	87	1343	0	0	71482960	27879871	61941109
4	92.83	0.83	1.16	93	1500	0	0	497692042.2	440112343.2	2095132858
5	90.33	2.33	1.66	85.66	2.66	1.33	1680	82962450.83	2055 5596.83	57375613.17
6	102.33	1.33	2.66	97	0	0	3750	83979470.83	75553970.83	55503766.17
7	94.5	0.5	0.5	90.5	0.5	0.5	3313	124490390	38894251	64118743
8	86.33	3.33	2.66	92.33	0.33	0.66	1500	77394251.33	18133979.33	26530914.67
9	102.83	0.83	3.16	92	0	0	1600	136743469.3	52591813.33	119377644.7
10	102.5	0.5	0.5	96.33	1.33	0.66	1725	71533303.67	322922 03.67	57999041.33
11	95.5	2.5	0.5	79	0	0	1920	93931521.5	32759865.5	59851928.5
12	78.16	2.16	0.83	91	0	0	4433	80608666.67	32905666.67	7894.33
13	104.66	1.66	2.33	104.33	1.33	0.66	2500	81318447	40482291	70233678
14	88.5	2.5	1.5	95	0	0	2800	58841917.83	19884666.83	23593549.17
15	81.66	3.66	2.33	93.83	1.83	1.16	1630	67658394.67	21270151.67	54420348.33
16	89	2	2	85.33	0.33	0.66	1127	65516 280.33	32077593.33	57141322.67
17	91.5	1.5	1.5	104	0	0	3400	125551611.2	75560965.17	163604623.8
18	114.33	3.33	2.66	94.33	2.33	0.66	1304	78379833	46487464	52020714
19	96.33	2.33	2.66	98	0	0	4206	105183991.8	50050519.83	159270923.2
20	86.66	1.66	1.33	01	0	0	1340	84203387.33	35893023.33	33078687.67
21	69.16	1.16	1.83	90.16	1.1667	0.83	1393	63480967.83	25531087.83	42072685.17
22	113.33	5.33	5.66	123	1	1	2191	1070 27548.2	29761595.17	36600655.83
23	79	1	1	100	0	0	2140	136266225	42751019	149072659
24	87.33	1.33	1.66	93.5	0.5	0.5	1231	67564034.33	62464054.33	35970829.67
25	98.33	1.33	1.66	90	0	0	1960	89732 318.33	29169845.33	43037370.67
26	74.66	1.66	2.33	85	3	1	3375	72153562.17	27478172.17	4 1472834.83
27	104.33	3.33	3.66	102.5	1.5	0.5	2540	92589909.17	31236367.17	94422444.83
28	99.16	2.16	2.83	97.5	1.5	0.5	1603	452222971.8	381361846.8	479307836.2
29	73.83	4.83	2.16	79	2	2	2300	83384457.33	23299737.33	27313481.67
30	89.5	2.5	2.5	92	0	0	2930	78813784.17	21245567.17	26894182.83

Table 3: The triangular fuzzy inputs for 30 branches of insurance organization.

	Om1	$O\alpha_1$	$O\beta_1$	$Om2$	$O\alpha_2$	$O\beta_2$	$Om3$	$O\alpha_3$	$O\beta_3$	$Om4$	$O\alpha_4$	$O\beta_4$
1	58449.33	781.33	1153.66	49.16	17.16	2	1136.16	19.16	11.83	211.16	22.16	64.83
2	37044.83	122.83	134.16	21.83	7.83	14	8795.83	160 .83	123.16	230.83	55.83	68.16
3	34438.33	9078.33	5010.66	31.66	11.66	7	6599.16	11.16	4.83	427.66	112.66	157.33
4	36651.83	404.83	371.16	41	20	0	9406	1326	1415	234.5	93.5	94.5
5	46389.33	10018.33	9692.66	40.5	12.5	19	9940.16	242.16	103.83	278.16	63.16	105.83
6	71808.66	2737.66	1196.33	15.83	15.83	7	8010.66	142.66	109.33	382.5	160.5	261.5
7	38667.66	1191.66	871.33	104.83	31.83	26	13781.83	660.83	1482.16	344.5	0.5	0.5
8	50189.16	2095.16	1954.83	20	8	5	1565.33	12 .33	9.66	391.5	130.5	242.5
9	88309.5	3778.5	4446.5	68.16	68.16	45	11802.5	1012.5	3599.5	425.33	26.33	190.66
10	49309	2352	4248	30.16	11.16	10	7617.5	753.5	7 61.5	226.33	39.33	81.66
11	35972	4418	3369	212.66	42.66	89	12639	297	239	254.83	127.83	110.16
12	31291.16	4279.16	8319.83	26	5	10	7793.5	241.5	202.5	181	21	64
13	61404.5	1823.5	1547.5	47	16	0	7409.33	231.33	22 2.66	326.66	72.66	93.33
14	90320.66	9895.66	3621.33	45.83	9.83	1	717.66	44.66	30.33	229.83	57.83	82.16
15	48643.83	4338.83	2321.16	20.66	3.66	1	10290.16	4.16	2.83	134.83	55.83	137.16
16	43741.5	3944.5	6113.5	31.5	5.5	11	7851.33	354.33	345.66	191.66	16.66	40.33
17	77586.66	23987.66	5336.33	24	10	0	5081.33	122.33	123.66	206.33	56.33	78.66
18	79290.66	6737.66	6534.33	28.33	9.33	6	4953.16	674.16	335.83	215.66	100.66	108.33
19	73663	26766	13700	23.33	10.33	0	1083.33	258.33	522.66	238	107	292
20	32189.16	3334.16	1447.83	59.66	36.66	0	14785.83	641.83	436.16	360.83	59.83	83.16
21	28340.83	512.83	400.16	27.5	12.5	15	953.16	15.16	14.83	241.5	81.5	18.5
22	105355.5	2112.5	3121.5	58	17	10	2658.5	9.5	9.5	362.66	107.66	105.33
23	34310.66	2051.66	1894.33	36.33	6.33	18	2273	67	59	463	40	126
24	58240.83	4306.83	3519.16	62.16	9.16 67	18	10337.5	112.5	124.5	201	33	77
25	83197.66	8395.66	3504.33	65.83	22.83	0	4772.66	204.66	237.33	286	75	72
26	44457	572	939	31.16	10.16	10	617.16	30.16	40.83	306.16	81.16	124.83
27	82569	482	579	55.16	14.16	15	9240.83	471. 83	318.16	302.83	84.83	253.16
28	69914.33	4925.33	4303.66	91	24	12	13219.16	447.16	488.83	212.16	71.16	48.83
29	38993.33	325.33	212.66	32.5	9.5	10	1494	19	2 2	107.16	45.16	109.83
30	62304.5	1440.5	2479.5	30.83	15.83	5	12275.33	393.33	345.66	262.5	75.5	143.5

Table 4: The triangular Fuzzy outputs for 30 branches of insurance organization.

	θ	θ_1	θ_2	θ_3	θ_4
1	0.6571	0.7000	0.7669	0.4073	0.7542
2	0.6611	0.8128	0.7143	0.3668	0.7506
3	1.0000	1.0000	1.0000	1.0000	1.0000
4	0.5334	0.6826	0.7139	0.6543	0.0827
5	0.8014	0.7971	0.8614	0.6508	0.8964
6	1.0000	1.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000
10	0.6937	0.6210	0.6343	0.5515	0.9679
11	1.0000	1.0000	1.0000	1.0000	1.0000
12	0.5327	0.7212	0.6675	0.1876	0.5546
13	0.8049	0.7866	0.8021	0.6307	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000
15	0.8509	0.9814	0.9284	0.7036	0.7902
16	0.7841	0.7197	0.7730	0.8261	0.8175
17	0.6791	0.8571	0.7406	0.5425	0.5764
18	1.0000	1.0000	1.0000	1.0000	1.0000
19	0.6538	0.7933	0.7573	0.4091	0.6554
20	1.0000	1.0000	1.0000	1.0000	1.0000
21	0.6304	0.6482	0.5027	0.5388	0.8317
22	1.0000	1.0000	1.0000	1.0000	1.0000
23	1.0000	1.0000	1.0000	1.0000	1.0000
24	1.0000	1.0000	1.0000	1.0000	1.0000
25	1.0000	1.0000	1.0000	1.0000	1.0000
26	0.7591	0.8804	0.8010	0.3550	1.0000
27	0.9000	0.9393	0.9351	0.7256	1.0000
28	1.0000	1.0000	1.0000	1.0000	1.0000
29	0.5210	0.6017	0.5605	0.4894	0.4326
30	1.0000	1.0000	1.0000	1.0000	1.0000

Table 5: The Non-radial Efficiency.

6 conclusion

The purpose of this study was to develop the DAE models to DMUs with fuzzy data that since the level of inputs and outputs for DMU_o are not know exactly, we respect to a given linear ranking function then, in this cases, the DAE models solve when inputs and outputs data are fuzzy ,we try to use a ranking function to solve fuzzy nonradial models.

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