

## Fuzzy Numbers: Positive and Nonnegative

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### Abstract

In this paper we emphasize that the definition of positive fuzzy number in D.Dubois, H. Prade, Fuzzy Sets and Systems: Theory and Applications, Academic Press, New York, 1980 (page 49), and also recently quoted for solving fully fuzzy linear systems in M. Dehghan et al., Applied Mathematics and Computation 179 (2006) 328-343, is in fact a definition for the nonnegative fuzzy number. Moreover, we give two different definitions for nonnegative and positive triangular fuzzy numbers to eliminate this shortcoming.

**Keywords:** Fuzzy number, Nonnegative and positive fuzzy numbers.

### 1 Introduction

After the successful applications of fuzzy sets theory on the controller systems, this theory have applied in other areas. In the most these applications fuzzy numbers are one way to describe the data vagueness and imprecision. They can be regarded as an extension of the real numbers. In the literature of fuzzy sets there is some shortcoming in the definition of fuzzy number. Frankly some authors do not distinguish between positive and nonnegative fuzzy numbers. Here, we focus on the triangular fuzzy number and give two different definitions for nonnegative and positive triangular fuzzy numbers to eliminate this shortcoming.

## 2 Fuzzy Numbers

Here, we give some necessary definitions of fuzzy set theory.

**Definition 2.1.** A fuzzy set  $A$  in  $R$  (real line) is defined to be a set of ordered pairs  $A = \{(x, \mu_A(x)) | x \in R\}$ , where  $\mu_A(x)$  is called the **membership function** for the fuzzy set.

**Definition 2.2.** A fuzzy set  $A$  is called **normal** if there is at least one point  $x \in R$  with  $\mu_A(x) = 1$ .

**Definition 2.3.** A fuzzy set  $A$  on  $R$  is **convex** if for any  $x, y \in R$  and any  $\lambda \in [0, 1]$ , we have  $\mu_A(\lambda x + (1 - \lambda)y) \geq \min\{\mu_A(x), \mu_A(y)\}$ .

**Definition 2.4.** A **fuzzy number** is a fuzzy set on the real line that satisfies the conditions of normality and convexity.

Let us assume that the membership function of any fuzzy number  $\tilde{a}$  is:

$$\tilde{a}(x) = \begin{cases} \frac{x}{a^\alpha} + \frac{a^\alpha - a^m}{a^\alpha}, & x \in [a^m - a^\alpha, a^m] \\ \frac{-x}{a^\beta} + \frac{a^m + a^\beta}{a^\beta}, & x \in [a^m, a^m + a^\beta] \\ 0, & \text{ow.} \end{cases} \quad (1-1)$$

We denote a triangular fuzzy number  $\tilde{a}$  by  $\tilde{a} = (a^m, a^\alpha, a^\beta)$ .

**Definition 2.5.** A fuzzy number  $\tilde{a}$  on  $R$  is called positive (negative), shown as  $\tilde{A} > 0$ , if its membership function  $\mu_{\tilde{A}}(x)$  satisfies  $\mu_{\tilde{A}}(x) = 0, \forall x < 0 (x > 0)$ .

## 3 Discussion and results

In [2] and recently quoted in [1] the positive fuzzy number is defined as following:

A fuzzy number  $M$  is called positive, denoted by  $M > 0$ , if its membership function  $\mu_M(x)$  satisfies  $\mu_M(x) = 0, \forall x < 0$ . Now according to Definition 2.9 in [1] we may write the LR type fuzzy number  $M$  symbolically as  $M = (m, \alpha, \beta)$ , where  $m$  is the mean value of  $M$  and  $\alpha$  and  $\beta$  are left and right spreads, respectively. We know that any crisp number  $a$  is a fuzzy number with this form  $\tilde{a} = (a, 0, 0)$ . Usually,  $\tilde{a}$  is a positive (nonnegative) fuzzy number, if and only if  $a$  is positive (nonnegative) crisp number. Now the fuzzy number  $\tilde{0} = (0, 0, 0)$  that is not positive in real concept, with Dubois's definition is a positive fuzzy number and this is not reasonable. Moreover, authors in [1] did not distinguish between positive and nonnegative fuzzy number and they use a same definition for them while in Theorem 3.1 they need exactly the fuzzy numbers with nonnegative concept. In addition, without the separation of the positive and nonnegative fuzzy numbers, Definition 2.6 and 2.12 and also Theorem 3.1 are not match and also there is not difference between positive and nonnegative matrices.

After this short comment we emphasize that it is necessary to define two different definitions for positive and nonnegative fuzzy numbers and then we present the following definitions.

**Definition 3.1.** A fuzzy number  $M$  is called positive, denoted by  $M > 0$ , if its membership function  $\mu_M(x)$  satisfies  $\mu_M(x) = 0, \forall x \leq 0$ .

**Definition 3.2.** A fuzzy number  $M$  is called nonnegative, denoted by  $M \geq 0$ , if its membership function  $\mu_M(x)$  satisfies  $\mu_M(x) = 0, \forall x < 0$ .

**Remark 3.1.** With the new definitions the fuzzy number  $\tilde{0} = (0, 0, 0)$  is

nonnegative. And it is necessary to in [1] we let the notation of “ $\geq$ ” (“ $\leq$ ”) in stead of “ $>$ ” (“ $<$ ”) and the main aim will be changed to find a nonnegative solution of fully fuzzy linear system of equations in stead of the positive solution. Moreover, the correct form of Theorem 3.1 is as following.

**Theorem 3.1.** Let  $\tilde{A} = (A, M, N)$  and  $\tilde{b} = (b, g, h)$  be a nonnegative fuzzy matrix and a nonnegative fuzzy vector, respectively, and  $A$  be the product of a permutation matrix by a diagonal matrix with positive diagonal entries. Moreover let  $h \geq MA^{-1}b, g \geq NA^{-1}b$  and  $(MA^{-1} + I)b \geq h$ . Then the fully fuzzy linear system  $\tilde{A}\tilde{x} = \tilde{b}$  has a nonnegative fuzzy solution.

Note that, other details in [1] similar to the above discussion must change.

## References

- [1] M. Dehghan, B. Hashemi, and M. Ghatee, “Computational methods for solving fully fuzzy linear systems”, *Applied Mathematics and Computation* 179 (2006) 328-343.
- [2] D. Dubois, H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, Academic Press, New York, 1980.

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