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Metrological Evaluation of a Bourdon Manometer from the Ordinary Least Squares Method

Cristian Pedraza-Yepes[1], José Daniel Hernández-Vásquez[1], Fernando Pastor Forero[1], Leonardo Pérez Manotas[1] and Jorge González-Coneo[2]

[1] Program of Mechanical Engineering, Universidad del Atlántico
Puerto Colombia - Atlántico, Colombia

[2] Department of Energy. Universidad de la Costa
Barranquilla-Atlántico Colombia

Abstract

The objective of this work is to evaluate the metrological reliability of a bourdon manometer for applications associated with a drop and film heat exchanger. The methodology consists of applying the ordinary least squares method to estimate a polynomial that better adjusts the experimental data in order to reduce the maximum error of the calibration, the measurement uncertainty and the total error associated with the calibration. The results confirmed that it is possible to decrease the uncertainty associated with the measurement by means of a first degree polynomial. Finally, the metrological reliability of the evaluated instrument was estimated at 2.42 psi, for a confidence level of 95%.

Keywords: Metrology, Heat Transfer, Ordinary Least Squares, Uncertainty

1 Introduction

The calibration of a particular measuring instrument consists of comparing the value indicated by this instrument with the value indicated by the standard used in its calibration. The difference between these values is what is called a "systematic measurement error". Based on a statistical treatment of these data performed in the light of the International Guide for Expression of Measurement Uncertainty, ISO GUM [1] and known the Calibration Certificate of the standard used in the calibration (which specifies the measurement uncertainty of the standard and the factor of freedom associated with the degrees of freedom of the calibration process) it is possible to evaluate the *uncertainty associated with measurement* performed by the measuring instrument.

A complete procedure to identify the linearity of the calibration curves is defined by the ordinary least squares method (OLSM). The method should include an adequate experimental design, the estimation of parameters and the treatment of outliers [2]. The method of ordinary least squares (OLSM) is one of the numerical regression techniques most used in the calibration processes of measurable instruments in different areas of industry (e.g.: mining, oil, energy, among others).

The main objective of this method is to determine the adjustment curve of the experimental data with the lowest mean square deviation (s), this parameter is usually associated with the standard deviation of the measured data. In the sequence, this method is defined and its principal characteristics are illustrated.

2 Ordinary Least Squares: Theoretical Fundamentals

A polynomial of degree $\boldsymbol{n}$ (Equation 1) is usually used to correlate the value read by the instrument against the respective value indicated by the standard, for each experiment of the calibration performed. The polynomial that best represents the physical nature of the calibration is selected from those that offer the least uncertainty of fit [3].

$$y(x_i) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + +a_3 \cdot x^3 + \cdots + a_n \cdot x^n \tag{1}$$

In the above expression, $\boldsymbol{x}$***:*** denotes the measurement result indicated by the instrument and $\boldsymbol{y}(\boldsymbol{x_i})$ the value adjusted by the polynomial, which relates the reading of the instrument to the value given by the standard [4-5].
The coefficients $\boldsymbol{a_0}$, $\boldsymbol{a_1} \cdots \boldsymbol{a_n}$ are determined by applying the Ordinary Least Squares (OLS) method, i.e.: solving the matrix system given by Equation 2. The algebraic work of solving Equation 2 may, however, be avoided by making use of the statistical tool available in Excel software.

$$\begin{bmatrix} \sum_{i=1}^{n} x_i^0 & \sum_{i=1}^{n} x_i^1 & \cdots & \sum_{i=1}^{n} x_i^n \\ \sum_{i=1}^{n} x_i^1 & \sum_{i=1}^{n} x_i^2 & \cdots & \sum_{i=1}^{n} x_i^{n+1} \\ \vdots & \vdots & & \vdots \\ \sum_{i=1}^{n} x_i^n & \sum_{i=1}^{n} x_i^{n+1} & \cdots & \sum_{i=1}^{n} x_i^{2n} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} x_i^0 \cdot y_i \\ \sum_{i=1}^{n} x_i^1 \cdot y_i \\ \vdots \\ \sum_{i=1}^{n} x_i^n \cdot y_i \end{bmatrix} \tag{2}$$

Although a polynomial of degree 1 (n = 1) may constitute a good representation of the physical nature of the calibration of some types of instruments, the choice of higher degree polynomials may result in smaller uncertainties of the fit. For this reason, it is a good calibration practice to test at least three degrees of polynomial to define the one that offers the lowest uncertainty of fit and, therefore, the lower uncertainty associated with the measurement. Thus, once the coefficients for each polynomial are determined, the adjusted values for each polynomial are calculated; i.e., successively considering n = 1 to 4 in Equation 1 (to define the polynomial) and in Equation 2 (to calculate the coefficients).

The application of a polynomial (said adjustment interpolator polynomial) allows: (i) to correct on-line the experimental results measured by the measuring instrument; (ii) reduce the systematic error inherent in the measurement process and (iii) estimate the adjusted values for any indication of the instrument, provided it is within the range of its calibration. In spite of these advantages of using the polynomial to correct the value indicated by the instrument, it must be borne in mind that its use introduces a new source of uncertainty (adjustment uncertainty), which polynomial reduces the Total Error associated with the measurement [6].
The value of this adjustment uncertainty is calculated (for each polynomial evaluated) by applying Equation (3).

$$u_s = \sqrt{\left(\frac{1}{n-c}\right) \cdot \sum_{i=1}^{n} [y(x_i) - y_i]^2} \tag{3}$$

Then, (i) we choose the interpolator polynomial associated with the least adjustment uncertainty and (ii) determine the coverage factor k (for a confidence level of 95.0%), from the probability distribution t -student and the number of degrees of freedom (φ), given by Equation (4):

$$\varphi = \mathrm{n} - \mathrm{c} \tag{4}$$

this equation, ***n*** denotes the number of experimental points of the calibration and c is the number of coefficients estimated by the polynomial. The expanded uncertainty associated with the adjustment is given by Equation (5):

$$\mathrm{U_E} = \mathrm{k} \cdot \mathrm{u}_c \tag{5}$$

3 Experimental Methodology: Condensation heat exchanger in drop and film

This test bench is focused on the experimental study of the phenomenon of drop and film condensation. The phenomenon of condensation has been widely observed in several stochastic processes and physical contexts, such as, for example, mass

transport models [7] and fluid flow in pipes [8]. Condensation is the physical phenomenon in which matter changes from a gaseous phase to a liquid phase. This phenomenon usually occurs when a cold fluid comes into contact with a hot surface, the heat transfer between the fluid and the surface provides an increase in temperature of the fluid and consequently a change from the gas to liquid phase. In industrial systems, the surfaces that are most frequently in contact with a given fluid are pipes, storage tanks and transport pumps.
The type of condensate generated by the heat transfer depends on the geometric and chemical characteristics of the surfaces. This test bench is focused on the experimental study of condensation in pipes.

In pipes with a natural finish, film-like condensation is noticeable, and drop condensation takes place in pipes that have some finish on their surface. Figure 1 illustrates the configuration of the test bench located in the CELTI of the Universidad del Atlántico.

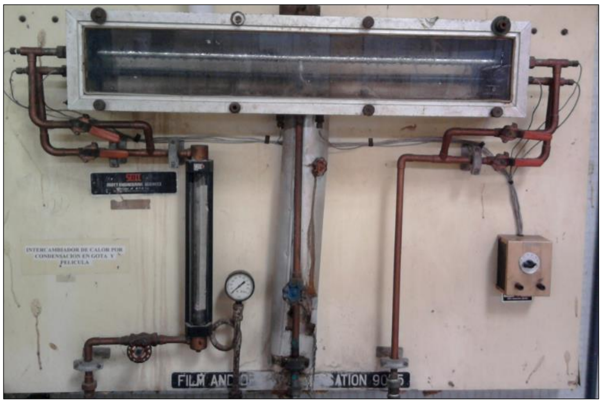

Figure 1. Condensation heat exchanger in drop and film

From the above described elements, it is observed that the measuring instruments to be metrologically evaluated are manometer, thermocouple and rotameter. The function of each of the measuring instruments is described below.

- **Manometer:** The task of this element is to control and measure the variation of the gauge pressure of the water vapor at the inlet of the heat exchanger.
- **Thermocouple:** The objective of this instrument is to determine the temperature variation during the heat transfer process
- **Rotameter:** This instrument is responsible for measuring the variation of the flow that runs through the pipes of the heat exchanger

The metrological reliability of the thermocouple was not evaluated since the scope of study of this degree work is limited by the analysis of the analytical measuring instruments. Before proceeding with the calibrations, it was determined whether the analytical measuring instruments (manometer and rotameter) had the minimum standards necessary to evaluate their metrological reliability. In relation to the rota-

meter it was observed that its current state does not meet the minimum conditions for calibration and therefore for its optimal operation.
Concerning the manometer (figure 2) it was observed that it is in good physical condition and the metrological analysis of it could be performed (Range: 0 – 100 psi; Resolution: 2 psi).

Figure 2. Manometer of Test bench Condensation heat exchanger in drop and film

4 Results and Discussion

As mentioned previously, this test bench was subjected to the metrological reliability analysis of the pressure gauge. Figure 3 illustrates the typical configuration between the measurement pattern and the instrument to be analyzed metrologically.

Figure 3. Experimental calibration of manometer

It was determined from the experimental data and using the Chauvenet criterion if the measurements made by the pattern have outliers (i.e.: atypical values). To be followed, as an illustrative example, the procedure applied for the calculation of the average (arithmetic mean) and the standard deviation in the experimental point number two (highlighted in Red in Table 3) is detailed. For calculating the average (upload load):

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{3}(10{,}2 + 10{,}1 + 10{,}1) = 10{,}1333$$

For the calculation of the standard deviation (ascending load):

$$s = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{3-1}[(10{,}2 - 10{,}1)^2 + (10{,}1 - 10{,}1)^2 + (10{,}1 - 10{,}1)^2] = 0{,}0577$$

For the subsequent points, the same rationale was applied, once the Chauvenet criterion was applied. From the results obtained it is inferred that the pressure measurements made by the standard instrument do not have atypical values. Next, the calibration process begins with the graph of the data measured by the pattern based on the measurements made by the instrument to be calibrated. Using each adjustment equation, an adjusted pressure value was determined as detailed below. For all cases, ***y:*** denotes the adjusted value; ***x:*** denotes the value of the medium indicated by the instrument:

- Grade 1:

$$y = 0{,}9999x + 0{,}0652 = 0{,}9999 \cdot 10{,}0 + 0{,}0652$$
$$y = 0{,}9999 \cdot 10{,}0 + 0{,}0652$$
$$y = 10{,}06$$

- Grade 2:

$$y = -1 \cdot 10^{-5} \cdot x^2 + 1{,}0013 \cdot x + 0{,}0442$$
$$y = -1 \cdot 10^{-5} \cdot 10{,}0^2 + 1{,}0013 \cdot 10{,}0 + 0{,}0442$$
$$y = 10{,}06$$

- Grade 3:

$$y = -4 \cdot 10^{-5} \cdot x^3 - 8 \cdot 10^{-6} \cdot x^2 + 1{,}001 \cdot x + 0{,}0457$$
$$y = -4 \cdot 10^{-5} \cdot 10{,}0^3 - 8 \cdot 10^{-6} \cdot 10{,}0^2 + 1{,}001 \cdot 10{,}0 + 0{,}0457$$
$$y = 10{,}05$$

- Grade 4:

$$y = -6 \cdot 10^{-8} \cdot x^4 + 1 \cdot 10^{-5} \cdot x^3 - 0{,}0008 \cdot x^2 + 1{,}0162 \cdot x + 0{,}002$$
$$y = -6 \cdot 10^{-8} \cdot 10{,}0^4 + 1 \cdot 10^{-5} \cdot 10{,}0^3 - 0{,}0008 \cdot 10{,}0^2 + 1{,}0162 \cdot 10{,}0 + 0{,}002$$
$$y = 10{,}09$$

Next, it was calculated the mean square deviation (u_s) for each adjustment polynomial, in order to know the contribution of the ordinary least squares method in the uncertainty associated with the measurement. In this sense, the polynomial that provides the least uncertainty is considered the one that best represents the physical nature of the problem and, consequently, the one that contributes the least uncertainty to the measurement process.

For all cases, ***m:*** the total of experimental points (ascending and descending load); ***n:*** denotes number of estimated coefficients for each adjustment polynomial.

- Grade 1:

$$u_s = \sqrt{\sum \frac{[y(x) - y]^2}{m - n}}$$

$$u_s = \frac{(0{,}07 - 0{,}00)^2}{22 - 2} + \frac{(10{,}06 - 10{,}1)^2}{22 - 2} + \cdots + \frac{(100{,}06 - 100{,}0)^2}{22 - 2}$$

$$u_s = 0{,}0054\, psi$$

- Grade 2

$$u_s = \frac{(0{,}04 - 0{,}00)^2}{22 - 2} + \frac{(10{,}06 - 10{,}1)^2}{22 - 2} + \cdots + \frac{(100{,}07 - 100{,}0)^2}{22 - 2}$$

$$u_s = 0{,}0060\, psi$$

- Grade 3:

$$u_s = \frac{(0{,}05 - 0{,}00)^2}{22 - 2} + \frac{(10{,}05 - 10{,}1)^2}{22 - 2} + \cdots + \frac{(100{,}03 - 100{,}0)^2}{22 - 2}$$

$$u_s = 0{,}0058\, psi$$

- Grade 4:

$$u_s = \frac{(0{,}00 - 0{,}00)^2}{22 - 2} + \frac{(10{,}09 - 10{,}1)^2}{22 - 2} + \cdots + \frac{97{,}62 - 100{,}0)^2}{22 - 2}$$

$$u_s = 1{,}3576\, psi$$

The results confirmed that the polynomial of degree 1 generates the smallest mean square deviation, consequently the smallest possible adjustment uncertainty is obtained:

$$u_s = 0{,}0054\, psi$$

In order to calculate the uncertainty associated with the pressure measurement process, the uncertainty of the pattern (u_p) and the uncertainty of the measurement object (u_{ins}) must be known. The uncertainty up is obtained by dividing the resolution of the pattern on 2 (coverage factor k). The uncertainty u_{ins} is acquired by dividing the resolution of the object on $\sqrt{3}$ (associating, in this way, the experimental data to a rectangular probability distribution). The calculation of the combined uncertainty (u) using equation 5 is shown below

$$u_c = \sqrt{{u_p}^2 + {u_{inst}}^2 + {u_s}^2}$$

$$u_c = \sqrt{0{,}050^2 + 1{,}155^2 + 0{,}0054^2} = 1{,}16\, psi$$

The sources of uncertainty considered for the analysis were: resolution of the instrument, measurement pattern, adjustment by the ordinary least squares method. The contribution of the uncertainties for repeatability and that associated with the hysteresis of the manometer were also considered and it was verified that their

contribution is minimal when compared with the combined uncertainty associated with the measurement.
For a degree of reliability of 95% and a number of experimental points equal to 22 (ascending and descending load), the coverage factor (t-student) is 2.09 and the Expanded Uncertainty is, $\mathbf{U_E = 2.42}$ **psi**.

5 Conclusions

In accordance with the calibration curve obtained from the best fitting polynomial for the measuring instrument analyzed (bourdon type manometer), it was found that the use of the ordinary least squares method is a useful procedure that aids in the reduction of the uncertainty associated with the measurement of any physical quantities.
To provide the measurement uncertainty of the instruments installed in the drop test and evaluated film, contextualizes the science of the measurements, in relation to the metrological reliability of the test bench. The foregoing is impregnated of great importance since the greater the metrological reliability of the measuring instruments, the greater the understanding of the phenomena studied by each test bench will be the student. Finally, it is concluded that the application of the ordinary least squares method allowed to decrease the expanded uncertainty associated with the measurement, the maximum error of the instrument and, consequently, the total error associated with the pressure measurement.

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