Contemporary Engineering Sciences, Vol. 11, 2018, no. 96, 4761 - 4768
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ces.2018.89513

Application of the Gradient Method in the Economic Dispatch

Diana Marcela Devia Narváez

Department of Mathematics and GIMAE
Universidad Tecnológica de Pereira
Pereira, Colombia

Germán Correa Vélez

Department of Mathematics and GIMAE
Universidad Tecnológica de Pereira
Pereira, Colombia

Diego Fernando Devia Narváez

Department of Mathematics and GREDYA
Universidad Tecnológica de Pereira
Pereira, Colombia

Abstract

Two variants of the gradient method are proposed for the solution of the economic dispatch problem, which aims to minimize the fuel consumption of the generator or the operating cost of the entire system by determining the power output of each generating unit under the conditions of restriction. of system load demands. Initially the theory is presented around the problem of economic dispatch, a brief description of the problem and the inherent variables is made, then the mathematical development of the gradient applied to the dispatch problem is exposed, starting with the Lagrange function, then the Comparative results of the two approaches of the gradient method, exposing the benefits of each one.

Keywords: Gradient method, economic dispatch, thermoelectric generation, Lagrange multipliers.

1 Introduction

Currently in the electric power production industry, new sources of energy have been incorporated to help cope with daily demand. In most European countries, renewable energy sources such as solar panels and wind turbines, among others, have already been implemented, however, traditional sources of generation, hydroelectric and thermoelectric plants, are not in disuse, despite innovations in energy resources the demand for energy grows and the use of thermoelectric units is necessary in many cases, so the study of the classic economic dispatch It is still necessary.

In a real energy system, the fuel costs of the thermoelectric plants are different, in addition, under normal operating conditions, the generation capacity is greater than the total load demand plus the losses, which leads to the need to schedule the generation [5]. The objective of the economic dispatch is to find the actual power programming of the power generating units that minimizes the operating cost, from the minimization of the objective cost function and thus the plants satisfy the load demand [9]. The economic dispatch is a mathematically complex and highly non-linear optimization problem, especially in large systems and different techniques have been reported in the literature. Many deterministic approaches have been applied, such as Lagrangian relaxation and nonlinear and dynamic programming techniques to find the optimal solution to the problem. The method used here is the gradient method, which is a special method for the optimization of quadratic functions. as those that describe the power delivery of the generators [7].

2 Economic Dispatch

The objective of the economic dispatch (ED) is to coordinate the generation of thermoelectric plants in order that the fuel consumption of the generator or the operating cost of the entire system is minimum, determining the power output of each generating unit under the conditions of restriction of the load demands of the system. This is also known as classic economic dispatch, in which the security restrictions of the lines are neglected [1]. The fundamental problem of economic shipping is the set of cost functions of the generating units [2].

2.1 Cost function of thermoelectric generators

The cost function of the generating thermal unit indicates the cost of generating real power, and therefore it is dependent on the fuel consumption of the same. This function describes the fuel input to the unit translates into cost per unit of heat (Btu), the final ratio expresses the cost of delivering real power. In addition to the cost of fuel consumption, the operating cost of a thermal plant includes the cost of labor, maintenance costs and the cost of transporting fuel [3] [6]. It is difficult to express these costs directly as a function of the production of the unit, so these costs are included as a fixed part of the operating cost [4].

The thermal unit system generally consists of the boiler, the steam turbine and the generator. The input of the boiler is combustible, and the output is the volume of steam. The input-output ratio can be expressed as a convex curve Figure 1. The input of the turbine generator unit is the volume of steam and the output is the electrical power. The cost function is inherent to the fuel input to the boiler until the power output of the generator. It is a convex curve, shown in Figure 1. It can be observed from the input - output characteristic of the generating unit that the output power is limited by the minimum and maximum capacity of the generating unit, which is:

$$P_{Gmin} \leq P_G \leq P_{Gmax}$$

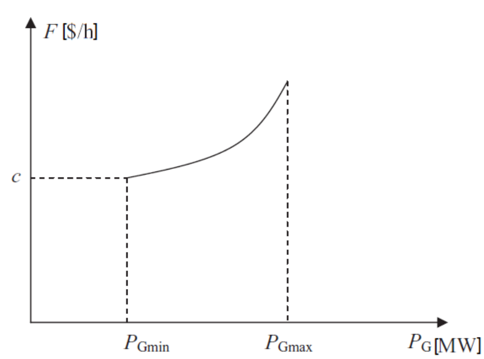

Figure 1: Curve of the cost function of thermoelectric generators.

The minimum output power is determined by the technical conditions, some mentioned above. The maximum output power of the generating unit is determined by the design capacity or speed capacity of the boiler, turbine or generator. Although these limitations on delivery power give more realism to the analysis of the generators, the purpose of this document is to observe the capacity of the gradient method, therefore, these limitations will be disregarded as indicated later [4] [10].

Generally, the cost function of the generating unit is non-linear. The widely used cost function of the generating unit is a quadratic function:

$$F = aP_G^2 + bP_G + c, \tag{1}$$

where a, b and c are the coefficients of the cost function and c is equivalent to the fuel consumption of the generating unit for the minimum output power (Fig. 1).

2.2 Calculation of the parameters of the cost function

The parameters of the cost function of the generating unit can be determined by the following approaches [3]:

1. Based on the experiments of the efficiency of the generating unit.
2. Based on the historical records of the operation of the generating unit.
3. Based on the design data of the generating unit provided by the manufacturer.

In practical energy systems, you can easily obtain statistical fuel data and statistical power output data. Through the analysis and computation of a data set (F_k, P_k), it is possible to determine the form of the cost function and the corresponding parameters. For example, if the quadratic curve is the best match according to the statistical data, it is feasible to use the least squares method to calculate the parameters [2].

3 Economic dispatch by the gradient method

Generally the cost function of a thermoelectric generator is a quadratic function, however, it can be a cubic or more complex function:

$$f_{G_i} = a_i + b_i P_{Gi} + c_i {P_{G}}_i^2 + d_i P_{G_i}^3 + \dots \tag{2}$$

For the functions of type (2) other methods are required to obtain the optimal solution [2], for quadratic functions the gradient method will be worked.

3.1 Gradient method

The gradient method is one of the most powerful numerical methods used for non-linear optimization and although it is a method for unrestricted optimization, it will later be shown that with a combination of concepts it is possible to work with restrictions [11].

As already mentioned above, the objective of the economic dispatch is to minimize the fuel consumption of the generator or the operating cost of the entire system by determining the output power of each generating unit under the conditions of restriction of the load demands of the system. [8] The cost function of the generating units has the form of a quadratic function (3):

$$f_i(P_{G_i}) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i \tag{3}$$

subject to

$$\sum_{i=1}^{N} P_{Gi} = P_D \tag{4}$$

Given the characteristics of the cost function of each generating unit, equation (3), one of the objectives is that they meet the demand of the P_D (rest) system. Therefore, I know that the cost function, equation (3), is subject to the condition of supplying the demand, equation (4). This is a constraint optimization problem, and can be solved by the Lagrange multipliers method [5] [8]. First, the Lagrange function must be formed by adding the constraint function to the objective function after the constraint function has been multiplied by an undetermined multiplier λ:

$$L = F + \lambda \left(P_D - \sum_{i=1}^{N} P_{Gi} \right) \tag{5}$$

$$= \sum_{i=1}^{N} f_i \left(P_{Gi} \right) + \lambda \left(P_D - \sum_{i=1}^{N} P_{Gi} \right),$$

where P_D is the power load demand, N is the number of generators, λ is the incremental cost rate (Lagrange multiplier), and P_{Gi} is the power delivered by the $i - th$ generation unit.

4 Numerical results

Two approaches are used to compare the gradient methods: the first, widely used in the solution of the economic dispatch; solving the problem with the fixed descent parameter ($\alpha^k = \alpha$), and the second approach, finding the successive α^k from a linear search. These two approaches are used to solve a standard problem of three generating units as a test system [2], which is considered lossless and without restrictions on output power. Table 1 gives the values of the fuel cost coefficients. The power demand that the generating units must meet is $500MW$.

Generating Unit	Fuel cost coefficients			$P_D = 500MW$
i	a_i	b_i	c_i	$PG_i^0[MW]$
1	0.0006	0.5	6	200
2	0.005	0.6	5	200
3	0.0007	0.4	3	100

Table 1: **Parameters of the cost functions of the generating units.**

Output powers	$\alpha = 400$	$\alpha = 600$	$\alpha = 800$
P_{G1}	$172,70206$	$172,68034$	$172,8831$
P_{G2}	$107,89911$	$107,82668$	$107,4968$
P_{G3}	$219,39883$	$219,49298$	$219,62$
λ	$0,70749$	$0,70744$	$0,70746$
Fuel cost h	$310,26172$	$310,26169$	$310,2617$
Iterations	10	6	4

Table 2: **Results with fixed values of the descent parameter.**

Output powers	α^k
P_{G1}	$172,8356$
P_{G2}	$107,52852$
P_{G3}	$219,63588$
λ	$0,70747$
Fuel cost h	$310,26168$
Iterations	4

Table 3: **Results with variable descent parameter.**

In the results obtained previously it is possible to observe that table 1 contains the coefficients of the respective cost functions of the generating units, in addition to the initial power values of these.

Tables 2 and 3 show that the fuel cost obtained is the same for all the values of the descent parameter, however, it is found that for the fixed descent parameter there is a greater or equal number of iterations, than for the Variable descent parameter.

5 Conclusion

The approach of the gradient method with the fixed descent parameter can provide good convergence results if an appropriate choice is made, however, this is very unlikely and requires an unknown number of tests that do not guarantee obtaining the best parameter for a convergence fast As observed in the results, obtaining the descent parameter by means of an optimal search line guarantees the choice of the best descent parameter, since this varies in each iteration, improving the convergence of the method.

Finally, the importance of the solution to the problem of the economic dispatch through the gradient method performed in this document lies in the correct use of the gradient method, as it could be observed the parameter of descent is changing for each iteration, this approach is not used in the literature concerning the subject. The calculations made in this document contain several assumptions, in the performance of future work can include the losses of the generators, as well as the power intervals generated from these, in addition to the incorporation of other parameters and/or techniques that feed back to the process carried out.

Acknowledgements. We would like to thank the referee for his valuable suggestions that improved the presentation of this paper and our gratitude to the Department of Mathematics of the Universidad Tecnológica de Pereira (Colombia), the GEDNOL Research Group and GIMAE (Grupo de Investigación en Matemática Aplicada y Educación).

References

[1] L. Kirchmayer, *Economic Operation of Power Systems*, New York, Wiley, Vol. 707, 1958.

[2] J. Zhu, *Optimization of Power System Operation*, *John Wiley and Sons*, Vol. 47, 2015. https://doi.org/10.1002/9781118887004

[3] J. Zhu, *Power System Optimal Operation*, Tutorial of Chongqing University, 1990.

[4] F. Li, R.K. Aggarwal, Fast and accurate power dispatch using a relaxed genetic algorithm and a local gradient technique, *Expert Systems with Applications*, **19** (2000), no. 3, 159-16.
https://doi.org/10.1016/s0957-4174(00)00030-0

[5] P. Attaviriyanupap, H. Kita, E. Tanaka, J. Hasegawa, A hybrid EP and SQP for dynamic economic dispatch with non-smooth fuel cost function, *IEEE Transactions on Power Systems*, **17** (2002), no. 2, 411-416. https://doi.org/10.1109/tpwrs.2002.1007911

[6] A. Wood, Bruce F. Wollenberg, Gerald B. Sheble, *Power Generation, Operation and Control*, *John Wiley and Sons*, 2012.

[7] E. Birgin, Jose Mario Martinez, Marcos Raydan, Non-monotone spectral projected gradient methods on convex sets, *SIAM Journal on Optimization*, **10** (2000), no. 4, 1196-1211. https://doi.org/10.1137/s1052623497330963

[8] D. Kothari, Power system optimization, *2012 Computational Intelligence and Signal Processing (CISP)*, (2012). https://doi.org/10.1109/nccisp.2012.6189669

[9] X. Xia, A.M. Elaiw, Optimal dynamic economic dispatch of generation A review, *Electric Power Systems*, **80** (2010), no. 8, 975-986. https://doi.org/10.1016/j.epsr.2009.12.012

[10] P. Bertsekas, *Nonlinear Programming*, Belmont: Athena Scientific, 1999.

[11] D. Wismer, R. Chattergy, *Introduction to Nonlinear Optimization: A Problem Solving Approach*, Vol. 10, North Holland, 1978.

Received: October 8, 2018; Published: November 12, 2018