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Perspectives in the Development of Topological Notions Using Soap Bubbles

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Abstract

This work is developed within the framework of the Master in Mathematics Teaching and is called "Perspectives in the Development of Topological Notions in the First Grades of Schooling", from where the way in which the use of soap bubbles stimulates the development of topological notions in transitional children, first and second grade, therefore it is proposed as a goal: To stimulate the development of topological notions from the use of soap bubbles; specifically the notions of continuity, interiority, exteriority and border.

Keywords: Topological notions, soap bubbles, interiority, exteriority

1 Introduction

In its theoretical framework are integrated Piaget and Inhelder approaches who make important contributions to the study of the development of topological notions, although with some inaccuracies that can be attributed to the disciplinary domain of this type of geometry. It also integrates the reflections made by Duval associated with the construction of mathematical knowledge and specifically geometric knowledge, through semiotic representation systems, where visualization and reasoning processes become relevant in the cognitive and cognitive structuring of children. These two perspectives complement one side with the discussions of Gardner for whom there is a particular way of learning and developing spatial intelligence, not only for its value in mathematical training, but also for what it means in the daily life of people for develop autonomously even in the absence of strategies or modes of visual perception of the world [1].

On the other hand, the methodological approach that guides the research is exposed, which in general terms is of a mixed character with domination of quantitative analysis methods, whose view is complemented with a nested method of qualitative character. This chapter also explains the characteristics of the participants, as well as the methodological design used. The quantitative findings allow to recognize the fulfillment of the evaluation criteria that account for the activation of the topological notions enunciated through the comparison of the simple and percentage frequency derived from the application of the tests. In a complementary way to the qualitative analysis, categories and emerging subcategories are established that broaden the understanding of the phenomenon under study. These two forms of analysis enrich the discussion and allow for the elaboration of conclusions and recommendations in both didactic and methodological terms and future research perspectives [2].

2 Topological notions

Topology as a branch of mathematics uses concepts from other areas such as geometry and set theory with the objective of studying the properties of "figures" and the conservation of them when subjected to deformations. The "figures" are a set of points that meet axioms to be treated as a topological space, this is defined as follows:

Let X be a non-empty set, a collection τ of subsets of X is said to be a topology over X if:

1. X and the empty set $\emptyset$, belong to τ.

2. The union of any number (finite or infinite) of sets in τ belong to τ.

3. The intersections of any two sets of τ belong to τ.

The deformations to which the figures are analyzed are continuously stretched, wrinkled and flexed, without the figure tearing. This means that the form can be altered, but this can return to its original state intact or that the shape it takes to be deformed has no breaks. Due to the broad concept of topology, this has led to other areas finding Algebraic, Differential and Geometric Topology, however, they all share the same bases from which new concepts are developed [3].

A metric space is a Cartesian pair (X, d) where X is a non-empty set and a function $d : X \times X \to [0, \infty)$, also in a metric call that meets the following conditions with $a, b, c \in X$:

- $d(a, b) = 0$ if $a = b$.
- $d(a, b) = d(b, a)$.
- $d(a, c) \leq d(a, b) + d(b, c)$.

Now, when the axioms are fulfilled, it is said that the function d is a metric, however, in order to deal with topological concepts, it is necessary to use the concept of balls that is discussed below [6]:

- **Open ball**. An open Ball is a set of points that arise from a metric space (X, d) that has a radius r from a point p and is defined as follows:

  $$B_r = \{x \in X : d(x, p) < r\},$$

  that is to say that the set includes all the points minus the circumference of the ball. From this idea, a topology in a metric space is no more than a set of openings that are found in the topology and that we treat as open balls.

- **Continuity** The concept of continuity in topology is very linked to the handling in the area of continuous functions. A continuous function can be defined by observing if the curve it represents has no breaks or jumps, that is, it is a line or continuous line, to define this concept in the topology area, topological spaces and a function between these are used.

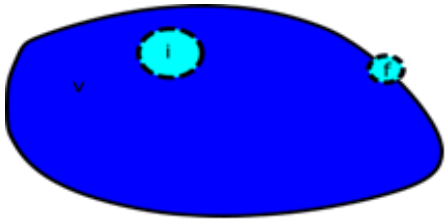

Figure 1: Relationship between interior, exterior and bundary.

3 Methodology

In quantitative terms a pre-experimental approach is proposed with the definition of a sample group of random selection (with pre-test, intervention and post-test application), which allows the construction and comparison of the levels of apprehension of topological notions from the evidence observed in the participants, before and after the application of an intervention designed for that purpose. On the other hand, the qualitative character is nested in the analysis of content applied to the intervention activities in the sample group recorded in video [4].

Both databases can provide different views of the problem considered. For example, in a "mixed" experiment the quantitative data can account for the effect of the treatments, while the qualitative evidence can explore the experiences of the participants during the treatments. Likewise, one approach can be framed within the other method

3.1 Population of the research

In the application of the pre-test, intervention and post-test, a random selection of eighteen (18) students of the transition degrees, first and second, is carried out. Two groups of three students without distinction of January were selected from each grade to carry out the application of the tests in a more dynamic way. The students selected in the same way participated in the intervention from which part of the qualitative analysis derives.

3.2 Design of the research methodology

From a quantitative perspective, it is proposed to compare the activation of the topological notions of continuity, boundary, interior and exterior that are evident in students from the application of a pre-test, an intervention and a post-test, from which is made a criterial evaluation that accounts for cognitive processes present or absent in the solution of visualization and reasoning tasks.

For this exercise, four assemblies are proposed that guide the questions. The pre-test is carried out in two moments, the first one in which the images are presented without a context of meaning, and then place them in a context of everyday life; that is, they are associated with situations that are close to the experience of children and children, and allow us to describe aspects of space outside the plane [5].

The intervention on its part focuses on visualization and reasoning experiences using soap bubbles, given the characteristics and possibilities of manipulation that are offered. Finally, in the post-test the manipulation of objects whose forms are proposed is proposed. they relate both to the forms presented in the pre-test and to the visualization experiences promoted from the intervention. Both in the pre-test and in the post-test, four assemblies are proposed, from which analytical criteria are defined that give evidence of the cognitive processes used by the students. In the particular case of assemblies 1 and 2 an additional criterion is applied to the post-test, whose evaluation is only possible through the manipulation of objects.

- **Activity 1**. This activity aims to identify evidences of the topological notion of continuity and, to a lesser degree, the frontier. Therefore, questions of observing the images and objects are proposed so that the students can recognize the presence of more objects or if they recognize the image or the object as a continuous whole. The use of the meaning

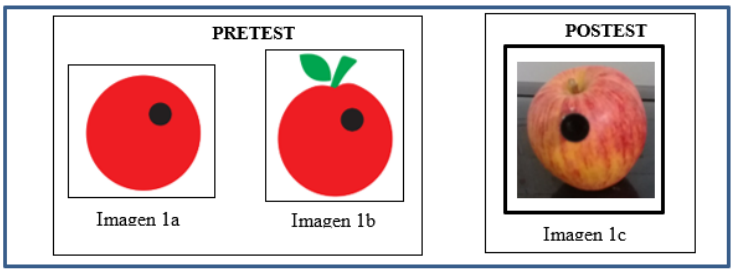

Figure 2: Activity 1 used during the application of the pretest and the post-test.

context of the apple presented in image 1b induces in the students an association with a hole (for example caused by an insect); in contrast to the presentation of the real object shown in the image 1c, and which consists of an apple with a small black cylinder just in a position that agrees with the image 1b.

- **Activity 2**. Activity 2 seeks to identify evidences of topological and interior notions of the interior and exterior, for which questions are posed

that show the place where students perceive the object when observing the images and objects, both in the pre-test as in the post-test.

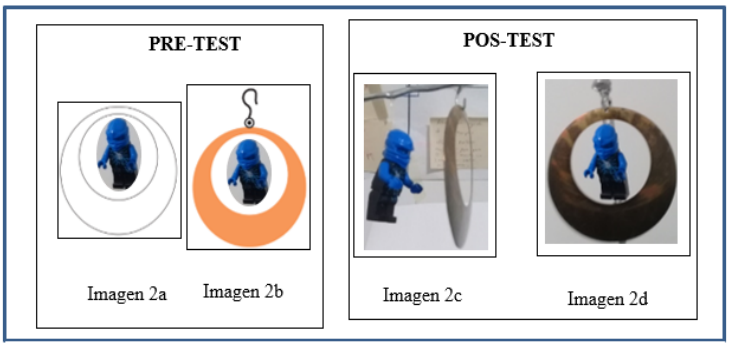

Figure 3: Activity 2 used during the application of the pretest and the post-test.

The context of significance of the accessory presented in image 2b promotes in the students the association with an element that can have an object inside, in this case, the lego character is centered on the image of the accessory, in contrast with the presentation in images 2c and 2d for the post-test, where the character of the layman is located behind the accessory in such a way that the students are shown two views, one frontal and one lateral.

- **Activity 3**. Activity 3 seeks to identify evidences of the topological notion of border, where with a series of three images the students are expected to associate them with the contact between two surfaces, one fixed and the other moving, this series of images Genes are accompanied by three questions for both the pre-test and the post-test.

  The context of meaning shown in image 3b induces the children the relationship with a bed the aesthetic in which two surfaces, that is to say the borders, come into contact with each other. In accordance with this sequence, in the post-test the instruments of image 3c are used, where children and children can observe the sphere bouncing on the bed the acoustic.

4 Results and discussion

Below is a diagram that results from the analysis of content made to students' discourse during their participation in the pre- and post-test. It shows the categories and subcategories that emerge from this analysis that broaden the

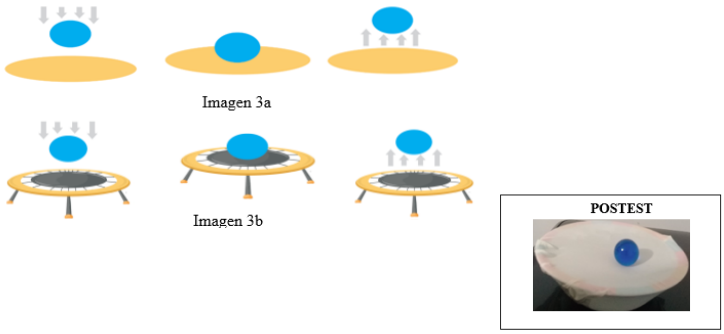

Figure 4: Activity 2 used during the application of the pretest and the post-test.

understanding of the phenomenon under study. In the figure 5 we can see

Figure 5: Categories and subcategories emerging from content analysis.

three categories, namely: Topic logic, Cognitive Dimension, Dynamic World Con- sciousness. Each of them encompasses a series of interrelated elements that are explained below.

Here, we try to answer the question posed in the initial chapter. In what way the use of soap bubbles stimulates the development of topological notions in the transition-years, 1 and 2 years of schooling ? In his reading, it is assumed as a thesis that topological notions are effectively stimulated by the use of soap bubbles; therefore, the emphasis of the response is especially focused on broadening the understanding of how the use and manipulation of bubbles, mobilize in the cognitive and cognitive structures of the participants the topological notions delimited in the methodological design. (continuity, interiority, exteriority and border). This hypo thesis is validated in the comparative anal-

ysis that derives from the qualitative method that guides the investigation; It shows how students achieve greater compliance with the criteria used to evaluate the activation of the topological notion in the cognitive structure from the processes of reasoning and visualization.

The soap bubble enriches the visualization experience since it allows the construction, destruction and transformation in a short but sufficient time to capture the attention of the children. The pomp is transformed, for example, into a sphere, into a single film, so it is not just manipulation in the direction of the hands, but in the operations that are achieved through visualization and to a lesser extent of the reasoning stated verbally. The pomp is an artifact to play but also to learn. The pomp of soap evolves in the topological space, and this is the natural geometric space in which the child moves. The translucent character of the face allows the recognition of shapes and structures that are otherwise impossible to observe, for example when working with opaque bodies.

5 Conclusion

In this research work, there is enough evidence to consider that enriching the experiences of visualization and reasoning using soap bubbles, allows students of the first grades to achieve better performance associated with tasks and processes of greater complexity such as the ana. Lysis of change of perspective, construction of inferences and explanations of cause and effect. On the other hand, it is recognized that the topological notions achieve a greater activation of the cognitive and cognitive structures, when these are approached from the natural geometric space of the children, that is to say the topological space, because as it shows In the qualitative analysis, the students reveal greater comprehension when compared to the tasks that are evaluated from a geometric context restricted to two dimensions. There are intrinsic characteristics of soap bubbles (translucency, ease of construction and transformation, tensability, reduction tendency to the minimum area, among others) that manipulated with pedagogical and didactic purposes can enrich experiences of visualization and reasoning, managing to mobilize the topological notions explored in this investigation; without leaving aside the enjoyment represented by the work with soap bubbles for children and girls given the lute dimension that characterizes this stage of child development.

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