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Comparison of the Radius of Curvature in the Cams which Used in the Displacement Law Beziér Curves and Cyclic and Harmonic Curves

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Abstract

The work shows the radius of curvature for different variations of the angle of rotation β and of the ratio of Ro/L, the study is shown for the case in which the laws of displacement of the cams have been designed by Bézier curves and these results are compared with those obtained by harmonic and cyclic curves.

Keywords: continuity, cam, follower

1 Introduction

The radius of curvature is an important parameter in the design of the cams profile. In design, it doesn't matter what type of follower you want to use, but for each type of follower there is to take into account the movement, the shape and position of this. To make a suitable design, one of the objectives, is to find the minimum radius of curvature of the pitch curve which must be greater than the radius of the follower. Mabie Y Reinholtz [1] and Cardona and Clos [2], they teach the methodology to obtain the profile of the cams and also they specify the definition of the law of motion and checking the profile of the follower cam mechanism. In addition, there are graphs that allow the value of the minimum radius of curvature to be found to design the cam profile; these graphs are in function of the laws of cycloidal displacement, harmonics and the primitive radius of the cam. The functions used in almost all cam designs are the polynomial, sinusoidal and cyclic functions, and the canonically based algebraic polynomials and Fourier based trigonometric polynomials are the traditional methods [3]. This document uses Bezier curves generated from grade three polynomial functions, five and seven that allow the representation of any displacement law and so avoid the unnecessary complication of mathematical calculations. These curves imitate the shape of the polygon that meets the property of invariance to avoid changes in points of continuity and have a smoother stroke.

2 Methodology

2.1 Radius of curvature

Different authors show the procedure according to the approach used for the design of the displacement law and so to find the value of the radius of curvature of the follower cam mechanism. The procedure of the velocity pole, the approximate method and the complex Krasnikov variables can be observed in Chen, [4] and the cases of translation and rotation followers are analyzed; Wilson and Sadler, [5] they replace in the parametric expression obtained from differential calculus of the appropriate derivatives of the expressions for the calculation of the profile coordinates, obtaining the equations of the radius of curvature according to the type of follower and thus finding its value; Shigley and Uicker, [6]; Norton, [7]; Mabie and Reinholtz, [1] they use the same procedure, through two vector closing equations to obtain the same expressions of the calculation.
There are graphs to find the value of the minimum radius of curvature of the cam profile, these are a function of the laws of cycloidal displacement, harmonics and the primary radius of the cam for the case of translation followers and several authors such as Shigley and Uicker, [6], Chen, [4], and Mabie and Reinholtz,[1], make them known.

2.1.1 General equation of the radius of curvature.

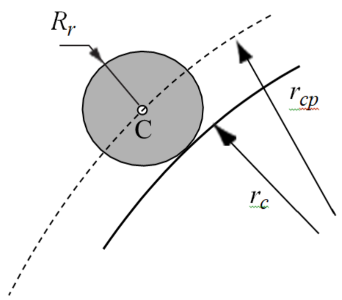

Figure 1. Radius of curvature of the pitch curve

To calculate the radius of curvature for a circular follower with a translational movement, the expressions for the calculation are given below, Zayas, [8]. Fig. 1 shows the radius of curvature of the cam profile and that of the pitch curve, the first curve is the offset of the second curve, and these radii differ in the radius of the follower.

$$r_c(\theta) = r_{cp}(\theta) - R_r \quad (1)$$

Where, $r_c(\theta)$ is the radius of curvature of the cam profile, $r_{cp}(\theta)$ is the radius of the pitch curve and is the radius of the roller R_r.
In the case of translation roller follower, authors Shigley and Uicker, [6], and Chen,[4], and Mabie and Reinholtz, [1], show graphs to calculate the value of the minimum radius of curvature of the profile in function of the harmonic and cyclic displacement laws and the primary radius of the cam. Fig. 2 shows the case of a circular follower with a roller radius R_r and centre C. The pitch curve is the displacement that describes the centres of the follower and its position is calculated using Fig. 2 and the cam profile is given by the displacement of a circumferences bundle in this case the roller follower.

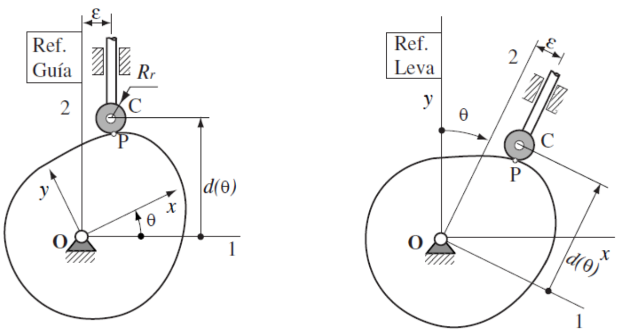

Figure 2. Kinematic inversion. Source: Zayas, [8].

The position of the follower is calculated as

$$\{\overline{OC}(\theta)\}_{1,2} = \begin{Bmatrix} \varepsilon \\ d(\theta) \end{Bmatrix}_{x,y} \quad (2)$$

Deriving equation (2) respect θ on mobile base 1.2

$$\{\overline{OC}'(\theta)\}_{1,2} = \begin{Bmatrix} d(\theta) \\ d'(\theta) - \varepsilon \end{Bmatrix}_{x,y} \quad (3)$$

Deriving equation (3) gives:

$$\{\overline{OC}''(\theta)\}_{1,2} = \begin{Bmatrix} 2d'(\theta) - \varepsilon \\ d''(\theta) - d(\theta) \end{Bmatrix}_{x,y} \quad (4)$$

Making parameters clockwise so that the normal (inward) component of the derivative is expressed as

$$\overline{OC''}|_n(\theta) = \frac{\overline{OC''}(\theta) \times \overline{OC'}_{eje3}}{|\overline{OC'}(\theta)|} \tag{5}$$

$$\overline{OC''}|_n(\theta) = (d'(\theta) - \varepsilon)(2d'(\theta) - \varepsilon) - d(\theta)(d''(\theta) - d(\theta)) \tag{6}$$

Replacing equations (3) and (6) in

$$r_c = \frac{|\overline{OC'}|^2}{|\overline{OC''}|_n}$$

it get the following

$$r_c(\theta) = \frac{(d^2(\theta) + (d'(\theta) - \varepsilon)^2))^{3/2}}{(d'(\theta) - \varepsilon)(2d'(\theta) - \varepsilon) - d(\theta)(d''(\theta) - d(\theta))} \tag{7}$$

Which corresponds to

$$r_c(\theta) = \frac{\left((R_o + s)^2 + \left(\frac{ds^2}{d\theta}\right)\right)^{3/2}}{(R_o + s)^2 + 2\left(\frac{ds}{d\theta}\right) - (R_o + s)\left(\frac{d^2s}{d\theta^2}\right)} \tag{8}$$

2.2 Calculation of the radius of curvature in the movement transition of complete height rise

To avoid double contact between the surface of the follower of roller and the surface of the cam, is required is find a minimum radius of curvature of the pitch curve and that this is greater than the radius of the follower in the convex sections of the cam. The follower will not perform the desired movements correctly, if this case happens and to solve it, the geometrical conditions of the cam or follower must be vary, in this case the radius of the roller follower to enforce $r_{c\,min} > R_r$.
The radius of curvature in the line of pitch is given by:

$$r_{cp} = r_c + R_r \tag{9}$$

r_{cp} is the radius of curvature at the cam pitch surface, r_c is the radius of curvature at the contour of the cam and R_r is the radius of the roller follower.

2.2.1 Transition movement of complete height rise

For the calculation of the radius of curvature of the transition movement complete height rise, there are Bezier curves of degrees 5, 7 and 9 and continuities C^2, C^3 y C^4

C^2

$$b(u) = L\,[\,10u^3 - 15u^4 + 6u^5\,]$$

$$b'(u) = \frac{L}{\beta}[\,30u^2 - 60u^3 + 30u^4\,]$$

$$b''(u) = \frac{L}{\beta^2}[\,60u - 180u^2 + 120u^3\,] \tag{10}$$

$$b'''(u) = \frac{L}{\beta^3}[\,60 - 360u + 360u^2\,]$$

C^3

$$b(u) = L\,[\,35u^4 - 84u^5 + 70u^6 - 20u^7]$$

$$b'(u) = \frac{L}{\beta}[\,140u^3 - 420u^4 + 420u^5 - 140u^6]$$

$$b''(u) = \frac{L}{\beta^2}[\,420u^2 - 1680u^3 + 2100u^4 - 840u^5] \tag{11}$$

$$b'''(u) = \frac{L}{\beta^3}[\,840u - 5040u^2 + 8400u^3 - 4200u^4]$$

C^4

$$b(u) = L\,[\,126u^5 - 420u^6 + 540u^7 - 315u^8 + 70u^9]$$

$$b''(u) = \frac{L}{\beta}[\,630u^4 - 2520u^5 + 3780u^6 - 2520u^7 + 630u^8]$$

$$b''(u) = \frac{L}{\beta^2}[\,2520u^3 - 12600u^4 + 22680u^5 - 17640u^6 + 5040u^7] \tag{12}$$

$$b''(u) = \frac{L}{\beta^3}[\,7560u^2 - 50400u^3 + 113400u^4 - 105840u^5 + 35280u^6]$$

When, $u = (\theta / \beta)$ and $\beta = (\theta_f - \theta_i)$.

The development of the C^2 continuity is the same of degree of the displacement law curve. The method used consist is to find the value of the minimum radius of curvature of the expression (8), for a swivel angle β, predetermined and for different values of the relationship R_o/L. For each of the sections, the swivel angle is varied from 10º until a value of a turning angle that tends to stabilize with increments of 10º. Making the calculation procedure more comfortable, the following variables are added and replaced in the equations (10).

$$A = (R_O + s) = \frac{L}{L}(R_o + 10u^3 - 15u^4 + 6u^5) \tag{13}$$

$$B = \left(\frac{ds}{d\theta}\right) = \frac{L}{\beta}(30u^2 - 60u^3 + 30u^4) \tag{14}$$

$$C = \left(d^2s\right) = L(60u - 180u^2 + 120u^3) \tag{15}$$

$\overline{d}\theta^2$ $\overline{\beta}^2$

Replacing the expressions (9), (13), (14), (15) and (10) in the equation (8).

$$\frac{r_c + R_r}{L} = \frac{(A^2 + B^2)^{3/2}}{A^2 + 2B^2 - A.C} \tag{16}$$

Table 1 is obtained by developing the equation (16). In the left column is observes the values of the rotation angle β, for intervals with continuity movement laws C^2, for transition union with full lift height. The intervals of the rotation angle, vary according to the position in the cam geometry and the following columns show results for each value of the column of the rotation angle with respect to the value of $(r_c + R_r)/L$.

Table 1. Radius of curvature in the movement transition of the complete height rise of bezier curve of continuity C^2

B	R_O/L 0,25	R_O/L 0,5	R_O/L 0,75	R_O/L 1,0	R_O/L 1,5	R_O/L 2,0	R_O/L 2,5	R_O/L 3,0	R_O/L 4,0	R_O/L 6,0	R_O/L 8,0	R_O/L 10,0
	$\frac{r_c+R_r}{L}$	$\frac{r_c+R_r}{L}$	$\frac{r_c+R_r}{L}$	$\frac{r_c+R_r}{L}$	$\frac{r_c+R_r}{L}$	$\frac{r_c+R_r}{L}$	$\frac{r_c+R_r}{L}$	$\frac{r_c+R_r}{L}$	$\frac{r_c+R_r}{L}$	$\frac{r_c+R_r}{L}$	$\frac{r_c+R_r}{L}$	$\frac{r_c+R_r}{L}$
10	0,02	0,03	0,03	0,04	0,06	0,08	0,10	0,13	0,19	0,32	0,50	0,70
20	0,06	0,08	0,10	0,12	0,18	0,24	0,31	0,38	0,56	0,99	1,52	2,15
30	0,11	0,14	0,18	0,23	0,33	0,44	0,57	0,71	1,03	1,80	2,73	3,80
40	0,16	0,22	0,28	0,35	0,49	0,66	0,85	1,06	1,52	2,62	3,88	5,28
50	0,22	0,30	0,38	0,47	0,67	0,89	1,14	1,40	2,00	3,35	4,85	6,47
60	0,25	0,38	0,48	0,59	0,83	1,11	1,41	1,73	2,42	3,96	5,63	7,38
70	0,25	0,45	0,57	0,70	0,99	1,31	1,65	2,01	2,78	4,46	6,23	8,07
80	0,25	0,50	0,66	0,81	1,14	1,49	1,86	2,26	3,09	4,86	6,70	8,60
90	0,25	0,50	0,75	0,91	1,27	1,65	2,05	2,47	3,34	5,18	7,07	9,00
100	0,25	0,50	0,75	1,00	1,38	1,79	2,21	2,65	3,56	5,43	7,36	9,31
110	0,25	0,50	0,75	1,00	1,49	1,91	2,35	2,80	3,73	5,64	7,59	9,55
120	0,25	0,50	0,75	1,00	1,50	2,00	2,47	2,93	3,88	5,81	7,77	9,74
130	0,25	0,50	0,75	1,00	1,50	2,00	2,50	3,00	4,00	5,95	7,92	9,90
140	0,25	0,50	0,75	1,00	1,50	2,00	2,50	3,00	4,00	6,00	8,00	10,0
150	0,25	0,50	0,75	1,00	1,50	2,00	2,50	3,00	4,00	6,00	8,00	10,0

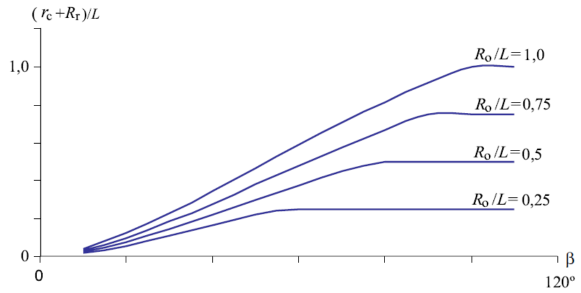

Figure 3. Radius of curvature, Bézier curve of continuity C^2. Relation R_o / L, from 0,25 until 1,0.

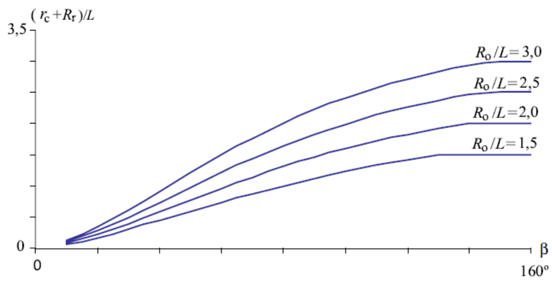

Figure 4. Radius of curvature, Bézier curve of continuity C^3. Relation R_o / L, from 1,5 until 3,0.

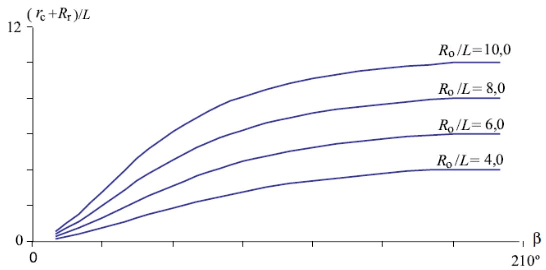

Figure 5. Radius of curvature, Bézier curve of continuity C^4. Relation R_o / L, from 4,0 until 10,0.

3 Conclusions

Analyzing Table 2. Shown below, six columns are observed, the first column on the left is read for transition joint sections of full rise height, some values taken from the relationship R_o / L and in the second, third, fourth and fifth columns and sixth is read for each relation of the first column its respective value of the radius of curvature plus roller radius $r_c + R_r$ of the Bézier curves grade five, seven and nine, harmonic curves and cycloidal curves. The values of the second, third and fourth columns are taken from the development of the graphs for the different continuities and the values of the fifth columns and sixth are taken from the book of Shigley Uicker [6] of the figures of minimum bending radius for cams with harmonic and full rise cycloidal movements.

Table 2. Comparative study of the minimum turning angle so as not to exceed the recommended value of 30° pressure angle.

$\frac{R_o}{L}$	Bézier n=5 Rotation angle β	Bézier n=7 Rotation angle β	Bézier n=9 Rotation angle β	Harmonic Curve Rotation angle β	Cyclic Curve Rotation angle β
1,5	95	111	117	80	100
2	75	88	95	60	80
3	53	62	69	45	56
4	41	48	54	35	43

Table 2 shows that the harmonic curve is the one that behaves best in comparison with the other curves because it is the one that presents the greatest value of the relationship. $r_c + R_r$ which reduces the chance of finding a value radius of curvature r_c negative for a value of roller radius given. In your order of best to worst ratio of bending radius plus roller radius follow the Bezier curves of degree n=5, the cyclic curve, the Bézier curve n=7 and finally the Bezier curve n=9.

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