Process Capability Proposal

with Polynomial Profile

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Abstract

This proposal for a capability indicator aims to evaluate the processes that have a polynomial profile, evaluated in terms of quality characteristics with a significant degree of association. Among the capacity proposals with a linear profile is that of Shahriari and Sarafian, where they demonstrate that the use of capability indices with simple profiles leads to better accuracy in the evaluation of the indicator. These study shows, in contrast to the simple linear profile indicator, that some processes are not suitable for simple linear association, the quadratic profile or the polynomial profile in general is more appropriate. The research shows a specific case study in a food company; the capability assessment with polynomial profile presents a better fit, in contrast to the capability indices with simple linear profile.

Keywords: Capability indices, Linear profiles, Quality statistical control

1 Introduction

Statistical process control (SPC) is a tool used as an analysis and monitoring tool
for variables that may cause inconsistencies within the process. This behavior can be corrected or prevented by using statistical quality measures such as capability indices; expressing with this indicator a possible effect on performance, generally caused by factors other than natural conditions. These factors prevent compliance with the minimum quality requirements, including design specifications, established for each of the variables or quality characteristics of the product, considered by an important number of the multivariate capability indicators, which are not correlated. This research presents a modification of the capability indices of simple linear profiles implemented by Nemati Keshelia, R. Baradaran Kazemzadeha and A. Amirib and R. Noorossan [9], applying indicators with polynomial profiles, in this way it is possible to make an unbiased estimate of the performance when the variables present an important association with a different profile. Linear profiles are commonly represented as parametric models, such as simple linear regression, multiple linear regression, polynomial regression, logistic regression and the new models. There are other investigations such as the one suggested by Woodall [11], which also presents an evaluation of the capability of the process using indicators with linear profiles. However, there are few documents on these types of indicators. This article aims to make a small contribution on this topic, with the purpose of opening the discussion of the applicability or advantage in its implementation.

2 Theory framework

2.1 Profile monitoring

In situations where the quality of a process or product is characterized by a functional relationship between a response variable and one or more independent variables, it is called profile monitoring. Most research on profile monitoring refers to the simple linear profile, i.e. a single independent variable. Guevara et al. [7]

The model of the single linear profile is defined by the following expression,

$$y_{ij} = A_0 + A_1X_i + \varepsilon_{ij}, \quad i = 1,2, ..., n, \quad j = 1,2, ..., k$$

(1)

where $\varepsilon_{ij}$ are the residuals defined as independent random variables normally distributed with mean zero and variance $\sigma^2 \ NID(0,\sigma^2)$. The slope and intersection are called regression coefficients or profile, it is assumed that the $x$ values are fixed and take the same set of values for each sample, Guevara et al. [6]

The capabilities indices are used to evaluate the performance of the process, initially presented by Kane [10], symbolizing it as $C_p$ in charge of measuring the potential without including the process mean,

$$C_p = \frac{USL-LSL}{\text{U.NETL-L.NETL}} = \frac{USL-LSL}{(\mu+3\sigma)-(\mu-3\sigma)}$$

(2)
where $\sigma$ is the process standard deviation, USL and LSL are the lower and upper specification limits respectively. UNTL is the upper natural tolerance limit and LNTL is the lower natural tolerance limit.

The other side, when considering evaluating the process displacement or position with respect to the centralization measure, the appropriate indicator is the $C_{pk}$ defined by the following equation [3]

$$C_{pk} = \min \left\{ C_{pu} = \frac{USL - \mu}{3\sigma}, C_{pl} = \frac{\mu - LSL}{3\sigma} \right\}$$  \hspace{1cm} (3)

Shahriari and Sarafian [3] present a method for calculating the process capability indices when monitoring a simple linear profile, considering the response variable as a characteristic with known distribution and specification limits. This $C_{pk}$ of the response variable is calculated for each level of the explanatory variable, then the $C_{pk}$ is entered as the process capability indice. Ebadi and Shahriari [4] [5] replaced the response variable with a predicted variable at each level of the explanatory variable, then used a multiple process capability indices to measure the process capability, resulting in underestimated $C_{pk}$ values at the explanatory variable levels in the study, and suggested a Bothe [1] method, which uses the proportion of nonconforming elements for its measurement, $P = \min\{P_1, P_2, \ldots, P_p\}$, the capability indice is defined,

$$C_p = \frac{1}{3} \phi^{-1}\{P\}$$  \hspace{1cm} (4)

where $\phi^{-1}$ being the inverse standard normal distribution.

The process capability indice $C_p$ that was defined in equation (2) is a comparison between the natural tolerance limits and the specification limits of a process. In a single linear profile $y = A_0 + A_1X$ is the process reference line, $a_0 + a_1x$ is the conditional mean of $y$ and $x$, then $\mu$ is calculated as follows:

$$\mu = a_0 + a_1x$$  \hspace{1cm} (5)

where $y$ is a normal random variable with mean $a_0 + a_1x$ and variance of $\sigma^2$, $a_0$ and $a_1$ are estimates of $A_0$ and $A_1$ and are calculated as $a_0 = (\Sigma_j^k a_{0j})/k$ $a_1 = (\Sigma_j^k a_{1j})/k$ respectively, $a_{0j}$ and $a_{1j}$ the intercepts and slopes estimated in the j-th sample profile.

The variance of the $\sigma^2$ process is estimated using MSE and calculated as $\text{MSE} = (\Sigma_j^k \text{MSE}_j)/k$, where MSE$_j$ is the estimated variance in the umpteenth sample profile. Therefore, you can define the tolerance limits UNTL and LNTL of the response variable as,
UNTLY(x) = \mu + 3\sigma = a_0 + a_1x + 3\sigma 

LNTL(x) = \mu + 3\sigma = a_0 + a_1x - 3\sigma 

UNTLY and LNTL are two parallel lines where the distance between them is equal to the capability 6\sigma. The specification limits of y are found as a function of the variable x as,

USLY(x) = a_{0u} + a_{1u}x 

LSLY(x) = a_{0u} + a_{1u}x 

The Cp of a single linear profile has a functional form, as presented in following equation,

C_p = \frac{USLY(x) - LSLY(x)}{UNTLY(x) - LNTL(x)} \quad x \in [x_l, x_u] 

By choosing C_p(x) as the process capability indices, it is possible to evaluate the capability at each level of the explanatory variable x, obtaining detailed process information. However, it is necessary in quality control to have a unique capability indices value to give an overall judgment of the process. It is therefore recommended to use the limited area between USLY(x) and LSLY(x) to calculate USLY(x) and LSLY(x) and also the limited area between UNTLY(x) and LNTL(x) to calculate UNTLY(x) and LNTL(x). To determine a single value for the Cp of a single linear profile is,

C_p (profile) = \frac{\int_{x_l}^{x_u} [USLY(x) - LSLY(x)] dx}{\int_{x_l}^{x_u} [UNTLY(x) - LNTL(x)] dx} \quad x \in [x_l, x_u] 

UNTLY(x) and LNTL(x) are two parallel lines, similarly USLY(x) and LSLY(x) are defined in equations (8) and (9). The distance of these parallel lines can be considered as their difference, so the capability indice of the linear profile C_p (profile) is calculated as follows,

C_p (profile) = \frac{a_{0u} - a_{0l}}{6\sigma} 

and Cpk is described as:

C_{pk}(x) = \min \left\{ \frac{USLY(x) - \mu_y(x)}{\mu_y(x) - LSLY(x)} \right\} \quad x \in [x_l, x_u] 

where \mu_y(x) is the function of the reference line. C_{pk}(x) gives the value of C_{pk} of
a single process at each level of \( x \), the \( C_{pk} \) of a single linear profile is calculated:

\[
C_{pk} = \min \left\{ \frac{\int_{x_l}^{x_u} [USL_y(x) - \mu_y(x)] \, dx}{\int_{x_l}^{x_u} [\mu_y(x) - LSL_y(x)] \, dx}, \frac{\int_{x_l}^{x_u} [\mu_y(x) - UNTL_y(x)] \, dx}{\int_{x_l}^{x_u} [\mu_y(x) - LNTL_y(x)] \, dx} \right\}
\] (14)

The capability indice of process \( C_{pk} \) when only the upper or lower functional specification limits are available can be calculated using the following equations:

\[
C_{pk(\text{profile})} = \frac{\int_{x_l}^{x_u} [USL_y(x) - \mu_y(x)] \, dx}{\int_{x_l}^{x_u} [\mu_y(x) - LSL_y(x)] \, dx}
\] (15)

\[
C_{pi(\text{profile})} = \frac{\int_{x_l}^{x_u} [\mu_y(x) - LSL_y(x)] \, dx}{\int_{x_l}^{x_u} [\mu_y(x) - LNTL_y(x)] \, dx}
\] (16)

when the value of \( LSL_y(x) \) is greater than \( \mu_y(x) \) in \( [x_l, x_m] \) and less than \( \mu_y(x) \) in \( [x_m, x_u] \), then the minimum capability indice is calculated as follows:

\[
C_{pk(\text{profile})} = \min \left\{ \frac{\int_{x_l}^{x_m} [USL_y(x) - \mu_y(x)] \, dx - \int_{x_m}^{x_u} [ \mu_y(x) - USL_y(x)] \, dx}{\int_{x_l}^{x_m} [ \mu_y(x) - LSL_y(x)] \, dx}, \frac{\int_{x_l}^{x_u} [\mu_y(x) - LNTL_y(x)] \, dx}{\int_{x_l}^{x_m} [\mu_y(x) - LSL_y(x)] \, dx} \right\}
\] (17)

### 2.2 Multivariate capability indice assuming independent variables

The authors Chen et al. [2] have developed capability indices that evaluate the quality characteristics that intervene within a productive process, showing in their proposal the degree or proportion with which a product can comply integrally with each of the specifications and requirements of the market. Based on the compliance ratio of each of the process quality characteristics, the capability indice for multiple characteristics are obtained as follows,

\[
C_{pk}^T = \frac{1}{3} \phi^{-1} \left[ \frac{P}{2} (2\phi(3C_{pk})-1)+1 \right]
\] (18)

where \( C_{pk} \) denotes the value of the j-th characteristic for \( j = 1, 2, \ldots, N \), \( p \) is the number of characteristics and \( \phi \) is the standard normal distribution.

### 2.3 Proposed polynomial capability indice

The new proposal presents a modification of equations (6), (7), (8) and (9) of R. Nemati et al. [9], the specification limits \( USL_y \) and \( LSL_y \) are obtained from the confidence intervals, for the polynomial case, defined as follows:
USL_{y}(x) = a_{0u} + a_{1u}x + a_{1u}x^2 \quad (19)

LSL_{y}(x) = a_{0l} + a_{1l}x + a_{1l}x^2 \quad (20)

\text{UNTL}_{y}(x) = \mu + 3\sigma = a_{0} + a_{1}x + 3\sigma \quad (21)

\text{LNTL}(x) = \mu + 3\sigma = a_{0} + a_{1}x - 3\sigma \quad (22)

3 Methodology

The evaluation of the multivariate capability indices is verified by a case study in a food company, where two variables chloride and Brix are selected. To determine the capability indice with linear profiles, proceed as follows: 1) Tabulate the data taken in the manufacturing process of the food product. 2) Calculate the traditional univariate capability indice $C_{pk}$ for both variables. 3) Evaluate the capacity indices taking the proposals of Chen et al. [2], considering the independent variables, Nemati et al. [9], with a simple linear profile and the proposal of this study that considers a polynomial profile.

The information comes from a food company, the variables studied were °Brix and Chloride shown in Table 1, whose specifications are °Brix [29.0-32.0] and Chloride [2.30 - 2.80]. The physicochemical characteristics of the final product and the data were collected over a period of 9 months; see Table 1, R. Herrera et al. [8].

Table 1. Coded partial measurements of the Historical °Brix and Chloride variables. By the Authors

<table>
<thead>
<tr>
<th>No.</th>
<th>°Brix</th>
<th>Chloride</th>
<th>No.</th>
<th>°Brix</th>
<th>Chloride</th>
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<td>2.32</td>
<td>55</td>
<td>29.4</td>
<td>2.61</td>
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</table>
4 Results

Initially the traditional univariate capability indices of equation (1) were determined for each variable independently, as shown in Figures 1 and 2, obtaining, evaluated in equation (1), $C_p$ variable °Brix: 0.59 and a $C_p$ of the Chloride variable: 0.77

Figure 1. Índice de capability for variable Brix $C_p = 0.59$

![Figure 1](image1.png)

Figure 2. Índice de capability for variable Chloride $C_p = 0.77$

![Figure 2](image2.png)

Both [1] indicators show a lower result than the unit, in this case the process doesn't meet the specifications in the two (2) variables.
Table 2. Comparison of the values obtained in the capability indices of the °Brix and Chloride variables. By the authors

<table>
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<th>Multivariate</th>
<th>Univariate capability indice</th>
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</thead>
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<td></td>
<td></td>
</tr>
<tr>
<td>R. Namiti</td>
<td></td>
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<tr>
<td>Cp(profile)</td>
<td>1.68</td>
<td>Brix</td>
</tr>
<tr>
<td>Cpk(profile)</td>
<td>1.68</td>
<td>Chloride</td>
</tr>
</tbody>
</table>

In contrast to the polynomial profile, the variables have a strong relationship with an R-squared of 0.951 which guarantees a high correlation between them. The equation of the polynomial model adjusted for the case study is,

\[
\text{Brix} = 26,948 + 1,66607 \times \text{Chloride} - 0,218416 \times \text{Chloride}^2
\]

The multivariate capability index is calculated by modifying the proposal of R. Nemati et al.[9] of the process, by a polynomial model that is denoted as \( C_{pk} \), for this purpose the upper and lower specification limits of the process (USL and LSL) and the upper and lower process limits (UNTL and LNTL) are determined in the following way, the USL and LSL limits are obtained with the confidence intervals of the process which are:

\[
\begin{align*}
\text{USL} &= 29,113 + 2,6482 x - 0,1611 x^2 \\
\text{LSL} &= 24,7831 + 0,683946 x - 0,275733 x^2 
\end{align*}
\]

The specification limits based on equations (19) and (20) USL = 29,113 + 2,6482 x – 0,1611 x^2 and LSL = 24,7831 + 0,683946 x – 0,275733 x^2, where \( \mu \) is the equation of the variable \( Y \), \( \sigma \) is the root of the mean square of the error which is 0,9251. The equations of the process limits equations (21) and (22) evaluated in the information are respectively for the upper and lower limit UNTL = 29,723 + 1,66607x – 0,218416x^2 and LNTL = 24,172 + 1,66607x – 0,218416x^2.

In this way, the process capability index is calculated taking into account the specifications of the company's Chloride variable which are [2.3-2.8] and based on equation (14), (15) and (16) modifying a linear polynomial model, equations (19), (20), (21) and (22).
Performing the calculations corresponding to the above ratio gives a capability indice with a polynomial profile of $CpP = 1.816$, indicating that the process satisfactorily meets the established specifications. $CpkP$ is calculated, equations (19), (20), (21), (22) and (23), in order to identify where the process is displaced. The values of the centering indicators $CpkP = \min\{1.816, 1.814\}$, indicate a centered process.

5 Discussion

The univariate capability index for each of the variables indicates that the design specifications are not met. The multivariate capability indice for independent events gives the same result as the univariate capability. However, applying the simple linear profile, as well as the polynomial regression profile, the case study results indicate that the process is capable of meeting the design requirements. The capability indice of the polynomial profile presents satisfactory results according to the experience of those responsible for evaluating the quality of the product, which indicates that this technique is more accurate for the calculation of this indicator.

References


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