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Stability Analysis of a Magnetic Levitator

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Abstract

In this article we present a stability analysis based on the theory of dynamic systems, for a magnetic levitator system, which is used as a principle of operation of different engineering applications, such is the case of the Maglev train.

Keywords: Stability analysis, magnetic levitator, Maglev train

1 Introduction

The study of the stability of linear and non-linear systems is of great importance in most engineering applications, for example, when you want to make designs for control systems the stability of the system plays a very important role. The first step to perform a stability analysis of a system is to obtain a set of equations through laws of mechanics, electrical, fluid mechanics, etc., which describe the dynamics of the system, which is known as mathematical modeling. This model may or may not consider a certain number of system characteristics, but in a first approximation it is preferable to use a simplified model to obtain in a simpler way a general idea about the solution, and then increase the complexity of the system if necessary [1,2].

Magnetic levitation has been considered of great interest since the 1930s. It serves to illustrate many of the fundamental principles of electrical and electronic engineering such as electromagnetism and electrodynamics, control theory and analog and digital design circuits. Mathematical modeling is an essential part that registers the approximate behavior of a real levitator, and with classic analysis techniques that model is clearly and precisely developed [3].

2 Magnetic levitator

The magnetic levitator turns out to be an unstable and strongly non-linear system. The principle of operation is based on a coil that generates a magnetic force of attraction by circulating a current through it. The objective of the system is to keep levitating a sphere of ferromagnetic material by means of said force and that in addition the levitation remains stable [4].

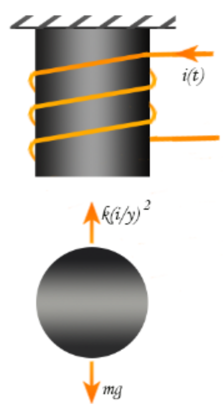

Figure 1: Magnetic levitator system.

3 Mathematical modeling of the magnetic levitator

Figure 2 shows the free body diagram of the sphere that is presented in the system of a magnetic levitator for which it is desired to obtain the second order differential equation that describes its behavior (movement), said equation will be used to perform the stability analysis of the system.

Figure 2: DCL sphere.

It makes use of Newton's second law to mathematically describe the system, as shown below [5,6].

$$+\downarrow \sum Fy = ma$$

$$mg - \frac{k}{m}\left(\frac{i}{y}\right)^2 = m\ddot{y}$$

$$\ddot{y} = g - \frac{k}{m}\left(\frac{i}{y}\right)^2$$

For this problem, we take the following values:

$$k = 2, \quad m = 1kg, \quad g = 10\frac{m}{s^2}, \quad i = 5A$$

The previous differential equation of second order describes the behavior of the system. In this case the excitation or input (current) is considered constant.

3.1 State-space system

$$x_1 = y$$

$$x_2 = \dot{y}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = g - \frac{k}{m}\left(\frac{i}{x_1}\right)^2$$

Here, critical points are given by

$$\dot{x}_1 = x_2 = 0, \dot{x}_2 = 0$$

$$mg - \frac{k}{m} * \frac{i^2}{x_1^2} = 0 \rightarrow x_1 = \pm\sqrt{\frac{ki^2}{mg}}$$

For the values of the parameters selected above, the solutions are obtained:

$$x_1 = \pm\sqrt{5}$$

Then, the critical points or equilibrium points are then determined as:

$$\left(\sqrt{5}, 0\right); \qquad \left(-\sqrt{5}, 0\right)$$

It can be seen that for point $x_1 = 0$, a singularity (Asymptote) is presented. We now want to linearize the system around the equilibrium points to know how stability is defined in these points. For this it is necessary first to obtain the Jacobian matrix of the system [7], as shown below.

$$J = \begin{bmatrix} 0 & 1 \\ 100/x_{10}^3 & 0 \end{bmatrix}$$

Now, for $\left(\sqrt{5}, 0\right)$ we have

$$A = \begin{bmatrix} 0 & 1 \\ 4\sqrt{5} & 0 \end{bmatrix}$$

$$\det\left(\lambda I - A\right) = 0 \rightarrow \lambda^2 - 4\sqrt{5} = 0$$

$$\lambda = \pm 2.99 \rightarrow \text{saddle point}$$

Now, for $\left(-\sqrt{5}, 0\right)$ we have

$$A = \begin{bmatrix} 0 & 1 \\ -4\sqrt{5} & 0 \end{bmatrix}$$

$$\det\left(\lambda I - A\right) = 0 \rightarrow \lambda^2 + 4\sqrt{5} = 0$$

$$\lambda = \pm 2.99i \rightarrow \text{center point}$$

Therefore, using the Poincaré-Bendixon theorem we have the following. The set selected in this case is $F \equiv 1 \leq (x_1 + 2)^2 + x_2^2 \leq 2$, said set is compact (closed and bounded as can be seen in Figure 3). To verify that it is positively invariant, the scalar product $< x, v(x) >$ is performed and evaluated at the borders of field F to see where the field lines are directed.

$$\langle x, v(x)\rangle = \langle (x_1, x_2), (x_2, 10 - \frac{50}{x_1^2})\rangle$$

$$\langle x, v(x)\rangle = x_1x_2 + 10x_2 - 50\frac{x_2}{x_1^2}$$

Transforming the previous equation to polar coordinates, taking $x_1 = r\cos(\theta)$, $x_2 = r\sin(\theta)$ we obtain

$$\langle x, v(x)\rangle = r^2 sen(\theta)\cos(\theta) + 10r sen(\theta) - \frac{50}{r}\frac{\tan(\theta)}{\cos(\theta)}$$

Evaluating r at the borders we have:
$r = 1$,

$$\langle x, v(x)\rangle = sen(\theta)\cos(\theta) + 10sen(\theta) - 50\frac{\tan(\theta)}{\cos(\theta)}$$

$r = \sqrt{2}$

$$\langle x, v(x)\rangle = 2sen(\theta)\cos(\theta) + 10\sqrt{2}sen(\theta) - \frac{50}{\sqrt{2}}\frac{\tan(\theta)}{\cos(\theta)}$$

Now, we perform the test for different values of θ and r, where the direction of the field changes in the borders, for example:

$$r = 1 \;\; y\; \theta = \frac{\pi}{9} \rightarrow \langle x, v(x)\rangle < 0$$

$$r = \sqrt{2} \;\; y\; \theta = \frac{\pi}{9} \rightarrow \langle x, v(x)\rangle > 0$$

The above indicates that in $r = 1$ the field lines are directed outwards and for $r = \sqrt{2}$ inwards, so it is concluded that there is a closed trajectory in F, in the sense that it remains in the set for all time instant. Since the set F does not contain any equilibrium point according to the Poincaré-Bendixon theorem, it is said that the system has closed orbits totally contained in F.

4 Conclusion

The behavior of the system, the points of equilibrium (see Figure 3), the field flow and the types of solution can be appreciated graphically, verifying the existence of a periodic solution in the system.

From the previous analysis the importance in the study of the stability of the systems stands out because of the great help that it presents when one wants to predict the behavior of these systems, especially when the systems

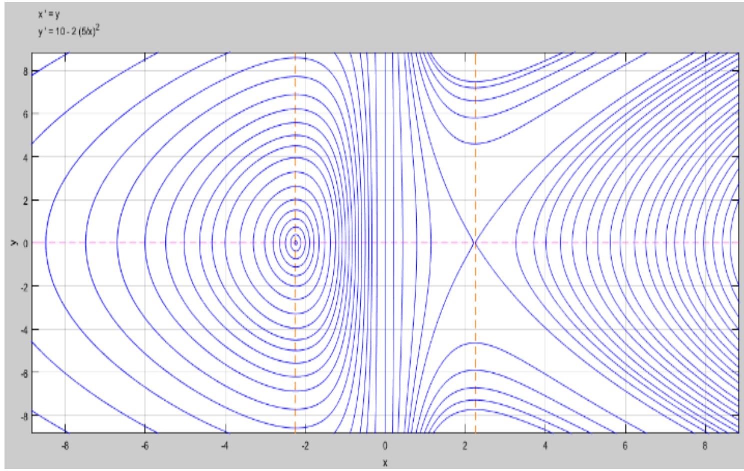

Figure 3: Phase portrait of the magnetic levitator.

are non-linear, since such behavior can generally be chaotic.

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