Approach to Solving the Multi-Objective Vehicles Routing Problem with Capacity Restrictions

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Abstract

The CVRP is a problem that aims to determine the routes that each of the vehicles in a fleet should follow in order to reach their destination points in a distribution network, so that the total distance travelled or the time spent by the set of vehicles is the least. The CVRP is considered a combinatorial problem, given that as the number of clients to be served increases, the number of possible routing solutions also grows exponentially. This exponential growth immediately exceeds the processing capacity of current computers, so it is advisable to use approximate methods, which do not ensure finding the optimal solution, but if they generate very good solutions (such as heuristics and metaheuristics), and on the contrary the use of exact methods is unsuccessful, since the computational time consumption is very high. In essence, this article presents a proposal to address the problem of vehicle
Routing, based on heuristic and metaheuristic methods, competitive in terms of response quality and computational time.

**Keywords:** Heuristics, metaheuristics, combinatorial problems, exact methods, vehicle routing

1. Introduction

The present article, fruit of a basic research work, analyzes and proposes an alternative to give solution to the vehicles routing problem with capacity restrictions (CVRP) based on Heuristic and Metaheuristic techniques. These techniques are characterized for being procedures based on experience that do not guarantee to find the optimal solution, but obtain very good solutions for complex problems in a reduced computational time.

The characteristics of the routing problem that was addressed to give rise to the approach were 1) a single tank, all vehicles start from the same place, 2) Homogeneous Fleet, all vehicles will have the same load capacity, 3) Weight and/or volume restrictions for the vehicles, 4) The number of vehicles to serve is not pre-established, i.e. the number of vehicles will be calculated before determining the routes of each one, 5) The total demand of the clients visited by a vehicle must not exceed its capacity, i.e. a client must not be visited by more than one vehicle, 6) Deterministic, the data of the problem will be known and constant.

The designed approach aims to minimize the number of vehicles needed to meet the demand of all customers and the total distance traveled by the vehicles. In order to achieve these objectives, Sweeping was used, assigning to each vehicle the clients that it must attend according to its capacity and the demand of the clients; then a combination is used between the Nearest Neighbor heuristic and a GRASP procedure to generate initial solutions for each route; finally, an optimization procedure is carried out with Tabu Search Metaheuristics.

After the design of the approach, an experimentation process was carried out to define the operating parameters that provide the best results for the heuristics used. Finally, a process of validation of the approach was carried out, which consisted in giving solution to instances studied by other authors whose optimal solution is known, carrying out an analysis and comparison of the results obtained in order to measure their effectiveness to give solution to the problem studied.

2. State of the Art

The Vehicle Routing Problem, VRP is one of the most important problems within
combinatorial optimization problems. Its simplest version is the problem of routing vehicles with capacity restrictions (CVRP). Its introduction dates back to 1959 when (Dantzig & Ramser, 1959) [3] describe a real application for gasoline distribution, proposing a mathematical formulation of the problem and an algorithmic approach. Later (Clarke & Wright, 1964) [2] they developed an algorithm that improved the response of the algorithm of (Dantzig, Ramser). From these works a line of research has been generated that has gained strength over the years and that today continues to be the subject of research due to the degree of application to real problems and their great complexity. In recent decades different approaches to the CVRP have been explored (Toth and Vigo, 2002 [11], Golden et al. 2008 [7], Juan et al. 2011, Faulin and Juan, 2008 [9]). These approaches have a broad spectrum that starts with the use of pure optimization methods such as linear programming to solve small problems with relatively simple constraints, to the use of heuristics and metaheuristics that offer almost optimal solutions to medium and large problems that can handle more complex constraints at the same time (Cáceres et al. 2012) [1]. With respect to recent works related to the present research work and with similar methodologies, we can highlight the study carried out by (Daza, Montoya, & Narducci, 2009) [4], which developed an "alternative" procedure to solve the problem of routing vehicles with capacity limitations and homogeneous fleet (CVRP). The study proposed a metaheuristic algorithm consisting of two phases: Route design and fleet planning. The first phase of route design is composed of heuristic and metaheuristic procedures where an initial solution is built that are later improved by the taboo search metaheuristics obtaining solutions not dominated in the polynomial calculation time (Pareto front). In the second phase of the planning (sheduling) the problem is approached taking as reference a problem of sequencing of identical parallel machines where the resources are the vehicles or transporters, and the works, the routes to which they must serve, the processing times for this particular case are replaced by the time it takes a transporter to serve all customers previously established in the route. Two instances were proposed, one random and one real; in such instances very good results were obtained in terms of computational time (much less than one minute) and also in terms of optimizing the number of trucks in the fleet which is reflected in the final costs. For its part (Gallart Suárez, 2009) [5] designed and implemented two GRASP algorithms (Greedy randomized adaptive search procedure), for the problem of routing vehicles with capacities and homogeneous fleet considering that a customer may have greater demand than the capacity of a delivery vehicle. The first algorithm was used to optimize the packaging space of the vehicle in three dimensions, considering the order of recipients, weight and dimensions of the packages, while the second algorithm was used to optimize the routes and then used a local search algorithm 2-
opt to improve the solution obtained with the GRASP. The results obtained are compared with a random solution, a voracious solution and a hybrid solution (random and voracious), proving that the GRASP obtains better solutions than the mentioned algorithms.

3. Parameter Determination

The approach uses three parameters belonging to the different heuristic and metaheuristic methods that make it up, such as alpha (α), elitism parameter in the GRASP function, the size of the tabu list (L) and the stop criterion, which will be the number of iterations without improvement (I), the latter two belonging to the metaheuristic taboo search. These parameters can take any positive value, unlike alpha (α) which must be in a range of 0 to 1. It is worth clarifying that the value of these parameters that have no restriction should be proportional to the size of the instance to be resolved. In order to estimate the values that these parameters should take, in such a way that they yield better results in terms of solving the vehicle routing problem, a variance analysis was carried out. For this purpose, an experiment was designed in which three levels were established for each parameter. The levels used are presented below:

<table>
<thead>
<tr>
<th>Factor</th>
<th>Low level</th>
<th>Medium level</th>
<th>High level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tabú list (L)</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Iterations without improvement (I)</td>
<td>20</td>
<td>35</td>
<td>50</td>
</tr>
<tr>
<td>Alpha (α)</td>
<td>0.2</td>
<td>0.5</td>
<td>0.8</td>
</tr>
</tbody>
</table>

With these levels for the parameters, an experiment 33 is generated that results in 27 scenarios. Four runs were made for each scenario of the experiment. In order to carry out the experiment and obtain the most suitable parameters to work with the proposed approach, an instance of 47 clients developed by (Augerat, 2000) with their respective depot and truck capacity equal to 100 was taken. The optimal solution of the instance includes 7 trucks or routes and a total distance travelled of 1073. ANOVA splits the total distance variability into separate parts for each of the effects, then tests the statistical significance of each effect by comparing its average square against an estimate of the experimental error. Thus, the effect that has a P-value less than 0.05 indicates that they are significantly different from zero with a
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confidence level of 95.0%. After applying the analysis of variance to our experiment, it is found that the parameters taking into account have an effect on the results of the approach, but not on their interactions. By optimizing these parameters, it is found that the values for which the best solutions to the studied problem are obtained are the following:

Table 2. Optimal Values for Metaheuristic Parameters

<table>
<thead>
<tr>
<th>Factor</th>
<th>Level</th>
<th>High</th>
<th>Optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tabú list</td>
<td>5</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>Iterations without</td>
<td>20</td>
<td>50</td>
<td>43</td>
</tr>
<tr>
<td>improvement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alpha</td>
<td>0,2</td>
<td>0,8</td>
<td>0,8</td>
</tr>
</tbody>
</table>

4. Results

Once the adequate parameters of the metaheuristics are established, the results of the approach are compared with the selected instances to observe and analyze the results obtained and thus measure the effectiveness and efficiency of the proposal. For this process three instances of different authors were chosen, which have established the optimal values of the two objectives to minimize, which are the number of trucks and the total distance. Each instance was run 4 times in order to observe and compare the results obtained, in addition to this the instances were run with our proposal first with the routing axis rotated at 0°, and then making a rotation of the axis from all customers, i.e., the sweep starts at the angle of the selected customer, this to observe if there is any positive change in the responses. The instances studied were the following:

**Instance 1:** E-n22-k4 (Euclidian Distances, 1 depot, 21 customers, truck capacity = 6000), minimum number of trucks or routes = 4, optimum distance value = 375. Prepared by (Cristophides & Eilon, 2000).

**Instance 2:** P-n45-k5 (Euclidian Distances, 1 depot, 44 customers, truck capacity = 150), minimum number of trucks or routes = 5, optimum distance value = 510. Prepared by (Augerat, 2000).

**Instance 3:** P-n50-k7 (Euclidian Distances, 1 warehouse, 49 customers, truck capacity = 150), minimum number of trucks or routes = 7, optimum distance value = 554. Prepared by (Augerat, 2000).
In order to address these instances, the proposed approach was used, first applying the method known as sweeping from the zero degree of inclination, to define the number of routes or vehicles. Then, for each vehicle, a combination of the nearest neighbor heuristic with GRASP is applied to construct an initial route to each vehicle. Finally, the metaheuristics called Taboo Search is applied to look for an improvement in the built routes that reduces the total distance traveled by the vehicles. The three instances were approached in a second occasion, but applying a Cartesian Rotation, to repeat the sweep and in this way look for new configurations of vehicles that reduce their quantity, without neglecting the total distance traveled. The results are presented below:

Table 3. Best solution found doing the sweep without Cartesian rotation

<table>
<thead>
<tr>
<th>Instance</th>
<th>Optimal Solution</th>
<th>Approach Solution</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vehicles</td>
<td>Distance</td>
<td>Vehicles</td>
</tr>
<tr>
<td>E-n22-k4</td>
<td>4</td>
<td>375</td>
<td>4</td>
</tr>
<tr>
<td>P-n45-k5</td>
<td>5</td>
<td>510</td>
<td>5</td>
</tr>
<tr>
<td>P-n50-k7</td>
<td>7</td>
<td>554</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 4. Best solution found doing the sweep with Cartesian rotation

<table>
<thead>
<tr>
<th>Instance</th>
<th>Optimal Solution</th>
<th>Approach Solution with Rotation</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vehicles</td>
<td>Distance</td>
<td>Vehicles</td>
</tr>
<tr>
<td>E-n22-k4</td>
<td>4</td>
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<td>P-n45-k5</td>
<td>5</td>
<td>510</td>
<td>5</td>
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<td>P-n50-k7</td>
<td>7</td>
<td>554</td>
<td>7</td>
</tr>
</tbody>
</table>

It can be seen that the proposed approach has a very good performance since its results are closer to the optimal solutions in more than 93% for all instances evaluated. This aspect is very important for practical cases since this would guarantee a good assignment of trucks to different groups of clients.

5. Conclusions

After the successful completion of this research, the following conclusions are drawn:
• The best results obtained by the proposed tool were presented when the Cartesian rotation was used, moving away from the best solution in the best of cases only 3.7% of the optimal value for instance 2. Giving to understand the quality of the response obtained and the need to include this strategy in subsequent studies.

• The importance of the metaheuristic taboo search to improve the solutions obtained by the nearest metaheuristic GRASP+ Neighbor. It was repeated throughout the investigation to find very good values of improvement found by taboo search with respect to the constructive heuristics, reaching in some cases to surpass 40% of improvement.

• The good performance of the sweeping heuristics at the moment of routing, it was observed during the investigation that for all the cases the optimal value for the number of routes was found, and in addition to this balanced routes were presented without intercrossing between them.

• The importance of the application of the design of experiments (DOE) in this investigation to find the values of the parameters of the metaheuristics that minimize the values of distance of the solutions found.

• In spite of the fact that the computational time used was a variable that was not much talked about, we can say that the times used by the tool are considerable due to the great number of iterations that were carried out, the greater times were presented when the Cartesian rotation was used, since when using this strategy the procedure is repeated many more times than when it is rotated from 0° only. As a recommendation for further research, it is recommended to use another faster programming platform to obtain better results in terms of computational time used.

References


Received: September 12, 2018; Published: September 26, 2018