Hydrodynamic Control of the Growth of the Two Dimensional Boundary Layers around a Flat Plate Placed in the Center of Convergent: Concept of Uniform Accessibility from the Point of View of the Diffusion Matter

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Abstract

The authors indicated numerically, that a plane surface placed in the center of convergent and crossed by a Newtonian fluid, in laminar and permanent flow is uniformly accessible from certain distance of the entering edge when the velocity at the outer boundary of the boundary layer that develops on its increases linearly with the longitudinal component $x$. They explained the concept of hydrodynamic and mass uniform accessibility and set its validity area according to the Reynolds number. Such study can assist in the understanding of vapor phase deposition phenomena of semiconductors.

**Keywords:** forced convection, vapor deposition, uniformly accessible surface, mass transfer, semiconductors
1. Introduction

The control of the boundary layer finds numerous, different applications in the industrial sector. One of the applications of this process is the technique of organometallic vapor phase epitaxy (OMVPE) [1, 2]. In fact, in this technique, it is sought to standardize on a fairly large extent the deposition thicknesses of metal compounds heated to high temperature. For example, let us consider the deposit of the metallic compounds obtained from gaseous mixture consisting essentially of hydrogen acting as a carrier gas with in low concentrations of trimethylgallium, trimethylaluminium and trimethyllantimony. The gas mixture, which should be extracted at ordinary temperature of 25°, is pushed inside the channel. If the conditions are well chosen, in immediate contact with the plane plate carried out in a temperature of 800°C, the organometallic gases break down and the metal atoms are deposited on the plate to form the desired solid compound. In reality, the reaction mechanism is poorly known and it is assumed that above the plane plate the following chemical reaction occurs [3, 4]

$$
\sigma Al + (1 - \sigma) Ga + \frac{1}{4} Sb_4 \rightarrow Ga_{1-\sigma} Al_\sigma Sb
$$

Through this study, we ask whether it is possible to control the flux of the material diffusion on the plane plate by imposing a velocity on the external border of the boundary layer which develops on the latter, a law of variation with the x coordinate in the following form: $U_e(x) = C x$. In other words we ask whether it is possible to control the thicknesses of the metallic compounds deposited on the substrate by imposing a variation law of the velocity at the external border of the boundary layer which develops on the substrate.

The theoretical works of [5] indicated that it is possible to control the material diffusion flux on the plate provided that it is placed in the center of a channel with a hyperbolic walls, whose wall profile is given by: $r^2 \sin[2\theta] = Cte$ which results in linear increase of the longitudinal component of the velocity according to direction of the $Ox$. In this case, the velocity profile of the border in the boundary layer that develops on the plane plate is given by [6,7] $U_e(x) = C x$.

The results presented by the cited authors [5] are insufficient to explain the concept of the uniform accessibility. Indeed, they do not specify the conditions of the uniform accessibility according to all parameters of the studied geometric, namely: (i) the link between the uniform accessibility and the geometric parameters of the studied system, (ii) the relationship between the uniform accessibility of the plane surface and the Reynolds number.
Figure 1: Schematic representation of the duct and of the co-ordinates

2. Mathematical formulation of the problem

The geometry of the studied problem is presented in figure 1. It is a flat plate placed in the center of a channel and a Cartesian reference \((O, x, y, z)\). Forcing a Newtonian fluid in the case of hydrogen to circulate in laminar and permanent flow inside the channel follows the \(Ox\) direction. Adding the following simplifying assumptions: (i) the viscous friction and the radiation transfers are negligible, (ii) the mass fractions are constant and uniform, (iii) the diffusion coefficients of the reagents are equals.

2.1 Continuity equation:

\[
\frac{\partial U^*}{\partial x^*} + \frac{\partial V^*}{\partial y^*} = 0
\]  

(2)

2.2 The conservation equation of the movement quantity:

\[
(U^* \frac{\partial U^*}{\partial x^*} + V^* \frac{\partial U^*}{\partial y^*}) = U_e^* \frac{dU_e^*}{dx^*} + \frac{1}{Re} \frac{\partial^2 U^*}{\partial y^*^2}
\]  

(3)

2.3 Diffusion equation of matter:

\[
(U^* \frac{\partial C_k^*}{\partial x^*} + V^* \frac{\partial C_k^*}{\partial y^*}) = \frac{1}{Pe_m} \frac{\partial^2 C_k^*}{\partial y^*^2}
\]  

(4)

2.4 Boundary conditions:

- On the surface of the substrate \((y^* = 0)\)
  
  \[
  U^* = 0; \quad V^* = 0; \quad C_k^* = 1
  \]
- At the external border of the boundary layer

\[ U^* \rightarrow U_e^*; C_k^* \rightarrow 0 \]

3. Numerical processing

The transport equations, after being discretized, can all be put in the following unique form:

\[
\phi_{i,j}^{k+1} = (1 - \beta)\phi_{i,j}^k + \beta \left[ \frac{(a_{i+1,j}\phi_{i+1,j}^k + a_{i,j+1}\phi_{i,j+1}^k + a_{i-1,j}\phi_{i-1,j}^k + a_{i,j-1}\phi_{i,j-1}^k) + (\Delta x_i \Delta y S_{\phi}^*)}{(a_{i+1,j} + a_{i-1,j} + a_{i,j+1} + a_{i,j-1})} \right]
\]

The \( \phi^* \) are the dimensionless unknowns composed of quantities \((C_k^*, U^*, V^*)\), the \( a_{i,j} \) are given as functions of the network steps, the dimensions of the controlled volume and the physical constants of the problem. They are calculated using the numerical scheme PLDS (Power low Differencing Scheme) of Patankar [8, 9].

4. Results and discussion

4.1 validation of the calculation code

After validating our calculation code, we impose a profile of velocity \( U_e^*(x^*) = 1 \) at the border of the boundary layer which develops on the flat plate. Our calculation code gives us the results shown in figure 2; it is well known in the literature because it is the Blasius flow [10].

4.2 Flow study: Definition of hydrodynamic boundary layer and hydrodynamic entry distance: Concept of uniform accessibility from hydrodynamic point of view

Figure 3 shows the variation of the longitudinal component of velocity \( U^* \) in terms of the dimensionless ordinate \( y^* \) for different values of the dimensionless abscissa \( x^* \) and the Reynolds number \( Re \). In general, when \( y^* \) increases, the component \( U^* \) differ from 0 value on the flat plate to value \( U_{max}^* \) independent of \( y^* \). It is also possible to define, on the flat plate, a hydrodynamic boundary layer with thickness \( \delta_H^* \) equal to the value of \( y^* \) from which \( U^* = U_{max}^* \).
In figure 4 which illustrates the variations of $\delta_H^*$ in terms of $x^*$ for $Re = 1500$, we see in particular that from certain hydrodynamic distance $x_{eH}^*$, the thickness $\delta_H^*$ does not depending on $x^*$ anymore and takes a maximum value $\delta_{Hmax}^*$; then we can say that the flat plate is uniformly accessible from the hydrodynamic point of view. Table 1 summarizes the values of $\delta_{Hmax}^*$ and $x_{eH}^*$ for different Reynolds number values.

<table>
<thead>
<tr>
<th>$Re$</th>
<th>$\delta_{Hmax}^*$</th>
<th>$x_{eH}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0,053</td>
<td>0,222</td>
</tr>
<tr>
<td>300</td>
<td>0,041</td>
<td>0,395</td>
</tr>
<tr>
<td>500</td>
<td>0,038</td>
<td>0,405</td>
</tr>
<tr>
<td>800</td>
<td>0,030</td>
<td>0,438</td>
</tr>
<tr>
<td>1500</td>
<td>0,023</td>
<td>0,401</td>
</tr>
<tr>
<td>2000</td>
<td>0,016</td>
<td>0,533</td>
</tr>
</tbody>
</table>

Table 1: Values of $x_{eH}^*$ and $\delta_{Hmax}^*$ as a function of Reynolds

The local values variations of the coefficient of friction represented in figure 5 reaffirm that for $x^* > x_{eH}^*$ the longitudinal component of the velocity increases linearly with the longitudinal component $x^*$.

4.3 Study of mass transfer: Definition of the mass boundary layer and the mass entry distance: Concept of uniform accessibility from the point of view of matter diffusion

The curves represent the variations of $C^*(k)$ depending on $y^*$, for different values of $x^*$ and the Reynolds number (figure 6). In general terms, when $y^*$ increases the mass fraction $C^*(k)$ decreases, it goes from a value equal to the unit away from the plate to a zero value on the surface of the flat plate. There is a critical value of $x^*$, defining a mass input distance $x^* > x_{ec}^*$, from which $C^*(k)$ depends more on $x^*$. This result is confirmed by the figure which represents the variations of Sherwood number according to $x^*$. Thus, for $x^* > x_{ec}^*$, it can be concluded that the flat plate is uniformly accessible from the matter diffusion point of view (the thickness of the metal compound which is deposited on the substrate is constant). The entry distance from which uniform accessibility is observed depends on the Reynolds number and it gets even smaller when the Reynolds number decreases. Table 2 gives the values of $x_{ec}^*$ according to the Reynolds number.
Table 2: Values of $x^*_ec$, as a function of Reynolds.

<table>
<thead>
<tr>
<th>$Re$</th>
<th>$x^*_ec$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.231</td>
</tr>
<tr>
<td>300</td>
<td>0.287</td>
</tr>
<tr>
<td>500</td>
<td>0.325</td>
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<td>800</td>
<td>0.377</td>
</tr>
<tr>
<td>1500</td>
<td>0.403</td>
</tr>
<tr>
<td>2000</td>
<td>0.581</td>
</tr>
</tbody>
</table>

5. Conclusion

At the end of this numerical study we can say that a flat surface is uniformly accessible from hydrodynamic and mass point of view from certain distance of the input edge when the velocity at the border of the boundary layer that develops on it is linear increases with the dimensionless abscissa $x^*$. Such case is obtained when the plane surface is placed in the center of the channel whose wall profile is an equilateral hyperbole with equation $r^2 \sin(2\theta) = Cte$. The entry distance from which the uniform accessibility is observed depends on the Reynolds number and it gets even smaller when the Reynolds number decreases.

Figure 2: Dimensionless velocity $U^*$ versus the dimensionless normal coordinate $y^*$ for different values of Reynolds
3. Variations of the dimensionless longitudinal velocity $U^*$ versus the dimensionless normal co-ordinate $y^*$ for different values of $x^*$ and Reynolds

4. Variations of $\delta H^*$ versus $x^*$ for Re =1500
Figure. 5. Variations of the concentration of the diffusing substance $C^*(k)$ versus the dimensionless normal co-ordinate $y^*$ for different values of $x^*$ and Reynolds

Figure. 6. Variations of the local sheer stress versus the dimensionless normal co-ordinate $x^*$ for different values of Reynolds
Figure 7. Variations of the Sherwood numbers versus the dimensionless normal co-ordinate $x^*$ for different values of Reynolds.

Nomenclature.

$Al$ : Chemical symbol of aluminum.
$C_f$ : Coefficient of local friction ($C_f = (\partial U^*/\partial y^*)$)
$Ga$ : Chemical symbol of Gallium
$L$ : Length of channel.
$Pem$ : Number of Peclet mass ($Pem = Re.Sc$)
$Sb$ : Chemical symbol of antimoine.
$Sher$ : Number of local Sherwood. $Sher = \left[-\frac{\partial C_n^*}{\partial y^*}\right]_{y^*=0}$
$Sc$ : Number of Schmidt ($Sc = \theta/D_n$)
$Re$ : Number of Reynolds.
$C_k^*$ : Mass fraction of metallic gas.
$C_k$ : Dimensionless mass fraction of metallic gas.
$U_e$ : Speed of the gas mixture at the inlet of the duct.
$U$ : Component of the following speed $x$.
$U^*$ : Dimensionless component of the following velocity $x$ ($U^* = U/U_e$)
$V$ : Component of the following velocity $y$.
$V^*$ : Dimensionless component of the following velocity $y$ ($V^* = V/U_e$)
$x$ : Cartesian coordinate defined in Fig.
$x^*$ : Dimensionless coordinate following $x$ ($x^* = x/L$)
$x_{ec}$ : Dimensionless mass input distance.
$y$ : Cartesian coordinate defined in figure 1
$y^*$ : Dimensionless coordinate following $y$ ($y^* = y/L$)
Greek letters

\( v \): Kinematic viscosity of hydrogen
\( \beta \): Coefficient of over-relaxation.
\( \varphi^* \): Unknown dimensionless \((C_k^*, U^*, V^*)\)
\( \sigma \): Constant that express the number of moles in chemical reaction

References


Hydrodynamic control of the growth ...


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