Minimum Solution of a Function Using
Particle Cluster Optimization

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Abstract

This document presents an explanation of the particle accumulation algorithm in optimization problems to calculate the minimum value of a function. The minimum value found in this document is that of a non-linear function. The non-linearity of the objective function makes the problem more complex compared to quadratic forms used exclusively in gradient or descent methods.

Keywords: Accumulation algorithm, PSO, cluster optimization
1 Introduction

Optimization is an area of applied mathematics that allows modeling and solving real-life problems; its principles and methods are used to solve quantitative problems in disciplines such as Physics, Biology, Engineering and Economics. The main objective of optimization is the best use of available resources to accomplish a certain task. Finding a solution includes the study of optimality criteria for problems, the determination of algorithmic solution methods, the study of the structure of such methods, and computer experimentation with methods both in simulations and in real-life problems.

One of the algorithms of optimization is the method of particle accumulation (PSO, Particle Swarm Optimization), which seeks to minimize objective functions of any nature, from the evaluation of each of the particles generated randomly by the algorithm. Of course the algorithm iterates by means of the speeds and update of variables that have better position or closer to the minimum of the objective function.

2 Preliminaries

There are many methods to find the minimum of a function. The algorithms of the gradient allow finding a response for quadratic functions but it is only limited to functions of that type since the gradient methods seek the solution by means of the most pronounced descent or maximum descent that throws the evaluation of the derivative in a candidate point optimum. On the other hand, the gradient methods can find optimal local solutions because the maximum direction of descent attracts the algorithm to the minimum but it can be a local minimum, that is, the solution found by the gradient methods although they are calculated in a few iterations they depend on the initial point of departure and decay quickly without contemplating other solutions in the neighborhood.

3 Particle Cluster: Particle Swarm Optimization

It is an optimization method based on heuristic techniques. The idea of the method is the search for the solution of a mathematical model within a cluster of particles that simulate the activities of a colony. Each particle in the cluster develops around a space where the solution is found. During the iterative process, each particle adjusts its speed and position according to its experience (The best Local solution) and its neighbor experiences (The best Global solution). To improve the performance of the algorithm, a variable is selected
as “the leader”, which commands the best solution within the iterative process (This variable can be different at each instant within the search of the solution), performance is also improved when guarantees a spread of the initial population and the following populations since the particles occupy more places within the solution space, this is done in order that each particle “live” a better experience and all the best answers can be evaluated to avoid select a response from a local minimum in an objective function of convex nature, for example.

The particle cluster (Swarm) is a system composed of a finite number of particles (responses), each particle moves through the search space and store information (communicate) which is the best solution. Each particle generates a number that is the evaluation in the objective function, the movement of the particles in the solution space is guided by candidate particles to optimal (best) in the current iteration [1].

4 Definition of the problem

To use PSO, the following items must be determined:

- Problem: Define the number of variables involved in the objective function since each particle has the dimensions of the variables and must be evaluated in the objective function to determine if it satisfies or not the problem (mimicization or maximization).

- Initial population: The initial population contains a number of randomly chosen individuals within a solution space, this in order to evaluate them in the objective function and obtain the information of which individual is better located. Some of the particles minimize the objective function in comparison with the others, that is, \( FO(x_n) < FO(x_{n-1}) \).

- Initial velocity: The velocity is updated according to equation (1), which depends on a social factor and cognitive factor (Eq. 3). In the literature [2], [3] the sum of these two factors should not be greater than a constant equal to 4, the initial velocity can be equal to zero or random but regardless of any of these two cases, the velocity is updated with the objective function and the particles.

\[
V_{i+1} = V_i + \gamma_1(i)(Pbest_i - X_i) + \gamma_2(i)(Gbest_i - X_i) \quad (1)
\]

\[
X_{i+1} = X_i + V_{i+1} \quad (2)
\]

\[
\gamma_1 + \gamma_2 < 4 \quad (3)
\]

where
- \( i \): Current iteration number.
- \( i + 1 \): Next iteration.
- \( V \): Velocity of the \( i-th \) particle. The velocity vector \( V \) stores the direction towards which each particle is directed.
- \( X \): Position of the \( i-th \) particle. The vector \( X \) stores the current position (location) of the particle in the search space.
- \( P_{best} \): Best position found by the \( i-th \) particle Personal Best. The \( P_{best} \) vector stores the position of the best solution found in that iteration.
- \( G_{best} \): Best position found in the GlobalBest cluster. The best particle found within the entire iterative process.
- \( \gamma_{1,2} \): Random numbers between \([0,1]\).

### 4.1 Description of the algorithm

```plaintext
Algorithm 1 PSO
1: \( X \leftarrow \text{crearCémlulodepartículas.} \)
2: while \( \text{iter} \neq \text{MaxIter} \) do
   3: \quad for \( i = 1 : \text{size}(x, 2) \) do
   4: \quad \quad Evaluar cada partícula \( x_{delCémlu} \)
   5: \quad if \( FO(x_i) < FO(P_{best_i}) \) then
   6: \quad \quad \( P_{best_i} \leftarrow x_i; FO(P_{best_i}) \leftarrow FO(x_i) \)
   7: \quad end if
   8: \quad if \( FO(P_{best_i}) < FO(G_{best}) \) then
   9: \quad \quad \( G_{best} \leftarrow P_{best_i}; FO(G_{best}) \leftarrow FO(P_{best_i}) \)
10: \quad end if
11: end for
12: for \( i = 1 : \text{size}(x, 2) \) do
13: \quad \( v_{i+1} = v_i + \gamma_1(P_{best_i} - x_i) + \gamma_2(G_{best} - x_i) \)
14: \quad \( x_{i+1} = x_i + v_{i+1} \)
15: end for
16: end while
17: return \( G_{best} \leftarrow kf \leftarrow \text{mejorrespuesta.} \)
```

Figure 1: Algorithm PSO.

### 5 Problem and results

It is required to obtain the minimum value of the following expression in \( \mathbb{R}^3 \).
Number of particles: 500. Maxiter: 500. Solution space: \([x_{max}, y_{max}] = [20, 20]\) and \([x_{min}, y_{min}] = [-20, -20]\). Now
\[ FO = -ae^{-b\sqrt{\frac{x}{n}}} - e^{-\frac{\cos(cx)\cos(cy)}{n}} + a + e, \]

where \( a = 20 \), \( b = 0.2 \), \( n = \text{size}(x, 2) \) and \( c = 2 \cdot \pi \).

Figure 2: Average value and initial best (iteration 20).

Figure 2 shows the best 20 values of the objective function. It should be noted that the value of the objective function decreases as the iterations increase.

Figure 3: Initial distribution of the particles.
The objective function has as a characteristic candidates for optimum distributed in the solution space, there is only a global minimum around zero.

![Figure 4: Evolution of the objective function.](image)

In iteration 55 it can be seen that the objective function decreases which is a qualitative signal that indicates that the algorithm is looking for a candidate to optimum in each iteration and that candidate is different.

![Figure 5: Profile of the objective function.](image)

Figure 5 shows the profile of the objective function, in this graph you can see the irregularity in the form of the function, this is due to the non-linearity of the exponential and trigonometric functions. The PSO algorithm distributes the particles along the profile, and these move according to the direction that is calculated with equation 1.
6 Conclusion

The PSO method as a tool to minimize an objective function is a heuristic algorithm that allows the evaluation of a finite quantity of particles that allow a global optimum to be determined in a solution space. Optimal candidates are updated iteration by iteration and search addresses point to the best candidate to optimum without the need to calculate derivatives as gradient methods do. The PSO algorithm is not only useful for nonlinear problems but also for linear programming problems and to solve such problems it is necessary to limit or reduce the solution space by means of linear constraints.

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References


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