Solution of the Maxwell’s Equations of the Electromagnetic Field in Two Dimensions by Means of the Finite Difference Method in the Time Domain (FDTD)

Diana Marcela Devia Narvaez
Department of Mathematics and GEDNOL
Universidad Tecnológica de Pereira
Pereira, Colombia

Fernando Mesa
Department of Mathematics and GEDNOL
Universidad Tecnológica de Pereira
Pereira, Colombia

Pedro Pablo Cárdenas Alzate
Department of Mathematics and GEDNOL
Universidad Tecnológica de Pereira
Pereira, Colombia

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Abstract

The solution of the Maxwell’s equations by means of the finite time domain difference method (FDTD) is used to describe the propagation of the electromagnetic field [1]. The method consists of transferring the differential equations of Maxwell to a discrete formulation, both spatially and temporally, that allows coding them and implementing them
in an algorithm that provides a numerical solution, which provides a visual overview of the physical behavior of electromagnetic waves for two dimensions. The applications of this method to solve different types of problems are diverse and offer adaptability for cases where the geometry of the problem becomes more complex to analyze. So this technique provides a tool capable of being applied to various disciplines of science with which it is possible to study systems and quantify the phenomena produced by electromagnetic activity successfully.

Keywords: Maxwell’s equations, discrete formulation, electromagnetic activity successfully

1 Introduction

The use of Maxwell’s equations is useful when you want to know the reflected value of the electromagnetic field at an arbitrary point in space, which in turn is restricted by the difficulty that the geometry of the problem can present and that by means of the traditional methods of differential equations it is not possible to provide a reliable solution, so that the problem is restricted to a computational solution. That is why it is of interest to mention the use of the finite difference method in the time domain FDTD [2] as a computational solution that allows the description of the electromagnetic field. It is necessary to bring these to a discrete formulation to implement the computational code that allows to study the physical behavior of the field, finally in the development of this article will be mentioned the respective necessary considerations such as Gaussian units and boundary conditions that must be taken into account for formulate the problem completely.

2 Propagation in two dimensions (2D)

In the development of the implementation of the method of finite differences in the domain of time FDTD for the solution of the Maxwell’s equations is necessary to make the numerical solution considering two dimensions should bear in mind the following aspects that are necessary for the application of the method, these aspects are related to two groups of vectors which are [3]:

- Mode (TM) - Which is formed by the group of vectors \((E_z, H_y, H_x)\). In this group we have that the magnetic field component in the direction of propagation \((z)\) is not present or zero.

- Mode (TE) - Which is formed by the group of vectors \((E_x, E_y, H_z)\). In this group the electric field component in the direction of propagation
(z) is absent or zero.

\[ D(\omega) = \varepsilon_r(\omega) * E(\omega) \]

To start with the numerical solution we have the following equations:

- For the electric flux density in the \( \omega \) domain

\[ D(\omega) = \varepsilon_r(\omega) * E(\omega) \]

- For the electric flux density vector:

\[ \frac{\partial \vec{D}}{\partial t} = \frac{1}{\sqrt{\varepsilon_0 * \mu_0}} * (\nabla \times \vec{H}) \]

- For the magnetic field vector:

\[ \frac{\partial \vec{H}}{\partial t} = -\frac{1}{\sqrt{\varepsilon_0 * \mu_0}} * (\nabla \times \vec{E}) \]

According to the previous equations, we have the mathematical basis to start the development of the equations for the electromagnetic field in two dimensions, starting this study with the magnetic transverse mode.

### 3 Magnetic transverse mode (TM)

The magnetic transverse mode (TM) [4] is composed of the following group of vectors \((E_z, H_y, H_x)\) for which the respective equation will be developed for each of these components. This study will begin with the magnetic field component in the direction \(x\), \(H_x\):

- **Component \(H_x\)** - To make this component we will quote the equation for the magnetic field. The equation for the magnetic field in component \(x-H_x\) is as follows:

\[ \frac{\partial H_x(z,t)}{\partial t} = -\frac{1}{\sqrt{\mu_0 * \varepsilon_0}} * \left( \frac{dE_z(z,t)}{dy} \right) \]

- **Component \(H_y\)** - To make this component we will quote the equation for the magnetic field. The equation for the magnetic field in component \(y-H_y\) is as follows:

\[ \frac{\partial H_y(z,t)}{\partial t} = \frac{1}{\sqrt{\mu_0 * \varepsilon_0}} * \left( \frac{dE_z}{dx} \right) \]
**Component $E_z$** - To make this component we will quote the equation for the vector of electric flux density and with this fact we will obtain the equation for the field $E_z$. The equation for the electric flux density vector $D$ in the $z$-$D_z$ component is as follows:

$$\frac{\partial \tilde{D}_z(z,t)}{\partial t} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \left( \frac{d}{dx} H_y - \frac{d}{dy} H_x \right)$$

Now, taking $D_z$ we obtain the mathematical form of the electric field $E_z$ defined in the following way:

$$E_{x(k)} = (Q) \ast (D_{x(k)} - P_{x(k)})$$

In this case we have that the electric flux density vector component is given in the direction $z$ in the same way for the electric field vector $E$ and the auxiliary vector $P$, these terms are defined as follows:

$$R = \frac{\sigma \ast \Delta t}{\varepsilon_0}$$

$$P_{x(k)} = P_{x(k)} + \left( R \ast E_{x(k)} \right)$$

The term for $Q$ of the equation is given by

$$Q = \frac{1}{\varepsilon_r + \frac{\sigma \ast \Delta t}{\varepsilon_0}}$$

Here the expression for the field $E$ is expressed totally in the terms that compose it where in turn is the vector $D$. The equations must be discretized by means of the finite differences method centered to implement them within a computational program that provides a numerical solution.

Then the discretization process [5] is developed for which it is necessary to describe the intercalation of the electric and magnetic fields in two dimensions:

- **For the $H_x$ component.** The discrete form of the equation is as follows taking into account that time is implicit in the formulation of the FDTD method.

$$H_{x(i)(j)} = H_{x(i)(j)} + \frac{1}{2} \ast [E_{z(i)(j)}(j) - E_{z(i)(j + 1)}]$$

- **For the $H_y$ component.**

$$H_{y(i)(j)} = H_{y(i)(j)} + \frac{1}{2} \ast [E_{z(i + 1)(j)}(j) - E_{z(i)(j)}]$$
Maxwell’s equations ...

• For the $H_z$ component.

$$E_z(i)(j) = Q \ast [D_z(i)(j) - P(i)(j)]$$

Next, the process of discretization of the equations will be carried out by means of the method of finite differences centered in order to implement them within a computer program that provides a numerical solution.

• For the $H_x - E_x$ component.

$$\frac{\partial \tilde{D}_x(z, t)}{\partial t} = \frac{1}{\sqrt{\varepsilon_0 \ast \mu_0}} \ast \left( \frac{d}{dy} H_z \right)$$

Now, applying centered finite differences we obtain:

$$\frac{\tilde{D}^{n+1}_{x(i,j)} - \tilde{D}^{n-1}_{x(i,j)}}{\Delta t} = \frac{1}{\sqrt{\varepsilon_0 \ast \mu_0}} \ast \left[ \frac{H^n_{z(i,j+1)} - H^n_{z(i,j-1)}}{\Delta y} \right]$$

$$\tilde{D}^{n+1}_{x(i,j)} = \tilde{D}^{n-1}_{x(i,j)} + \frac{1}{\sqrt{\varepsilon_0 \ast \mu_0}} \ast \frac{\Delta t}{\Delta x} \ast [H^n_{z(i,j+1)} - H^n_{z(i,j-1)}]$$

and therefore we get

$$\tilde{D}^{n+1}_{x(i,j)} = \tilde{D}^{n-1}_{x(i,j)} + \frac{1}{2} \ast [H^n_{z(i,j+1)} - H^n_{z(i,j-1)}]$$

The discrete form of the equation is of the following form taking into account again that the time is implicit in the formulation of the FDTD method.

$$D_x(i)(j) = D_x(i)(j) + \frac{1}{2} \ast [H^z(i)(j) - H^z(i)(j - 1)]$$

With $D_x$ defined it is possible to express mathematically the expression for the electric field vector $E_x$:

$$E_x(i)(j) = Q \ast [D_x(i)(j) - P_x(i)(j)]$$

In the same way we obtain:

• For the $D_y - E_y$ component.

$$E_y(i)(j) = Q \ast [D_y(i)(j) - P_y(i)(j)]$$

• For the $H_z$ component.

$$H_z(i)(j) = H_z(i)(j) + \frac{1}{2} \ast [-E_y(i)(j) + E_y(i - 1)(j) + E_x(i)(j + 1) - E_x(i)(j)]$$
4 Results

Next, the way in which the method presents the solution is shown for different test signals as the time progresses. Electric field graphs in $z$ direction ($E_z$) in two dimensions and Transverse Magnetic (TM) mode without absorbing boundary conditions.

Figure 1: Field $E_z$ for $T = 10s$

Figure 2: Field $E_z$ for $T = 20s$
Maxwell’s equations ...

Figure 3: Field $E_z$ for $T = 30s$

Figure 4: Field $E_z$ for $T = 40s$

Figure 5: Field $E_z$ for $T = 70s$
5 Conclusion

In the present document we have presented a basic formulation of the finite difference method in the time domain FDTD applied to electromagnetism, with which we propose to propose an initial solution analyzing a particular case for two dimensions of the propagation of the electromagnetic field. Likewise this method offers flexibility to study the spatial and temporal evolution of field in two and three dimensions with the necessary considerations that would be the object of another more detailed study, finally it is important to mention the potentiality of the FDTD method to analyze several cases that not only they compete to the subject treated here besides that the programming of the method does not present major disadvantages to implement it in other programming languages as they are it C++ or phyton among others, this if it is not counted on matlab or it is preferred to make use of another programming language.

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References


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