Modeling and Numerical Simulation of Fluid Flow in a Porous Tube with Parietal Suction

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Abstract

The aim of this study is to model and simulate numerically the laminar fluid flow in porous tube with parietal suction using the computational fluid dynamics (CFD) techniques. The finite element method is used to solve the coupled Navier-Stokes and Darcy’s law equations in the tube with porous walls. The porous walls were made from compacted exfoliated clays (CECs) for sealing applications. Some parameters effect on the performance of porous tube was studied, as geometrical dimension and Reynolds number. Some comments on designing of such tubes are suggested.

Keywords: Porous tube, Computational fluid dynamics (CFD), finite element method

Nomenclature:

- $e$: Tube thickness, m
- $k$: Porous walls permeability
- $L$: Length of tube, m
- $P$: Pressure, Pa
- $P_0$: Inlet pressure, Pa
- $P_e$: External pressure, Pa
- $r$: Radial coordinate, m
- $R$: Inner radius of the tube, m
- $R_e$: Reynolds axial number
- $R_{ew}$: Reynolds filtration number
- $U$: Axial velocity, $m. s^{-1}$
- $U_0$: Inlet average axial velocity, $m. s^{-1}$
**1 Introduction**

Porous media are widely used and play an important role in many industrial sectors and natural phenomena. These include, as typical examples: petroleum engineering, chemical engineering, hydrogeology, civil engineering, medicine... By their implication in these various areas and phenomena, porous media have been intensively studied during the past 50 years, and constitute a separate discipline in many field researches [1-3]. The geometric areas containing porous borders are important domains in physics. They have several industrial applications, such as biomedical engineering [4-6], drinking water treatment [7], and processing of beverages [8, 9]. Filtration processes involve the fluid flow tangentially over a porous wall. In general the flow is driven by a pressure differential. Analysis of the problem involves the prediction of the velocity distribution. In the past, researchers used approximate methods for the analysis of the problem [10-14]. Numerous approximate one-dimensional models have been performed [15]. The first simulation of flow in a membrane was undertaken under laminar conditions in channels with porous walls [10]. Investigation of laminar flow in a porous pipe with variable wall suction or variable radial mass flux was done by Galowin and De Santis [16]. A summary of the recent developments, up to 1989, on the role of fluid mechanics in membrane filtration was presented by Belfort et al. [17]. Many authors are very interested in using this method to optimize membrane processes [18]. Nassehi et al. [19] used Darcy’s equation to represent the porous wall conditions. They used the finite element method in their simulation and presented a more robust simulation comparing to other previous works. Damak et al. [20] simulated a laminar, incompressible and isothermal flow in a cylindrical tube with a permeable wall using a finite difference scheme. In this paper, a two-dimensional flow field model based on the Navier-Stocks equations coupled with Darcy’s law was developed. The finite element method, is used to solve these equations in a porous tube. The effect of various physical parameters on the porous tube performance is investigated.

**2 The mathematical model and numerical simulations**

**2.1 Governing equations**

The proposed study is to put in place the equations and the appropriate boundary conditions to determine the fields which are established within a tube with porous walls. This tube is subjected to parietal suction during the fluid flow,
as shown in Figure 1. We are interested to determine the velocity and pressure fields. The geometric structure of the tube with porous walls has an axial symmetry; therefore, the study will be performed in the plane \((r, z)\). The simulation is based on the following assumptions:

- The flow is assumed laminar, permanent, and isotherm.
- Neglecting the volume forces.
- The solid matrix is incompressible.
- The inertial terms can be neglected in the Navier-Stokes equation.
- Constant viscosity and density of the fluid
- A fully developed velocity profile at the tube inlet.

Governing equations are described as the following:

- Continuity equation:
  \[
  \nabla (\rho \vec{V}) = 0
  \]
- the Navier-Stokes equations
  \[
  \rho \frac{d\vec{V}}{dt} = -\nabla p + \mu \Delta \vec{V}
  \]

### 2.2 Boundary Conditions

- Inlet boundary condition, \(z=0\): We assume that the flow is hydrodynamically established at the entrance of tube with porous walls. Therefore, the profile of the axial velocity at the inlet is the Poiseuille parabolic profile, and the radial velocity component is zero:
  \[
  U(0, r) = 2U_{\text{max}} \left( 1 - \left( \frac{r}{R} \right)^2 \right), V(0, r) = 0, P = P_0
  \]
- Outlet boundary condition, \(z=L\): fully developed flow condition is considered at the tube exit plane. The condition for the axial and radial velocity is:
  \[
  \frac{\partial U(L, r)}{\partial z} = 0, \frac{\partial V(L, r)}{\partial z} = 0
  \]
- At axisymmetric axis, \(r = 0\): The boundary conditions on the tube axis are the axisymmetric conditions:
  \[
  \frac{\partial U(z, 0)}{\partial r} = 0, V(z, 0) = 0
  \]
- In the inner surface of the porous tube, \(r=R\):
  \[
  \frac{\partial U(z, R)}{\partial r} = 0, V(z, R) = V_w
  \]

In the above equation, \(V_w(z)\) is determined by Darcy equation:
\[
V_w = -\frac{k}{\mu} \nabla p
\]
If the external pressure $P_e$ is uniform along the tube and is equal to atmospheric pressure, the boundary condition at the wall is written:

$$V_w = \frac{k}{\mu} (P - P_e)$$

Where $k$ is the porous walls permeability and $e$ is the tube thickness.

The problem under consideration is schematically shown in figure 1.

Beavers and Joseph [21] show that there is a slip velocity at the interface porous-fluid. Schmitz and Prat [13] showed that the effect of the velocity slip is practically negligible at the inner surface of the porous tube. Thus, we assume that the assumption of no-slip remains valid at the wall.

Fig. 1. Simulated geometry for cylindrical porous tube

### 2.3 Solving

The coupled Navier–Stokes and Darcy’s law equations are solved using SIMPLE algorithm based on finite element method. The criteria for convergence are $10^{-6}$ for residuals of continuity and velocity components.

The fluid used for simulation is the air, whose main characteristics are recalled in Table 1. The porous wall is made of the compacted exfoliated clay [1, 22], whose its permeability is equal to $1.85 \times 10^{-17}$ m$^2$ [23].

<table>
<thead>
<tr>
<th>Density Kg.m$^{-3}$</th>
<th>Dynamic viscosity Pa.s</th>
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</thead>
<tbody>
<tr>
<td>1.29</td>
<td>$1.87 \times 10^{-5}$</td>
</tr>
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### 3 Results and discussion

The proposed numerical model allows the study of porous tube performance. Also, the effects of various operating conditions such as geometrical dimensions and Reynolds number on the good working of the porous tube are studied. The porous tube length is equal to 50 times the diameter. The porous wall has a permeability assumed constant and is equal to $1.85 \times 10^{-17}$ m$^2$ [23]. For more understanding of operating conditions, we define non-dimensional quantities as the followings:
The results were presented as flow velocity profiles, pressure drop and permeation velocity.

### 3.1 Evolution of the axial and radial velocity

The axial velocity component $U$ is greatly influenced by the value of $R_{e_w}$, i.e. by the permeation velocity. Indeed, it decreases along the tube (Figure 2) and this decrease is particularly pronounced as the $R_{e_w}$ number is high. A comparative study of the flow evolution between a tube with impermeable wall and a tube with porous wall shows that the penetration of fluid through the wall is accompanied by a modification of the velocity profile (Figure 3). The velocity profile is modified especially as the $Re$ number is small (Figure 4).

Figure 5 displays the variation of the radial velocity according to the radial coordinate $r/R$ for different values of $R_{e_w}$ and $Re = 500$. Figure 5 shows that the radial velocity increases until a maximum value near the wall. This evolution appears to be related to the cylindrical geometry of the tube as already mentioned by Granger et al. [24].

![Fig. 2. Evolution of the axial velocity on the tube axis, Re=500](image-url)
Fig. 3. Profiles of the axial velocity at the outlet of the porous tube, Re=500

Fig. 4. Profiles of the axial velocity at the outlet of the porous tube, Re=250

Fig. 5. Profiles of the radial velocity, Re=500 and z=L/2
3.2 Evolution of the axial pressure drop

The axial pressure drop along the tube with porous wall is shown in figure 6. When the $Re_w$ number increases, the axial pressure drop decreases. Indeed, the filtration causes a modification of the profile and the velocity intensity in the tube with porous wall. It follows a variation in the pressure field so the longitudinal pressure gradient.

![Figure 6: Variation of the axial Pressure drop, Re=500.](image)

3.3 Evolution of the permeation velocity

Contrary to the assumption which considers a constant permeation velocity $V_w$ [25], our calculations show that the permeation velocity values depend on the axial position and its evolution is linear (Figure 7). Indeed, this velocity decreases as the flow progresses through the porous tube because the pressure gradient in the tube wall decreases in the flow direction. This variation is independent of $Re$ number, i.e the input velocity.

![Figure 7: Variation of the permeation velocity along the porous tube, Re=500.](image)
4 Conclusions

In the present work, the modeling and numerical study of the parietal suction was performed using the Navier-Stokes equation and Darcy's law. This has done with a numerical finite element code, using SIMPLE algorithm. It allowed to characterize the flow of a fluid in a tube with porous wall. The numerical model successfully predicts the fundamental mechanisms involved in fluid flow with porous wall. The Reynolds filtration number influences the velocities profiles (axial and radial) and the pressure drop in the tube with porous wall. These numerical results show that a higher Reynolds filtration number leads to a decrease of the axial velocity component and the axial pressure drop.

References


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