An Integrated Inventory Location Routing Problem Considering CO₂ Emissions

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Abstract

This paper considers an integrated Inventory Location Routing problem ILRP which consists of optimizing the operating costs, risks and routing’s CO₂ emissions related to a Hazmat supply chain. The goal is to determine: the location of a set of capacitated depots to open, customers allocated to them, quantities delivered from these depots to customers per period, and the sequence in which customers are served by a heterogeneous fleet of vehicles. To solve the problem, an exact bi-objective model is designed and tested for small and medium size instances. A case study is proposed to prove industrial practicality of the model. The results show that the proposed solution resolves decently the medium size instances.

Keywords: Supply chain design, Inventory-Location-Routing Problem, Green Routing, Bi-objective model, Case study

1 Introduction

Logistics management is a making decision approach used to govern supply chain functions, in order to distribute goods according to customers demand and taking into consideration various constraints including cost reducing.
The design of efficient supply chains has become more challenging in view of environmental regulations, especially for Hazmat supply chain. Companies must making decisions on facility location, inventory management, and distribution while considering the environmental impacts of these decisions. Some decisions may conflict, such as storage and transport decisions. In fact, in order to comply with regulations and risk management policies at the storage level, fixed installations may reduce their fixed stocks by increasing the number of delivery which would increase the risk and CO₂ emissions at the transportation level (de Marcellis-Warin et al. 2013) [6]. That's why it's important to integrate the strategic, tactical, and operational decisions in order to minimize costs, risks, and CO₂ emissions.

In this paper we study an Integrated location routing problem (ILRP) with heterogeneous fleet and considering a multi-period decisions. The objective is to optimize location and opening costs of depots, distribution and inventory management, with considering transport and storage constraints of a Hazmat case and routing's CO₂ emissions.

Therefore, Section 2 reviews the literature on the integration of three decisions level (strategic, tactical and operational). The mathematical model of the problem and the computational study are provided in Section 3. A real case application is described in Section 4 and finally, Section 5 is devoted to conclusions and future research.

2 Literature review

Researchers classify the decisions for supply chain management into three categories: strategic, tactical, and operational. The classical approach for optimization of logistics systems is the decomposition into sub problems according to these decision levels. This approach has many advantages such as simplification of modeling and problem solving techniques. However, this can often lead to a sub optimality of solutions because it doesn't take account of the whole problem (Prodhon 2006) [15]. Several researchers studied the benefits of integration of supply chain decisions.

In the literature, many papers integrate two supply chain levels decisions. Location-Routing Problem (LRP) combines two decision levels, a strategic level which is the facility location and operational or tactical level associated to the vehicle routing problem (VRP). Laporte (1992) [11] presents a first classification of this problem; other reviews have been made in this direction by Nagy and Salhi (2007) [14], and more recently by Prodhon and Prins (2014) [16]. Another case is Inventory-Routing Problem (IRP), this problem integrates inventory management (tactical level) and vehicle routing (operational level), it was surveyed in (F. Baita et al. 1998) [3], A. Kleywegt et al. (2002) [10], M. Vidovic et al. (2014) [23]). Finally, the integration of facilities location problem and inventory management is named Inventory-Location Problem (ILP), Daskin et al. (2002) [5] and Shen (2005) [19] studied different variants of ILP.
Recently, the progress of operations research and computer science encourages researchers to think to solve more complex and larger problems, and thus have a more global vision of supply chain problems. This will help to anticipate interactions between decisions levels. The Inventory Location-Routing Problem (ILRP) can be seen as an approach to integrate the three components (depot location, vehicle routing and inventory management) into one model, in order to optimize the supply chain design and minimize the total costs. Considering stochastic demand, Lui and Lee (2003) [12] and Liu and Lin (2005) [13] presented a location-routing problems (LRP) including nonlinear inventory costs in the objective function. They consider a single product and multi-depot with unlimited capacity. To solve this problem, Lui and Lee (2003) [12] used a two-phase heuristic method, and Liu and Lin (2005) [13] proposed a hybrid heuristic based on the Tabu Search (TS) and Simulated Annealing (SA). Another nonlinear stochastic model is studied by Shen and Qi (2007) [20] to integrate the location, inventory, and the routing approximate costs. Javid and Azad (2010) [1] present a stochastic model to find simultaneously the decisions of location, allocation, inventory at depots and routing. The authors propose, an exact solution and then a heuristic method based on hybridization of tabu search (TS) and simulated annealing (SA). For deterministic demand, Ambrosino and Scutella (2005) [2] use a linear programming relaxation for the ILRP. Hiassat and Diabat (2011) [9] studied an inventory location routing problem with deterministic demand for perishable product. They propose a multi-period model formulated as Mixed Integer Program (MIP). Recently, Guerrero et al. (2013) [7] present an algorithm for a multi-period ILRP with deterministic demand. They propose a hybrid approach to optimize simultaneously location of depots, allocation of retailers to depots, inventory at both depots and retailers, and multi-period routing. The algorithm optimizes globally the components of the problem by sharing information between decisions levels. Guerrero et al. (2015) [8] investigate a relax and price heuristic for the same problem. They coordinate the generation of new columns in the pricing problem and update of Lagrangian multipliers. Results show important cost savings and efficient computation if compared to the previous heuristics.

We can note that little research has been done to study solution methods for the ILRP especially in comparison with the integration of two supply chain levels decisions. And there is no work, to our knowledge, that incorporate environmental aspects and a heterogeneous fleet.

In our case, we study an ILRP for Hazmat with inventory decisions at clients and considering multi-period decisions with random data inspired from our real case problem. The main contribution of this work is the integration of \( \text{CO}_2 \) emissions in the objective function and considering a heterogeneous fleet.

### 3 Problem definition and model formulation

We consider a supply chain design problem, in which, we have to distribute a hazardous product from a single manufacturer to a set of customers, through a set
of intermediate depots with heterogeneous fleet. The goals that will be taken into account in this model are to minimize both of the total cost and the routing CO₂ emissions. The first objective function includes the followings costs: (i) the opening and operating costs of depots, (ii) the delivering cost from depots to customers, and (iii) the inventory costs at customers level. The second objective function minimizes the CO₂ emissions.

Let I denote the set of m candidate depots, J the set of n customers, and T the set of p periods in the planning horizon, V a set of vehicles types indexed by v.

For each i ∈ I let Oᵢ be the cost of opening a depot at site i and let Capᵢ be its capacity. Each customer j has a deterministic and non-constant demand dᵢⱼₜ, ∀t ∈ T. Consider R as the set of all feasible routes with at most a fixed number of customers. A route starts from the depot, visits a subset of customers at most one time and then returns to the depot. The routes are generated before solving the model, and thus allows subtour constraints elimination. zᵢᵣ and wᵢᵣ are respectively defined as follows: for each route r ∈ R and customer j ∈ J, zᵢᵣ is equal to 1 if route r visits customer j and 0 otherwise, and for each route r ∈ R and depot i ∈ I, wᵢᵣ is equal to 1 if route r begin from depot i and 0 otherwise. A capacity Cᵢᵣ and a routing cost Cᵣ are associated with each route r ∈ R.

The following are the main assumptions of the presented model.

- The demand of customers is deterministic.
- Inventory level at customers and depots is limited in order to take into account the storage constraints of hazardous products.
- A route starts and terminates at the same depot, and visits at least one client.
- Vehicle fleet is heterogeneous (different capacities).
- A customer demand is supplied by only one distribution center.
- Depots and customers have a limited storage capacity.

Our model integrates the location problem with the inventory and routing problem, taking into account capacity constraints, and the distribution CO₂ emissions.

Parameters

Nᵥ Number of vehicle type v ∈ V
Oᵢ Opened cost of depot i ∈ I
Cᵣ Transportation cost of route r ∈ R
fᵢⱼₜ Inventory holding cost of customer j ∈ J in time period t ∈ T
Cᵢᵣ Capacity of route r ∈ R (equal to the capacity of the vehicle assigned to the route r)
zᵢᵣ = 1 If route r ∈ R visits customer j ∈ J
wᵢᵣ = 1 If route r ∈ R visits depot i ∈ I
dᵢⱼₜ Demand of customer j ∈ J in time period t ∈ T
Sᵢ₀ Inventory level at customer j ∈ J at the beginning of time period t = 1
Sᵢ max Maximum inventory level at customers
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Cpdi = Maximum storage capacity at depot i
bvr=1 If route r∈R is deserved by vehicle type v and 0 otherwise
ECO2, CO2 emissions quantity of a route r ∈R

Decisions variables

Xi = 1 if depot i ∈ I is opened and 0 otherwise
Yrt=1 if routes r ∈ R is selected in time period t∈T and 0 otherwise
Sjt= Inventory holding of customer j ∈ J in the end time period t ∈ T
Qjr= Quantity delivered to customer j ∈ J by route r ∈ R in time period t ∈ T

The hazardous materials regulations limit the storage and the transportation quantities and this increase frequency of transportation, and thus augment the environmental impacts due to CO2 emissions. That is why we decide to add this environmental impacts in the objective function. The estimation of CO2 emission for transportation requires complex calculations, which can only represent an approximation because of the difficulty of quantifying some variables as driving style, weather conditions, congestion, and the like (Palmer, 2007 [17], Van Woensel et al., 2001[22]). To calculate the emissions factor CO2 we use a similar method a as that used in S. Ubeda et al. (2011) [21].Table 1 shows the emissions factor for several capacity scenarios between full loaded and empty vehicles. To calculate the CO2 emissions quantity of a route r, we multiply the emissions factor by distance.

<table>
<thead>
<tr>
<th>State of the vehicle</th>
<th>Weight laden (%)</th>
<th>Consumption (l/100 km)</th>
<th>Fuel conversion factor (kg CO2/l)</th>
<th>Emission factor Kg CO2/km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty</td>
<td>0</td>
<td>29.6</td>
<td>0.773</td>
<td>x2.61</td>
</tr>
<tr>
<td>Low loaded</td>
<td>25</td>
<td>32.0</td>
<td>0.831</td>
<td></td>
</tr>
<tr>
<td>Half loaded</td>
<td>50</td>
<td>34.4</td>
<td>0.9</td>
<td></td>
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<tr>
<td>High loaded</td>
<td>75</td>
<td>36.7</td>
<td>0.958</td>
<td></td>
</tr>
<tr>
<td>Full load</td>
<td>100</td>
<td>39.0</td>
<td>1.018</td>
<td></td>
</tr>
</tbody>
</table>

Model

\[
\min \sum_{i \in I} O_i x_i + \sum_{r \in R} \sum_{t \in T} (C_r + ECO2_r) y_{rt} + \sum_{t \in T} \sum_{j \in J} f_j S_{jt}
\]  \hspace{1cm} (1)

Subject to:

\[
\sum_{r \in R} z_{jr} y_{rt} \leq 1 \quad \forall j \in J, \forall t \in T
\]  \hspace{1cm} (2)
\[
\sum_{j \in J} q_{jrt} \leq C_p r y_{rt} \quad \forall r \in R, \forall t \in T \tag{3}
\]
\[
y_{rt} \leq \sum_{i \in I} w_{ir} x_i \quad \forall r \in R, \forall t \in T \tag{4}
\]
\[
\sum_{r \in R} \sum_{j \in J} w_{ir} q_{jrt} \leq C_p d \quad \forall i \in I, \forall t \in T \tag{5}
\]
\[
S_{jt-1} + \sum_{r \in R} z_{jr} q_{jrt} = d_{jr} + S_{jt} \quad \forall j \in J, \forall t \in T \tag{6}
\]
\[
S_{jt} \leq S_{\text{max}} \quad \forall j \in J, \forall t \in T \tag{7}
\]
\[
\sum_{r \in R} b_{vr} y_{rt} \leq N_v \quad \forall t \in T, \forall v \in V \tag{8}
\]

Objective function (Equation 1) includes the sum of location transportation and inventory cost. The first term represents the total fixed cost to open the selected depots, the transportation and CO₂ emissions costs are minimized by second term, and the last term represents the inventory holding cost. Constraints (2) guarantee that each customer is visited once at most in any time period. Constraints (3) indicates the capacity constraints of vehicles. Constraints (4) indicates that the routes start and end with the opened warehouse. The capacity of each depot is presented in constraints (5). Equation (6) represents the inventory balance. Equation (7) ensure that the inventory level at a customer never exceeds maximum storage limits due to the hazmat regulations. Equation (8) is the fleet size constraints for each type of vehicle and per period.

4 Computational Results

The data was generated as follows:
- Number of customers: 4, 8, 10, 15 and 20.
- The maximum number of customers per route is 4.
- Number of depots: 2, 3 and 5.
- Number of time periods: 5
- Demand of a customer per time period: random integer in the interval [0, 5].
- Inventory cost for customer j: random number in the interval [4.6, 5].
- Location \((x_i, y_j)\) of customer j: random integer numbers in the interval [0, 500].
- Transportation cost: \(C_r = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}\)
- Capacity of the heterogeneous fleet and number of vehicles is a constant which is inspired from the real case.
- Maximum storage capacity value at depot and at client depends of the security regulation of hazmat.
In order to validate the model, we solve some scenarios of randomize generated data. We used the CPLEX Opl version 12.5.1. in a 2.40 GHz Intel(R) Core(TM) i3-2370 CPU. A summary of computational results is reported in the following table. In the first column of these tables we present the instance (costumers, depots, time period), the second column is the objective function parameters, the last columns present successively the objective function value, and the computational time. To tackle our multi-objective optimization problem we use a lexicographic weighting method. A pre-defined priority order is tested and established between objectives. For example, four weights are compared to determine the best one for the CO$_2$ emissions objective.

<table>
<thead>
<tr>
<th>Instances</th>
<th>O.F parameters</th>
<th>O.F values</th>
<th>CPU(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cr Eco2r Oi Cr Eco2r fjt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4C/2D/5P</td>
<td>1 0 9848858 10844 0 2437,6</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 1 9848858 11307 3056,375 2523,475</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 100 9848858 11765 2745,7 2522,325</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 1000 9848858 11765 2745,7 2470,975</td>
<td>2</td>
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<tr>
<td>8C/2D/5P</td>
<td>1 0 9848858 14672,6 0 6201,34</td>
<td>7,2</td>
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<tr>
<td></td>
<td>1 1 9848858 14421,4 3735,86 6528,06</td>
<td>9,2</td>
<td></td>
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<td></td>
<td>1 100 9848858 13976,2 3575,98 7445,24</td>
<td>17,4</td>
<td></td>
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<td></td>
<td>1 1000 9848858 13984,2 3576,08 7355,44</td>
<td>32</td>
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</tr>
<tr>
<td>10C/2D/5P</td>
<td>1 0 9848858 15326,6 0 11966,4 53,838</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 1 9848858 15974,8 3574,56 11949,2 67,022</td>
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<td></td>
<td>1 100 9848858 15890,4 3430,34 12197 445,2</td>
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<td></td>
<td>1 1000 9848858 15890,4 3430,34 12204,6 169</td>
<td></td>
<td></td>
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<td>15C/3D/5P</td>
<td>1 0 880000000 30246,25 0 17730,5 105</td>
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<td></td>
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<tr>
<td></td>
<td>1 1 880000000 30021,5 10525,175 17887,5 78</td>
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<td></td>
<td>1 100 880000000 29202,5 9433,725 18692,5 300</td>
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</tr>
<tr>
<td></td>
<td>1 1000 - - - - -</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20C/3D/5P</td>
<td>1 0 1260000000 34439 0 24474 2131,2</td>
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<td></td>
<td>1 100 1260000000 5560,9 24950 36958</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>1 1000 - - - - -</td>
<td></td>
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</tr>
</tbody>
</table>

IV case study

In this section, a real case application has been developed for supply chain design of a leading company in the drinking water distribution sector. This company uses chlorine for water disinfection. Chlorine is the most predominant disinfectant due to its low cost installation and residual biocidal effect.
(2001)) [18], and efficacy of chlorine disinfection for enteric viruses (Bigliardi, L et.al 2006) [4]. However, it is highly toxic and considered to be among the most dangerous of hazardous materials. To minimize the risks related to the use of chlorine and improve risk management of its business, the company should optimize chlorine supply chain while respecting storage and transport constraints. The problem consists of 20 delivery points (water treatment station) served directly from one supplier. The fleet of vehicles is composed of two types capacity trucks. The goal of this study is to determine the set of depots to open, they are the starting points of the delivery route. In practice, we note there is only routes that visit a small number of customers. Therefore, we generate all feasible routes with no more than three customers. The objective function consists to minimize total cost and CO$_2$ emissions. A pseudoreal data is generated, especially for the geographical coordinate of depots and inventory costs. The planning horizon is three time periods (a period is one month). Customers demands are deterministic, but may vary from one period to the next. The demand are generated as a uniform distribution in the range $[1, 9]$ for high season and in the range $[1, 5]$ for low season. The data of customers and depots are given in Table 3 and Table 4. The results of 12 simulations are reported in Table 4. The second column of this table indicate the scenarios obtained by selecting 5 among 6 depots. Columns called O.D, CR, ECO2, CI, CD respectively indicate the chosen depots, routing costs, CO$_2$ emissions quantity, inventory costs and the opening costs of depots. "OF" gives the value of the objective function, and finally the CPU time is given in seconds. The second scenario gives the best objective function for the low and high season. To ensure the robustness of the obtained solution, we also simulate a scenarios with a peak demand. Table 5 presents the results of this simulation that confirm the robustness of our solution. In this section, we tested a realistic datasets for our model. Results shows that our solution can be regarded as a simulation tool to help decision-makers to optimize a real situation.

Table 3- Depots data

<table>
<thead>
<tr>
<th>Depots</th>
<th>Coordinate</th>
<th>Capacity</th>
<th>Fixed cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>583900</td>
<td>394950</td>
<td>30</td>
</tr>
<tr>
<td>D2</td>
<td>319100</td>
<td>324100</td>
<td>10</td>
</tr>
<tr>
<td>D3</td>
<td>311950</td>
<td>160150</td>
<td>30</td>
</tr>
<tr>
<td>D4</td>
<td>74600</td>
<td>420600</td>
<td>10</td>
</tr>
<tr>
<td>D5</td>
<td>180220</td>
<td>110960</td>
<td>20</td>
</tr>
<tr>
<td>D6</td>
<td>454111</td>
<td>550636</td>
<td>10</td>
</tr>
</tbody>
</table>
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Table 4 - Results of low and high season scenarios

<table>
<thead>
<tr>
<th>SC</th>
<th>OD</th>
<th>CR</th>
<th>ECO2</th>
<th>CO2</th>
<th>CD</th>
<th>O.F</th>
<th>CPU</th>
</tr>
</thead>
</table>
| SC1 | D3-D4 | 21429| 4538.2| 3112.4| 1340000000 | 1340478363 | 576.50
| SC2 | D3-D4 | 20970| 4015.6| 3498.3| 1340000000 | 1340426032 | 205.11
| SC3 | D3-D6 | 21098| 4769.9| 3697.2| 1380000000 | 1380501784.6 | 192.30
| SC4 | D1-D4 | 22310| 4253.1| 3581.5| 1440000000 | 1440451197 | 158.81
| SC5 | D3-D4 | 21661| 4461.5| 3490.6| 1340000000 | 1340471299.9 | 119.04
| SC6 | D3-D4 | 19705| 4061.4| 3500.2| 1340000000 | 1340429349 | 473.24

<table>
<thead>
<tr>
<th>SC</th>
<th>OD</th>
<th>CR</th>
<th>ECO2</th>
<th>CO2</th>
<th>CD</th>
<th>O.F</th>
<th>CPU</th>
</tr>
</thead>
</table>
| SC1 | D1-D3-D5 | 31801| 4548.9| 4045.7| 2140000000 | 2140493178 | 569.97
| SC2 | D1-D3-D4 | 28750| 3886.5| 6357  | 2140000000 | 2140423761 | 238.00
| SC3 | D1-D3-D6 | 29099| 4240  | 6279  | 2180000000 | 2180459453 | 188.81
| SC4 | D1-D4-D5-D6 | 29518| 4870  | 5975.7| 2960000000 | 2960522489 | 166.25
| SC5 | D1-D3-D4 | 30284| 3878.4| 6435.7| 2140000000 | 2140424564.3 | 119.97
| SC6 | D3-D4-D5-D6 | 30167| 5761.9| 6233.3| 3640000000 | 3640757565 | 239.37

Table 5 - Robustness analysis

<table>
<thead>
<tr>
<th>SC</th>
<th>OD</th>
<th>CR</th>
<th>ECO2</th>
<th>CO2</th>
<th>CD</th>
<th>O.F</th>
<th>CPU</th>
</tr>
</thead>
</table>
| SC1 | D1-D3  | 21508| 3295.8| 5881.5| 1500000000 | 1500356972 | 517.14
| SC2 | D1-D3  | 22096| 3316.6| 5966.2| 1500000000 | 1500359718 | 286.12
| SC3 | D1-D3  | 22772| 3356  | 5311  | 1500000000 | 1500363684 | 313.44
| SC4 | D1-D4-D5 | 21560| 3664.5| 5460  | 2280000000 | 2280393470 | 373.84
| SC5 | D1-D3  | 22169| 3366.4| 5741  | 1500000000 | 1500364554 | 249.84
| SC6 | D3-D4-D5 | 23799| 4707.2| 5095  | 2180000000 | 2180499617 | 425.04

V Conclusion

This paper considers a bi-objective inventory location routing problem ILRP that optimizes the operating costs, risks and routing's CO2 emissions related to a Hazmat supply chain. The goal is to determine: the set of depots to open, customers allocated to them, inventory decisions, and routes with a heterogeneous fleet of vehicles so as to satisfy the customers’ demand in each period of the planning horizon. We modeled the problem as an exact bi-objective integer linear program where the main decision variables are binary variables for the selection of depots to open, inventory variables and the set of feasible routes. We have thus no subtour constraints. To validate the model, some tests data instances have been generated. We have also studied an industrial real case to illustrate the problem. The results show that the proposed model solves medium instances with a reasonable computational time. The future work is both the development of a
heuristic method, based on a guided generation of routes, and also the testing of some more large scale instances.

References


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