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Coherence Function in Analysis and Synthesis of Complex Systems

Alexander Danilov

Penza State University of Architecture and Construction
Street Titov 28, Penza, Russia, 440028

Irina Garkina

Penza State University of Architecture and Construction
Street Titov 28, Penza, Russia, 440028

Abstract

The paper provides the determination of information importance of control actions in a dynamic system based on private coherence functions. The possible applications to the compilation and analysis of cards vibration of different objects, selection of informative signals for the design of monitoring systems building structures, the design of simulation and learning complexes are provided.

Keywords: dynamical systems, quality control, selection of informative control signals, partial coherence function

Introduction

The solution of many practical problems are connected with the problem of the development of information systems models isomorphic to the set of states: the design of systems for monitoring various construction applications, simulation and creation of simulation and training systems for the training of operators ergatic systems, clinical diagnostics in medicine, vibrodiagnostics et al. [1 ... 4]. The simplicity of the information model is determined by how manages to highlight the most important parameters characterizing the state of the system (without losing their isomorphic). The solution of this problem, of course, requires a systematic approach to the formation of control actions to improve the quality of control (selection, ranking, the definition of hierarchical structure). This fully manifested in the design of simulators of human-machine systems. Thus, the interconnection of the operator and the object is carried out on information and awareness-executive channels. Information on the status of the facility and its systems is transmitted to the operator via the display or perceived them directly through visual, auditory, etc. receptors. As a result of information received by the operator in the central nervous system is formed by *the current information model* of object motion. On the basis of comparing it with *the conceptual model* (formed in the mind of the operator on the basis of education, training, experience) operator generates the control signals sent to the authorities (*executive information model*). Function of the operator is to form the control actions. To reduce the analytical activity of the operator only the most important parameters of the state of the system should be included in the information model (the set of information models of the object state to be isomorphic to the set of states of the controlled object). In this case, there is a possibility of making the simplest simulation and training systems for training of operators with a significant reduction of false cases of acquisition of skills. We assume that the investigated sources of information are formalized (unformalized sources are assumed *a priori significant*).

Coherence function in the selection of informative signals

As it turned out, for the selection of informative signals, you can use selective coherence function $\gamma_{xy}^2(f)=\dfrac{|S_{xy}(f)|^2}{S_{xx}(f)\cdot S_{yy}(f)}$ (allows you to determine the relative quality of dimension complex transfer function). Namely, to establish which part of the energy response is correlated with the energy input $x(t)$ (identified noise and other plural signal sources affecting the output signal $y(t)$). When using a single measurement coherence function registers only a single value for all frequencies, so it is advisable to determine average value for two or more measurements of input and output signals.

We emphasize *the coherence function is an analogue of the correlation coefficient in the frequency domain and reflects the degree of linear relationship harmonic components of these processes*. The closer the coherence function to a unit at a given frequency f, the more harmonic components to match this frequency.

The proposed method is convenient because it is based on the determination of sample elements of the spectral densities, which usually must be somehow obtained by statistical analysis of stochastic processes. The expediency of its use is defined and clarity of the results obtained with the inherent coherence function obvious physical meaning.

Operator estimates mode motion of the object by generalized state vector whose components can be as individual values (speed, acceleration, angular coordinates, etc.) easily formalized and whole images (sound, light, and others.); some of them may be difficult to formalize. Knowledge of the operator at each time an object state allows it to carry out a predetermined movement *approximately* based on the generation of control actions. If we restrict formalizable inputs $x_i(t), i=\overline{1,n}$, then this problem reduces to establishing their connection with each of the control actions $y_k(t), k=\overline{1,m}$.

It is known [5], for $\left|\sum f\right|>0$,

$$\sum(f)=\begin{bmatrix} S_{11}(f)\ldots S_{1n}(f) & S_{1y}(f) \\ \ldots & \\ S_{n1}(f)\ldots S_{nn}(f) & S_{ny}(f) \\ S_{y1}(f)\ldots S_{yn}(f) & S_{yy}(f) \end{bmatrix}$$

multiple coherence function at the frequency f :

$$\gamma^2_{y.1,2,\ldots,n}(f)=1-\frac{1}{S_{yy}(f)\cdot S^{yy}(f)}.$$

Here $S_{\alpha\beta}(f)$, $S_{\alpha y}(f)$ - mutual spectral density $x_\alpha(t)$ and $x_\beta(t)$, also, $x_\alpha(t)$ and $y_k(t)$ respectively;; $S_{yy}(f)$ - spectral density $y_k(t)$; $S^{yy}(f)$ - the last element of the main diagonal of the matrix $\sum^{-1}(f)$, inverse to $\sum(f)$; $S_{y\alpha}(f)=S^*_{\alpha y}(f)$, $S^*_{\alpha y}(f)$ is in a complex interfaced with $S_{y\alpha}(f)$.

Correct:

$$0\le\gamma^2_{y.1,2,\ldots,n}(f)\le 1.$$

The closer the multiple coherence function to 1, the closer the relationship between $y_k(t)$ and all $x_\alpha(t)$ to linear; if $\gamma^2_{y.1,2,\ldots,n}(f)=0$ a connection is absent.

When $\gamma^2_{y.1,2,\ldots,n}(f)=1$ bonds are linear:

$$y_k(t)=L_{y1}[x_1(t)]+L_{y2}[x_2(t)]+\cdots+L_{yn}[x_n(t)]+\omega(t);$$

$L_{y\alpha}$ - linear stationary operators; $\omega(t)$- a stationary process, not connected at all frequencies processes $x_\alpha(t)$ (function of multiple coherence between $\omega(t)$ and $x_\alpha(t)$ is equal to zero). The process $\omega(t)$ can be regarded as a process derived from $y_k(t)$ by an exception of linear effect $v(t)$ components $x_\alpha(t)$:

$$y_k(t)=v(t)+\omega(t).$$

The process $v(t)$ is fully coherent components $x_\alpha(t)$ and $\omega(t)$ is completely not coherent to $x_1(t),\ldots,x_n(t)$.

Theoretically, the value of the function of multiple coherence is not always a good characteristic of the relationship between $y_k(t)$ and $x_\alpha(t)$. It would seem that if the value of the coherence function between $x_1(t)$ and $y(t)$ is close to unity, there is reason to believe that they can be considered as the input and output of a linear system. But if there is a coherent process $x_2(t)$ with $x_1(t)$ and contributing to $y(t)$ after passing through a linear system, the high degree of coherence between $x_1(t)$ and $y(t)$ may reflect only the fact that the coherence between $x_1(t)$ and $x_2(t)$ is also high; and the process $x_2(t)$ associated with $y(t)$ via a linear system. In fact, $x_1(t)$ and $y(t)$ may not be bound by any physical system. Therefore, to obtain reliable results on the relationship between $y_k(t)$ and $x_\alpha(t)$ it is better to use of the *conditional (private)* multiple coherence function. It allows you to determine in what degree $y_k(t)$ at a frequency f is connected by the linear stationary operator $x_i(t)$ after excluding from $y_k(t)$ the impact of linear, time-independent relationships with other components $x_1(t),\ldots,x_{i-1}(t),x_{i+1}(t),\ldots,x_n(t)$. For the previous case the private coherence function between $x_1(t)$ and $y(t)$ will be close to zero, and the private coherence function between $x_2(t)$ and $y(t)$ will be close to unity.

The private multiple coherence function is defined as:

$$\sum\nolimits_{i|1,2,\ldots,i-1,i+1,\ldots,n}(f)=A-BD^{-1}C=\begin{bmatrix} a_{11}(f) & a_{12}(f) \\ a_{21}(f) & a_{22}(f)\end{bmatrix},\ a_{21}(f)=a_{12}^{*}(f);$$

$$A=\begin{bmatrix} S_{yy} & S_{yi} \\ S_{iy} & S_{ii}\end{bmatrix},\qquad B=\begin{bmatrix} S_{y1} & S_{y2} & \cdots & S_{y(i-1)} & S_{y(i+1)} & \cdots & S_{yn} \\ S_{i1} & S_{i2} & \cdots & S_{i(i-1)} & S_{i(i+1)} & \cdots & S_{in}\end{bmatrix},$$

$$C=\begin{bmatrix} S_{1y} & S_{1i} \\ S_{2y} & S_{2i} \\ \vdots & \vdots \\ S_{(i-1)y} & S_{(i-1)i} \\ S_{(i+1)y} & S_{(i+1)i} \\ \vdots & \vdots \\ S_{ny} & S_{ni}\end{bmatrix},\qquad D=\begin{bmatrix} S_{11} & S_{12} & \ldots S_{1(i-1)} & S_{1(i+1)} & \cdots & S_{1n} \\ S_{21} & S_{22} & \ldots S_{2(i-1)} & S_{2(i+1)} & \cdots & S_{2n} \\ & & \cdots & & & \\ S_{(i-1)1} & S_{(i-1)2} & \ldots S_{(i-1)(i-1)} & S_{(i-1)(i+1)} & \cdots & S_{(i-1)n} \\ S_{(i+1)1} & S_{(i+1)2} & \ldots S_{(i+1)(i-1)} & S_{(i+1)(i+1)} & \cdots & S_{(i+1)n} \\ & & \cdots & & & \\ S_{n1} & S_{n2} \ldots & S_{n(i-1)} & S_{n(i+1)} & \cdots & S_{nn}\end{bmatrix}.$$

From the foregoing it follows immediately:

$$\gamma^2_{iy|1,2,\ldots,i-1,i+1,\ldots,n}(f)=\frac{|a_{21}(f)|^2}{a_{22}(f)\cdot a_{11}(f)},$$

(to determine the transfer functions of a human operator is advisable to use a ratio $W_i(jf)=\dfrac{a_{21}(f)}{a_{22}(f)}$).

Algorithm selection of informative signals and their rankings

For selection of *informative signals and their rankings should*:

- produce synchronous signal measurement $x_i(t)$ and $y_k(t)$ during normal operation;
- determine the elements of the matrices A, B, C, D;
- calculate the matrix elements $\sum_{i|1,2,\ldots,i-1,i+1,\ldots,n}(f)$ and for each value i to define the functions of private coherence

$$\gamma^2_{iy|1,2,\ldots,i-1,i+1,\ldots,n}(f) = \frac{|a_{21}(f)|^2}{a_{22}(f)\cdot a_{11}(f)};$$

- arrange $x_i(t)$ *in order of decreasing values of the functions of private coherence* (than more information importance of the signal, the higher its rank).

We will notice that not always according to the normal operation can be set, which processes are input, which are output (the consequence is often accepted to the reason); require cognitive modeling with the construction of a digraph (applies not only to closed systems). In some cases, the direction of "input-output" can be set on the position of the maximum cross-correlation functions. If the cross-correlation function reaches a maximum $R_{x_i x_j}(\tau_0) = R_{\max}$ when $\tau_0 < 0$, the input is to be considered $x_j(t)$; when $\tau_0 > 0$ - $x_i(t)$.

We also emphasize that the linear dependences established for private coherence functions are not necessarily unique.

Conclusion

The above method is used effectively in the development of unique systems for operator training ergatic systems [3]. The possibility of adapting this method of rankings input signals for generating control actions are obvious and do not require further explanation.

References

[1] A.B. Voznuk, Monitoring of soil and other high-rise buildings by reason of wind fluctuations. Engineering and environmental studies in construction: theory, techniques, methods, and practice, Moscow: Sergeevskie reading, **8**. (2006), 271-274.

[2] F.N. Yudakhin, Microseisms - an important source of information / / Bulletin of the Ural Branch of the Russian Academy of Sciences, **3** (2010), 65-73.

[3] Lapshin E.V., Danilov A.M., Garkina I.A., Klyuyev B.V., Yurkov N.K., Flight Simulators of modular architecture: monograph / Editors Lapshin E.V.,

Danilov A.M. Penza: Information and publishing center, Penza State University, 2005, 146 p.

[4] A.P. Kulaichev, Coherent analysis of information content in the EEG // Journal of Higher Nervous Activity, **59** (2009), 766-775.

[5] N. Goodman, Calculation of the matrix of frequency characteristics and functions of multiple coherence // in: Bendat J., Piersol A., Measurement and analysis of stochastic processes, Moscow: Mir, 1974, 448-464.

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