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Residual Control Chart for Monitoring Pediatrics Hospital Admission Performances

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Abstract

This paper aims to demonstrate the use of pre-whitening (PW) technique to handle the presence of autocorrelation on the statistical control charts, for 3-year (2008-2010) daily pediatrics (less than 4 years old) hospital admission. The PW technique has been implemented as an alternative procedure to obtain residuals series which are statistically uncorrelated to each other. Results showed that there is a reduction in the number of out-of–control signals in residual series control chart as compared to the amount of the out-of–control signals on traditional statistical control chart before the use of PW technique. Thus, it is suggested that statistical control chart using residual series performs better when the original pediatric hospital admission series are auto-correlated. In addition, it can be concluded that the Phase II (monitoring period) performance process is likely to follow the similar pattern of the Phase I (baseline period) process except for only one day on the 13th October 2009 that exceeds the upper control limit. This means that the pediatrics hospital admission on that particular day has not improved and fundamentally changed from what are expected in stable process.

Keywords: Autocorrelation, control chart, pre-whitening

1 Introduction

Standard statistical control charts are based on the assumption that the observations obtained at each time period are independent, which implies that there is no relationship between adjacent observations. A positive autocorrelation leads to a tendency for high observations to follow high observations while low observations to follow low observations, whereas in a negatively auto-correlated process, high observations tend to follow low observations and vice versa. However, most data are serially correlated in discrete or continuous processes [1]. The presence of autocorrelation results in a number of problems such as an increase in type I error rate and thereby increases the number of false alarm [2]. For these reasons, handling and testing the auto-correlated process is an important procedure before constructing any statistical control chart.

In the relevant literature, there exists many different techniques to handle the effect of autocorrelation in process observations [3, 4]. Various authors have suggested fitting an appropriate time series model to the observations and then applying traditional control charts to the process residuals [5]. The autoregressive integrated moving average (ARIMA) model is one of the most popular methods to deal with autocorrelation in time series model. However, some of the model identification and model parameter estimation in ARIMA model are difficult to determined and time series modeling knowledge is needed for constructing the ARIMA model [6, 7].

Therefore, pre-whitening (PW) technique as proposed by Kulkarni & Von Storch [8] has been applied as an alternative approach for removing the effect of autocorrelation in the process observations. This technique is based on the assumption that the time series of observations could be adequately described by a lag-one autoregressive process, denoted as AR (1). It involves modification of original observation and the removal of AR (1) process in a time series. PW technique has been widely used in trend detection studies such as hydrological and meteorological fields. The aim of this paper is to use PW technique to handle the presence of autocorrelation on the statistical control charts of 3-year (2008-2010) daily hospital admission of children aged less than 4 years old. The PW has been performed as an alternative approach to obtain residuals series which are statistically uncorrelated to each other. Then, the Shewart control chart based on residual series is constructed to compare and monitor the performances of pediatrics hospital admission during baseline period (2008-2009) and monitoring period (2010). The daily inpatients' admission counts of the main public hospital in Klang Valley, Malaysia are collected from the Department of Statistics, Malaysia during 2008 – 2010 periods. This paper focuses on the daily hospital admission due to the respiratory disease (ICD-10 codes: J00-J99) and the cardiovascular disease (ICD-10 codes: I00-I99). The age group of pediatrics aged less than 4 years old is chosen since they have very different lung function and immune systems compared to the other age groups. Data are divided into baseline period (1 January 2008 to 31 December 2009 or 731 days) and monitoring period (1 January 2010 to 31 December 2010 or 365 days). All analyses were performed by using *R* software.

2 Methods

2.1 Pre-whitening technique

Before applying SPC control chart in any performance, it is important to satisfy the basic assumptions that the observations or time series data are distributed independent. However the independency assumption is not realistic in practice due to the presence of autocorrelation in the observations. In this paper, PW technique has been performed as an alternative approach to detect and remove the effect of autocorrelation in the process observations. This technique is based on the assumption that the observation could be adequately described by a lag-one autoregressive process (AR (1)) which assumed that the current observation is correlated with its previous observation. Firstly, to investigate the stationarity of AR (1) process, the trend slope is estimated using the Theil-Sen robust slope estimator method,

$$b = median\left(\frac{X_j - X_l}{j - l}\right), \quad \forall l < j \tag{1}$$

where b is the estimate of slope of the trend, X_j and X_l are the j th and l th observations respectively. Secondly, original observation is detrended by removing any significant linear trend, $Y_t = X_t - bt$.

Thirdly, the lag-1 serial correlation coefficient (r_1) of the detrended series, Y_t is computed as

$$r_1 = \frac{\frac{1}{n-1}\sum_{t=1}^{n-1}\left[Y_t - E(Y_t)\right]\left[Y_{t+1} - E(Y_t)\right]}{\frac{1}{n}\sum_{t=1}^{n-1}\left[Y_t - E(Y_t)\right]^2} \quad \text{with} \quad E(Y_t) = \frac{1}{n}\sum_{t=1}^{n} Y_t \,. \tag{2}$$

At the significance level of 0.10 for the two tailed–test, if the value of r_1 falls within the confidence intervals, $\frac{-1-1.645\sqrt{n-2}}{n-1} \le r_1 \le \frac{-1+1.645\sqrt{n-2}}{n-1}$, then the original observations are assumed to be serially independent and statistical control chart can be directly applied to the original observations. Otherwise, it is considered to be serially correlated and PW is used to eliminate the serial correlation from the detrended time series data as follows:

$$Y_t^{'} = Y_t - r_1 Y_{t-1}. \tag{3}$$

The residual series, $Y_t^{'}$ after performing the PW procedure should be an independent series.

2.2 Statistical process control

Statistical process control (SPC) can be defined as a method for monitoring, controlling and improving a process using statistical analysis. In practice, SPC is implemented in two phases, namely Phase I and Phase II. In Phase I, historical observations are used to set up trial control limits, assess process stability, detect and eliminate the out-of–control signals that are plotted beyond the trial limit in order to bring the process into a state of statistical control. Then, the control limits established in Phase I are used for ongoing monitoring in Phase II whether the future process has improved, remained in-control or worsened.

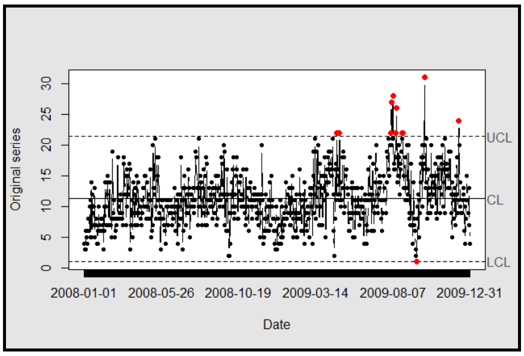
Since the first control chart has been proposed by Shewhart [9], various control charts have been developed for monitoring and controlling different process data. The primary goal of using control charts is to distinguish between common cause variability, which is the natural variability inherent to any process, and special cause of variability which refers to unnatural variation caused by a factor extrinsic to the process. Basically, the standard statistical control charts include a graph of the data over time with three horizontal lines called the centre line (CL), the lower control limit (LCL) and the upper control limit (UCL). CL usually based on the mean and both control limits (LCL, UCL) typically set at ± 3 standard deviations (SDs) from the mean. In daily pediatrics hospital admission context, the sample size used for process control is $n = 1$, that is, only one observation is available at each time period. Hence, the Shewhart control chart for an individual measurement is utilized to plot and to monitor the process observation over the study period. The formulae of CL, LCL and UCL are computed as follows.

$$CL = \overline{X}, \quad LCL = \overline{X} - 3\frac{\overline{MR}}{d_2}, \quad UCL = \overline{X} + 3\frac{\overline{MR}}{d_2}, \tag{4}$$

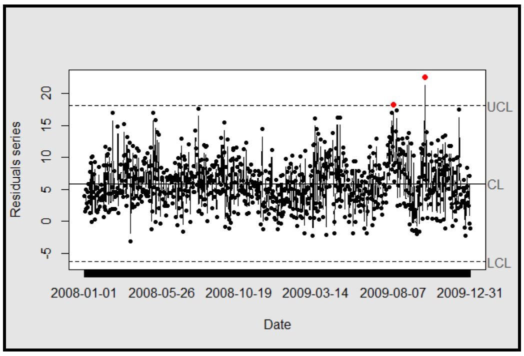
where $\overline{X}$ is the average of original observations, X_t, MR is the range between consecutive observations, $MR_t = | X_t - X_{t-1} |$, $\overline{MR}$ is the average moving average and $d_2 = 1.128$. If X_t are serially auto-correlated, the formulation in (4) is replaced by using residual series, $Y_t^{'}$ which is obtained after performing the PW procedure as discussed in the previous section.

3 Results and discussions

A total of 12821 pediatrics patients for respiratory and cardiovascular disease were admitted to the main public hospital in Klang Valley, Malaysia from 1 January 2008 to 31 December 2010 with a mean 11.7 and inter-quantile range (IQR) of 5 patients. There were 8232 pediatrics patients admitted during baseline period while 4589 pediatrics patients were admitted during monitoring period. Before constructing statistical control chart, the AR(1) was computed and tested to detect any presence of autocorrelation component in the daily pediatric hospital admission process. Results show-that the original series is statistically correlated since $r_1 = 0.4146$ which falls outside the confidence intervals (-0.0622, 0.0595). Then, the PW technique has been performed to obtain the residual series of the daily pediatric hospital admission. It is found that AR(1) for residual series is not significant with $r_1 = -0.0610$ and confidence intervals, (-0.0622, 0.0595). Therefore, the resulting residual series can be assumed to be independently distributed and these residual had no longer influenced by the effects of autocorrelation.



(a)



(b)

Figure 1 Control chart using (a) original series and (b) residual series

Figure 1 illustrates the control charts before and after performing the PW technique. The figure reveals that there are only 2 (or 0.27%) out-of–control signals in residual control chart compared to 12 (or 1.65%) out-of–control signals on the traditional control chart. It is possible to conclude that the out-of signals in Figure 1(a) are actually the signals of false alarm which may occur due to the influence of autocorrelation in the original series. In order to establish a stable and predictable control chart, the out-of-control signals in Figure 1(b) were eliminated until the process appeared to be in a statistical control. Once this was accomplished, the residual control chart is used for monitoring the performance of daily future residuals series in Phase II as illustrated in Figure 2.

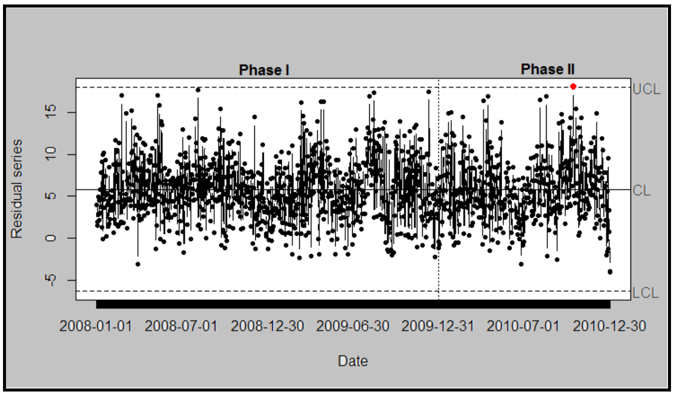


Figure 2 Residual control chart in Phase I and Phase II

It can be seen that there is only one out-of control signal in Phase II, which implies that the performance process of Phase II is likely to follow a similar pattern of the performance process of Phase I except for only one day, that is on the 13th October 2010 which exceeds the upper control limit. This means that the pediatrics hospital admission on that particular day has not improved and fundamentally changed from what is expected in a stable process. Therefore, further investigation and corrective actions are needed for improvement.

4 Conclusions

This paper demonstrates the use of pre whitening (PW) technique in handling the presence of autocorrelation on the statistical control charts of daily pediatrics (aged less than 4 years) hospital admission. The comparison of results between the original series and the residual serial shows that ignoring the autocorrelation in original series increases the number of false alarm in control chart, but after the PW technique is applied, the number of out-of–control signals in residual series control chart has been reduced as compared to a larger number of out-of–control signals on the traditional statistical control chart. Thus, it is suggested that statistical control chart using residual series performs better when the original series are auto-correlated. In addition, control charts can aid researchers and practitioners in monitoring the current process performance, and predicting future process performance.

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