Performance Analysis of SVD Based Single and Multiple Beamforming for SU-MIMO and MU-MIMO Systems with Various Modulation Schemes

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Abstract

Most of the wireless communication technologies are not able to provide substantial gain in terms of reliability and capacity due to the effect of multipath fading and limited spectrum. A best solution to overcome these problems is multiple antennas techniques. This paper investigates the effect of multiple antennas at the transmitter and receiver end of SU-MIMO and MU-MIMO, using a SVD based single and multiple beamforming technique, for various Modulation schemes namely ASK, BPSK, QPSK, 8-PSK and 8-QAM. The effectiveness of these approaches has been validated with help of MATLAB simulations.

Keywords: Single-User MIMO, Multi-User MIMO, SVD, Beamforming
1 Introduction

In recent years, MIMO antenna system is used to achieve greater data rate and higher reliability compared to single antenna system. MIMO System typically classified into two types Single-User MIMO and Multi-User MIMO. A Single-User MIMO uses a single system with multiple antenna dimensions, while a Multi-User MIMO has more than one system, each with multiple antennas. The knowledge of the channel at the transmitter and the receiver increases the performance gain of the system. When the Channel State Information is known, the Singular Value Decomposition of the channel can be performed, thus implementing the beamforming technique. Beamforming is of two types Single and Multiple. Transmitting the same symbol from all the antennas at the same time through a subchannel with the largest gain is called single beamforming. It reduces the Average Bit-Error Rate, thus increasing the reliability of the System. If more than one channel is used for the transmission of’s’ symbols simultaneously, then the technique used is multiple beamforming. It increases the data rate and reliability of the system [1].

2 SVD Based MIMO Beamforming System

Consider a MIMO system that has $N_t$ transmit antennas and $M_r$ receiver antennas. The transmission is done over the Rayleigh fading channel. We assume that the information about the channel is available at both the transmitter and receiver side. $s = s_0, s_1, ..., s_{N}$ are taken as the information bits. The bits are modulated with either ASK, BPSK, QPSK, 8PSK, 8QAM modulator which yield the symbol vector of $\bar{x} = \bar{x}_1, ..., \bar{x}_N$ where $N$ denotes the total number of symbols that are transmitted. The modulated symbol $\bar{x}$ is then precoded at the transmitter. The precoded symbols are transmitted through $N_t$ antennas over Rayleigh fading channel. We assume the presence of additive white Gaussian noise in the channel. If $x$ is taken as the precoded vector of size $N_t \times 1$, $H$ being taken as the channel matrix of size $M_r \times N_t$, and $n$ is considered as the Additive White Gaussian Noise (AWGN) introduced in the channel, then the received vector $y$ can be represented as

$$y = Hx + n$$  \hspace{1cm} (1)

the received symbol $y = y_1, y_2, ..., y_N$ is a vector of size $M_r \times 1$. The decoding process is done on $y$ to obtain a vector $\tilde{y}$. It is then demodulated and the information is obtained.

Mathematically, Singular Value Decomposition (SVD) is the process of factorizing a real or complex matrix into individual matrices. In a MIMO system,
SVD can be incorporated by breaking down the channel matrix

\[ H = U\sum V^H \]  

(2)

Where \(U\) and \(V\) are the two unitary matrices of size \(N_t \times N_t\) and \(M_r \times M_r\) respectively, and \((\cdot)^H\) denotes the conjugate transpose and \(\sum\) is the \(N_t \times M_r\) diagonal matrix with positive numbers on the diagonal, \(\sum = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{M_r})\) where \(\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{M_r} \geq 0\) are the singular values. The diagonal values are taken as the gain of the respective channels. We are decomposing the MIMO channel into independent and parallel subchannels. We are considering the independent and identical complex distribution (i.i.d) of the channel matrix [2].

2.1 Single User MIMO systems

![Fig.1: Single User MIMO systems](image)

In fig 1, the symbols to be transmitted are multiplied by \(V\) before transmitting and the received symbol matrix is multiplied by \(U^H\) to perform the transmit precoding and receiver shaping. The overall process can be expressed as follows.

\[
\bar{y} = U^H (Hx + n) 
\]  

(3)

\[
\bar{y} = U^H (U\sum V^H x + n) 
\]  

(4)

\[
\bar{y} = U^H (U\sum V^H x V x\bar{x} + n) 
\]  

(5)

\[
\bar{y} = \sum x + n_i 
\]  

(6)

For a Single-User MIMO, In case of Single beamforming the received signal can be represented as

\[
\bar{y} = \lambda_i x + n_i \quad \text{where } i = 1, 2, \ldots, N 
\]  

(7)

In case of multiple beamforming, \(R\) is the no of parallel data streams or symbol or subchannels simultaneously is used. Note that \(R \leq \text{min}(N_t, M_r)\) the optimal vectors to be used as weights at the transmitter side and receiver side are the first \(R\) columns of \(U\) and \(V\) corresponding to the first \(R\) largest singular values of \(H\)[3], [4]. Then, the input/output relation for the \(i^{th}\) subchannel for multiple beamforming
becomes \[ \bar{y}_i = (1/\sqrt{R}) x_i + n_i \] (8)

2.1 Multi User MIMO systems

In case of Multi-User MIMO, similar to Single user MIMO the single beamforming and multiple beamforming technique were incorporated [5].

In fig 2 the overall channel matrix H is decomposed into multiple subchannels matrix such as \( H_1, H_2, \ldots, H_k \). For each user within its own channel matrix all antennas transmit same symbol through its largest singular values of \( H_k \), thus implementing single beamforming technique. When every user transmits multiple symbols through its the first R largest singular values of own channel matrix then the concept of multiple beamforming is used. Multi-User MIMO exploits the multiuser diversity in spatial domain, thus resulting in significant gains over Single-User MIMO.

Fig. 2: Multi User MIMO systems

Fig. 3: Constellation diagram
3 Detection Algorithms for Different Modulation Techniques

From fig 3 we can say that received symbol from the channel is complex in nature. In the detection phase, at the ASK receiver, the threshold is kept as 0.5. Hence, if the symbol is greater than 0.5, it is considered as 1 else 0. Similarly, for BPSK the threshold is kept as 0. In QPSK, the real and imaginary parts of the received symbol are considered separately. The threshold is kept at 0+0j. If the real part and the imaginary parts are both positive, the bits are considered as ‘11’. If the real part alone is positive and the imaginary part is negative, then it is ‘10’. If the real part is negative and the imaginary part is positive, ‘01’ is considered. If both real and imaginary parts are negative, ‘00’ is considered.

<table>
<thead>
<tr>
<th>Threshold value</th>
<th>Information Bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re≥2 and Img&gt;0</td>
<td>000</td>
</tr>
<tr>
<td>Re≥2 and Img&lt;0</td>
<td>100</td>
</tr>
<tr>
<td>Re&gt;0 to Re&lt;2 and Img&gt;0</td>
<td>001</td>
</tr>
<tr>
<td>Re&gt;0 to Re&lt;2 and Img&lt;0</td>
<td>101</td>
</tr>
<tr>
<td>Re&lt;0 to Re&gt;2 and Img&gt;0</td>
<td>011</td>
</tr>
<tr>
<td>Re&lt;0 to Re&gt;2 and Img&lt;0</td>
<td>111</td>
</tr>
<tr>
<td>Re≤-2 and Img&lt;0</td>
<td>010</td>
</tr>
<tr>
<td>Re≤-2 and Img&gt;0</td>
<td>001</td>
</tr>
</tbody>
</table>

Table 2: **Threshold values for 8-PSK**

<table>
<thead>
<tr>
<th>Threshold value</th>
<th>Information Bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re&lt;0 to Re&gt;-2 and Img&lt;0</td>
<td>000</td>
</tr>
<tr>
<td>Re&lt;0 to Re&gt;-2 and Img&gt;0</td>
<td>010</td>
</tr>
<tr>
<td>Re≤-2 and Img&gt;0</td>
<td>011</td>
</tr>
<tr>
<td>Re≤-2 and Img&lt;0</td>
<td>001</td>
</tr>
<tr>
<td>Re&gt;0 to Re&lt;2 and Img&gt;0</td>
<td>110</td>
</tr>
<tr>
<td>Re&gt;0 to Re&lt;2 and Img&lt;0</td>
<td>100</td>
</tr>
<tr>
<td>Re≥2 and Img&gt;0</td>
<td>111</td>
</tr>
<tr>
<td>Re≥2 and Img&lt;0</td>
<td>101</td>
</tr>
</tbody>
</table>

Table 3: **Threshold values for 8-QAM**
4 Results

In this section, the simulation results of the BER performance of SVD based beamforming technique using various modulation schemes for both Single-User and Multi-User MIMO are shown in Fig.5 and Fig.6.

![Graph 1: BER Performance of SBF and MBF for Single User with N_t=3 M_r=3.](image1)

![Graph 2: BER Performance of SBF and MBF for Multi-User with N_t=3 M_r=3.](image2)

**Fig.5:** BER Performance of SBF and MBF for Single User with N_t=3 M_r=3.

**Fig.6:** BER Performance of SBF and MBF for Multi-User with N_t=3 M_r=3.
5 Conclusions

In this paper, we focused on SVD based single and multiple beamforming techniques for single and multi user MIMO systems with various modulation schemes namely ASK, BPSK, QPSK, 8-PSK and 8-QAM. Comparisons of BER performances with various modulation techniques using various numbers of antennas in the transmitter and receiver sides have been evaluated. It can be seen that from the modulation techniques used, BPSK has least BER and 8-PSK has the highest BER with single and multiple beamforming for both Single-User and Multiple-User MIMO. It can also be noted that with the increase in number of antennas in the transmit and receive sides, there is a reduction in BER and thus increase in reliability. With single beamforming, there is a further substantial decrease in the BER as compared to multiple beamforming.

References


