Design of State Observer for a Class of Non linear Singular Systems Described by Takagi-Sugeno Model

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Abstract
In this paper, we propose a new method to design an observer for a class of non linear singular systems described by Takagi-Sugeno (TS) model, with measurable decision variables. The idea of the proposed approach is based on the singular value decomposition. The convergence of the state estimation error is studied using the Lyapunov theory and the stability conditions are given in terms of Linear Matrix Inequalities (LMIs). Finally, an example is given to illustrate the proposed approach.

Keywords: observer, differential-algebraic system, descriptor system, Takagi-Sugeno model, linear matrix inequality (LMI)
1 Introduction

All recently there has been a great deal of interest in using dynamic Takagi-
Sugeno fuzzy models [11] to approximate nonlinear systems. This interest
relies on the fact that once the Takagi-Sugeno fuzzy models are obtained, lin-
ear control methodology can be used to design local state feedback controllers
for each linear model. Aggregation of the fuzzy rules results in a generally
nonlinear model, but in a very special form, which is exactly the same as a
time varying and nonlinear system described by a set of Polytopic Linear In-
clusions.

The Takagi-Sugeno model has been generalized to Singular systems. Many
physical systems are naturally modeled as systems of differential and algebraic
equations (DAE) such as chemical, electrical and mechanical engineering sys-
tems [8], [4], [5]. These systems are variously called descriptor systems, singular
systems, implicit, or differential algebraic equations (DAEs). This formulation
includes both dynamic and static relations. Consequently this formalism is
much more general than the usual one and can model physical constraints or
impulsive behavior due to an improper part of the system.

Many control issues have been extended to the descriptor case, in particular
the observer design for descriptor systems has been intensively addressed see
e.g. [6], [9], [1], a linear fractional transformation parameterization of linear
observers is done in [12] and [3] introduces the proportional integral (PI) ob-
server.

This paper presents a method for state-estimation of Takagi-Sugeno descrip-
tor systems (TSDS) with measurable premise variables. Under some sufficient
conditions, the design of the observer is reduced to the determination of some
matrices. The choice of these matrices is performed by solving strict LMIs.

The proposed observer design method is based on the use of the second method
of Lyapunov and a quadratic function.

The paper is organized as follows: the class of studied systems is defined in sec-
tion II and the main results about observers for Takagi-sugeno fuzzy descriptor
systems with measurable premise variables design are detailed in section III.
Section IV is devoted to a numerical example.

2 Takagi-sugeno descriptor systems

In this paper, we consider the following general form of singular nonlinear
systems:

\[
\begin{aligned}
E \dot{x}(t) &= f(x(t)) + g(x(t))u(t) \\
y(t) &= Cx(t)
\end{aligned}
\]
where $x \in \mathbb{R}^n$ is the state variable, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^p$ is the measured output. $f$ and $g$ are nonlinear functions. $E \in \mathbb{R}^{n \times n}$ is constant matrix with $\text{rank}(E) = r$.

To obtain a Takagi-Sugeno model of the system (1) we use the procedure of fuzzy model construction given in [10]. The multiple model structure is given by:

$$
\begin{align*}
E \dot{x}(t) &= \sum_{i=1}^{q} h_i(x(t))(A_i x(t) + B_i u(t)) \\
y(t) &= C x(t)
\end{align*}
$$

where $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$ are real known constant matrices. The $h_i(x(t))$ are the weighting functions that ensure the transition between the contribution of each sub model:

$$
\begin{align*}
E \dot{x}(t) &= A_i x(t) + B_i u(t) \\
y(t) &= C x(t)
\end{align*}
$$

The activating functions, denoted $h_i(x(t))$ for $i = 1, \ldots, q$, are normalised, and satisfy the following constraints:

$$0 \leq h_i(x(t)) \leq 1, \quad \sum_{i=1}^{q} h_i(x(t)) = 1, \quad \forall \ t$$

In the sequel, we assume the following Hypotheses (for each sub model (3), $i = 1, \ldots, q$):

**H1)** $(E, A_i)$ is regular, i.e. $\text{det}(sE - A_i) \neq 0 \ \forall s \in \mathbb{C}$

**H2)** All sub models are impulse observable, i.e.

$$\text{rank}\left( \begin{array}{cc} E & A_i \\ 0 & E \\ 0 & C \end{array} \right) = n + \text{rank}(E)$$

**H3)** All sub models are R detectable, i.e.

$$\text{rank}\left( \begin{array}{c} sE - A_i \\ C \end{array} \right) = n \quad \forall s \in \mathbb{C}$$

**H4)** $\text{rank}\left( \begin{array}{c} E \\ C \end{array} \right) = n$. 
3 Fuzzy observer design

In this section, our aim is to design a fuzzy observer for system described (2). The proposed observer which is not in descriptor form is given by the following equations:

\[
\begin{align*}
\dot{z} &= \sum_{i=1}^{q} h_i(x)(N_i z + L_{1i} y + L_{2i} y + G_i u) \\
\hat{x} &= z + by + Kdy
\end{align*}
\]

(5)

where \(N_i, L_{1i}, L_{2i}, G_i\) and \(K\) are unknown matrices of appropriate dimensions, which must be determined such that \(\hat{x}\) will asymptotically converge to \(x\).

: The estimation error of the observer (5) for (2) tends to zero if there exists a symmetric positive matrices \(P, Q\) and \(W\) verifying the following LMI for \(i = 1, \ldots, q\)

\[
A_i^T a^T P + P a A_i + (c A_i)^T Q^T + Q (c A_i) - C^T W_i^T - W_i C 2 \alpha P < 0 \quad (6)
\]

Proof: Since \(\text{rank}(\begin{bmatrix} E \\ C \end{bmatrix}) = n\), there exist a nonsingular matrix \(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\) such that:

\[
aE + bC = I_n \quad (7)
\]

\[
cE + dC = 0 \quad (8)
\]

The estimation error is given by:

\[
e(t) = x(t) - \hat{x}(t) \quad (9)
\]

Then by substituting (2) and (4) into (7) we obtain:

\[
e(t) = (I_n - bC - K dc)x - z \quad (10)
\]

From (7) and (8), we have:

\[
e(t) = (a + Kc) Ex - z \quad (11)
\]

Then, the dynamic of this observer error is:

\[
\dot{e}(t) = (a + Kc) E \dot{x} - \dot{z} \quad (12)
\]

By substituting (2) into (12) we obtain:

\[
\dot{e}(t) = \sum_{i=1}^{q} h_i(x) [(a + Kc) (A_i x + B_i u) - N_i z - L_{1i} y - L_{2i} y - G_i u] \quad (13)
\]
Observer for a class of non linear singular systems

By substituting (11), equation (13) can be written as:

\[
\dot{e}(t) = \sum_{i=1}^{q} h_i(x)[(a + Kc)A_i x + (a + Kc)B_i u + N_i e - N_i(a + Kc)Ex - L_{1i}Cx - L_{2i}CxG_iu]
\] (14)

Which is equivalent to:

\[
\dot{e}(t) = \sum_{i=1}^{q} h_i(x)[((a + Kc)A_i - N_i(a + Kc)E - L_{1i}C - L_{2i}C)x + ((a + Kc)B_i - G_i)u + N_i e]
\] (15)

Equation (15) reduces to the equation:

\[
\dot{e}(t) = \sum_{i=1}^{q} h_i(x)N_i e
\] (16)

Provided the matrices \(G_i, K, L_{1i}, L_{2i}\) and \(N_i\) satisfy:

\[
G_i = (a + Kc)B_i
\] (17)

and

\[(a + Kc)A_i - N_i(a + Kc)E - L_{1i}C - L_{2i}C = 0\] (18)

From (7),(8) and (18), we have:

\[
N_i = (a + Kc)A_i - L_{2i}C - (N_i(b + Kd) - L_{1i})C
\] (19)

Take:

\[
L_{1i} = N_i(b + Kd)
\] (20)

Then:

\[
N_i = aA_i + KcA_i - L_{2i}C
\] (21)

To prove the convergence of the estimation error toward zero, let us consider the following quadratic Lyapunov function:

\[
V(t) = e^T(t)Pe(t), \quad P = P^T > 0
\] (22)

Estimation error convergence is ensured if the following condition is guaranteed:

\[
\dot{V}(t) < 0
\] (23)
Its derivative with regard to time is given by:

\[
\dot{V}(t) = \dot{e}^T(t)P e(t) + e^T(t)\dot{P} e(t)
\]  

(24)

By using the dynamic of the state estimation error (16), (24) can be written as:

\[
\dot{V}(t) = \sum_{i=1}^{q} h_i(x)(e^T(t)N_i^T Pe(t) + e^T(t)P N_i e(t))
\]

(25)

\[
\dot{V}(t) = \sum_{i=1}^{r} h_i(x) e^T(t)(N_i^T P + PN_i) e(t)
\]

According to the convex sum property of the weighting functions \( h_i(x) \):

\[
\sum_{i=1}^{r} h_i(x) = 1 \text{ and } h_i(x) > 0
\]

the following inequalities are obtained:

\[
\dot{V}(t) < 0 \iff N_i^T P + PN_i < 0
\]

(26)

By substituting (21), equation (26) can be written as:

\[
(aA_i + KcA_i - L_2iC)^T P + P(aA_i + KcA_i - L_2iC)^T < 0
\]

(27)

Which be rewritten:

\[
A_i^T a^T P + (cA_i)^T K^T P - C^T L_{2i}^T P + PaA_i + PKcA_i - PL_{2i}^2 C < 0
\]

(28)

The matrices inequalities (28) are not linear with regard to the variables \( P \), \( K \), and \( L_{2i} \). In order to solve these matrices inequalities, it is necessary to linearize them to obtain LMIs. To do this, we will use some variable changes:

\[
\begin{cases}
Q = PK \\
W_i = PL_{2i}
\end{cases}
\]

(29)

which allows to obtain:

\[
A_i^T a^T P + PaA_i + (cA_i)^T Q^T + Q(cA_i) - C^T W_i^T - W_i C < 0
\]

(30)

To assure a speed of convergence of the error of estimation, we can define a surface \( S \) to the left of the complex plan bounded by a right of abscissa \(( -\alpha )\) where \( \alpha \in \mathbb{R}^+ \).

The LMIs (30) are then replaced by the following LMIs:

\[
A_i^T a^T P + PaA_i + (cA_i)^T Q^T + Q(cA_i) - C^T W_i^T - W_i C 2\alpha P < 0
\]

(31)

which completes the proof.
Theorem 3.1: There exists an observer (5) for (2) if the Hypotheses H1, H2, H3 and H4 holds and there exists a symmetric positive matrices P, Q and W verifying (6) for $i = 1, \ldots, q$

Finally, the design of the proposed observer is reduced to the following procedure.

Step 1. Verify that Hypotheses H1, H2, H3 and H4 holds.

Step 2. Compute $a, b, c$ and $d$ using (7) and (8).

Step 3. Solve the LMI (6) in $P, Q$ and $W_i$.

Step 4. Compute $K$ and $L_{2i}$ using (29). The matrices $N_i, L_{1i}$ and $G_i$ are derived from (21), (20) and (17) respectively.

4 An illustrative example

In this section, the proposed method is illustrated through an example. Consider a singular system represented by the circuit displayed in figure (1) [2]. Where a dc source with voltage $E$ is connected in series to a linear resistor, a linear inductor and a nonlinear capacitor with $q - v$ characteristic

$$q = z(v) = v - v_0^{1/3} + q_0$$

![Figure 1: A nonlinear circuit](image)

This system can be modeled as descriptor system by the following charge-flux description:

$$\begin{cases}
\dot{q} &= \frac{\phi}{L} \\
\dot{x}_2 &= \frac{-R}{L}\phi - v + E \\
0 &= v - v_0 - (q - q_0)^3
\end{cases}$$ (32)
where $\phi$ is the magnetic flux in the inductor. $R = 1, L = 0.5, E = 2, v_0 = 1, q_0 = 1$.

By choosing the state and input vector as following:

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} q - q_0 \\ \phi \\ v - v_0 \end{pmatrix}, \quad u = E - v_0.$$  

We assume that $x_1$ and $x_3$ can be measured. The system can be described by the following implicit model:

$$\begin{align*}
E \dot{x} &= f(x) + g(x)u \\
y &= Cx
\end{align*} \quad (33)$$  

with:

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad f(x) = \begin{pmatrix} x_2 \\ \frac{L}{-R}x_2 - x_3 \\ \frac{L}{x_3 - x_1^3} \end{pmatrix}, \quad g(x) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  

The fuzzy descriptor system obtain from (33) is given by:

$$\begin{align*}
E \dot{x}(t) &= \sum_{i=1}^{2} h_i(x(t))(A_i x(t) + B_i u(t)) \\
y(t) &= C x(t)
\end{align*} \quad (34)$$  

Where

$$A_1 = \begin{pmatrix} 0 & 2 & 0 \\ 0 & -2 & -1 \\ -4 & 0 & 1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 2 & 0 \\ 0 & -2 & -1 \\ 0 & 0 & 1 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  

The weighting functions are defined by:

$$h_1(x(t)) = \frac{x_1^2}{4}, \quad h_2(x(t)) = \frac{4 - x_1^2}{4}.$$
The fuzzy observer for system (34) is given by:

\[
\begin{aligned}
\dot{z} &= \sum_{i=1}^{2} h_i(x)(N_i z + L_{1i} y + L_{2i} y + G_i u) \\
\dot{x} &= z + by + K dy
\end{aligned}
\]  

(35)

with

\[
\begin{bmatrix}
0.5 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix},
\begin{bmatrix}
0.5 & 0 \\
0 & 0 \\
0 & 1
\end{bmatrix},
\begin{bmatrix}
-0.7071 & 0 & 0 \\
0 & 0 & -1
\end{bmatrix},
\begin{bmatrix}
0.7071 & 0 \\
0 & 0
\end{bmatrix}
\]

\[
P = \begin{pmatrix}
5.2250 \times 10^7 & 2.0767 \times 10^{-10} & -9.1492 \times 10^{-9} \\
2.0767 \times 10^{-10} & 5.2250 \times 10^7 & 1.1787 \times 10^{-10} \\
-9.1492 \times 10^{-9} & 1.1787 \times 10^{-10} & 5.2250 \times 10^7
\end{pmatrix}.
\]

\[
N_1 = \begin{pmatrix}
-10.5000 & 0.3745 & -1.9485 \\
-0.3745 & -10.5000 & -0.7127 \\
1.9485 & -10.5000 & -10.5000
\end{pmatrix},
N_2 = \begin{pmatrix}
-10.5000 & 0.3745 & 1.0825 \\
-0.3745 & -10.5000 & -0.7127 \\
1.0825 & -10.5000 & -10.5000
\end{pmatrix}.
\]

\[
L_{11} = \begin{pmatrix}
-6.2475 & -1.9485 \\
-44.6754 & -0.7127 \\
8.3545 & -10.5000
\end{pmatrix},
L_{12} = \begin{pmatrix}
-7.3276 & 1.0825 \\
-44.6754 & -0.7127 \\
5.8912 & -10.5000
\end{pmatrix}.
\]

\[
L_{21} = \begin{pmatrix}
1.8402 & 4.1134 \\
-0.0618 & -0.1782 \\
4.1134 & 8.9845
\end{pmatrix},
L_{22} = \begin{pmatrix}
10.5000 & 1.0825 \\
0.3745 & -0.1782 \\
1.0825 & 8.9845
\end{pmatrix}.
\]

\[
Q = 10^8 \begin{pmatrix}
0.2311 & -1.1312 \\
3.1404 & -0.0570 \\
-0.2633 & 0.7918
\end{pmatrix},
K = \begin{pmatrix}
0.4423 & -2.1649 \\
6.0104 & -0.1091 \\
-0.5040 & 1.5155
\end{pmatrix}.
\]

\[
W_1 = 10^8 \begin{pmatrix}
0.9615 & 2.1492 \\
-0.0323 & -0.0931 \\
2.1492 & 4.6944
\end{pmatrix},
W_2 = 10^8 \begin{pmatrix}
5.4862 & 0.5656 \\
0.1957 & -0.0931 \\
0.5656 & 4.6944
\end{pmatrix}.
\]

\[
G_1 = \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix},
G_2 = \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix},
x_0 = \begin{pmatrix}
0 \\
2 \\
0
\end{pmatrix}^T, x_{e_0} = \begin{pmatrix}
0 \\
2.5 \\
0
\end{pmatrix}^T.
\]
The simulation results are given in figure 2.

Figure 2 display the comparison of the state variable and their estimates supplied by the proposed observer.

5 Conclusion

In this paper a simple method is proposed to design observers for Takagi-Sugeno descriptor systems. This work is an extension of the observers developed for linear systems to nonlinear Takagi-Sugeno systems. The determination of the observer parameters is based on the singular decomposition and solving a system of LMI. The design example has illustrated the proposed method.
References


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