

# Mobility Models for Unstrained and Strained Silicon MOSFETS: A Review

**Amit Chaudhry**

Faculty of University Institute of Engineering and Technology  
Panjab University, Chandigarh, India  
amit\_chaudhry01@yahoo.com

**Sonu Sangwan**

University Institute of Engineering and Technology, Punjab University,  
Chandigarh, India  
sonuvlsi@gmail.com

**Jatindra Nath Roy**

With Solar Semiconductor Private Limited  
Hyderabad, India

## Abstract

In this paper, a critical review of various models for the effective mobility of charge carriers in metal oxide semiconductor field effect transistor (MOSFET) inversion layer (p and n type) has been done. Analytical, empirical and numerical models for carrier mobility have been reviewed and compared. Both strained silicon and unstrained silicon MOSFET cases have been studied. Depending upon their accuracy and simplicity, certain models have been identified to be applicable for mobility determination in strained and unstrained MOSFETs.

**Keywords:** Scattering; Electric field dependence; Strain; Analytical; Numerical; Empirical; Comparisons.

## 1 Introduction

As projected by Moore's law, the transistor density on a CMOS chip doubles approximately after every one and a half years [1]. Continuing with the Moore's law, the gate length of the MOSFET will eventually shrink to 10 nm in 2015 [2].

As the feature size of transistor decreases, the improvement in the VLSI MOSFET device models are required so that the exact behavior of deep sub-micron and nanometer scaled MOSFETs can be described with accuracy. The reduced dimensions of the device mostly affect the mobility in the inversion layer of the MOSFET. But as we scale down the MOSFET, carrier mobility decreases due to the high vertical electrical fields in the substrate. The reduction in carrier mobility is a major cause of drain current degradation. This reduces the speed of the device.

In this paper, a detailed comparative study of the different inversion layer mobility models for silicon and strained silicon MOSFET has been done. The paper is organized as follows. Section 2 deals with the numerical models. Then, section 3 gives a review of empirical models and section 4 elaborates analytical models. Section 5 compares the above mentioned models and conclusion is given in section 6.

## 2 Numerical Mobility Models

There are several numerical mobility models reported in the literature. Out of these, three main numerical models for the mobility of carriers in the inversion layers of MOSFET, as a function of electric field have been discussed here. In year 1979, Ken Yamaguchi [9], the effect of both parallel and perpendicular fields on the carrier mobility has been taken by using two dimensional numerical analysis of MOSFET. Variation of electron mobility with effective electric field for device parameters, substrate doping= $3.5 \times 10^{15} \text{cm}^{-3}$ , effective channel length= $2 \mu\text{m}$  and oxide thickness= $50 \text{nm}$  is shown in figure 1. Variation of hole mobility with effective electric field is shown in figure 2. This model gives excellent agreement with experimental results. The discrepancies are only 5 percent in the current saturation region. The limitation of this model is that, the local drain gate field becomes very strong at the same time and saturation velocity intensively depends on the perpendicular electric field. The 2<sup>nd</sup> model included here removes these limitations.

In year 1987, Nishida and Sah [10], carrier mobility for MOSFET has been modeled by including surface and bulk acoustic phonon-scattering, intervalley-phonon-scattering, bulk ionized-impurity scattering, surface oxide-charge-scattering and the surface roughness scattering using the theoretical and experimental results. Both electrons and holes mobility has been taken in [10]. The velocity saturation effect is also modeled. The comparison of this model with the Selberherr model gives excellent agreement in subthreshold region and a less than about 20 percent difference in the strong inversion range. This model is used for micrometer and sub-micrometer size silicon MOSFETs. Variation of electron mobility with effective vertical electric field is shown in figure 1.

Lastly, in 2009 Luca Donetti et al. [13], the effective mobility for holes in inversion layers is computed using a one-particle Monte Carlo simulator and the Poisson solver. The optical and acoustic phonon scattering, surface roughness and

the coulomb interaction due to impurities and trapped charges have been included in the model. The hole mobility versus effective electric field is shown in figure 2. Moreover, effect of uniaxial/biaxial stress on the hole mobility are also described in this model as shown in figure 3.

Effective mobility of these models are compared with the experimental results of models [5,6] in the inversion layer of MOSFETs. For micrometer size devices Ken Yamaguchi [9], gives the excellent agreement with experimental results for both electron and hole mobility.

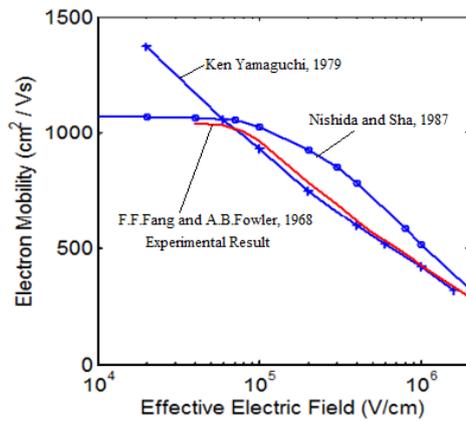


Figure 1: Variation of electron mobility with effective electric field for numerical models including experimental results of [5] is shown.

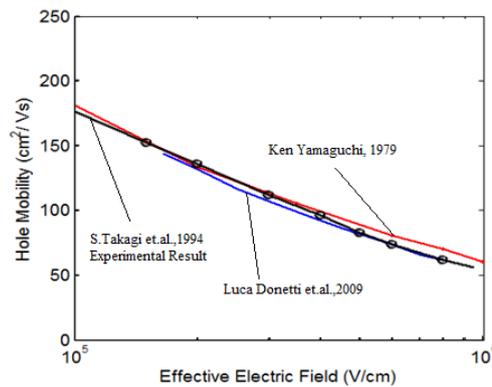


Figure 2: Variation of hole mobility with effective electric field for numerical models including experimental results of [6] is shown.

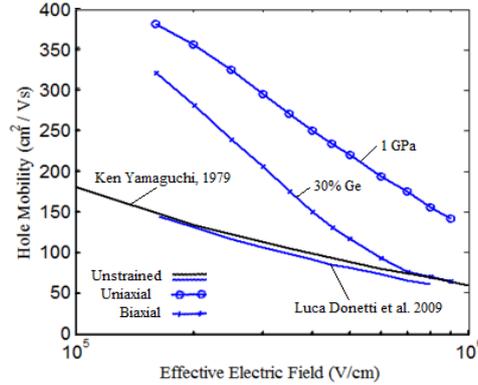


Figure 3: Variation of hole mobility of strained (biaxial for 30% Ge fraction and uniaxial for 1GPa compressive stress) and unstrained MOSFETs with effective electric field is shown.

### 3 Empirical Mobility Models

Several empirical models for the carrier mobility in the inversion layers of MOSFET, as a function of electric field have been reported. In 1972, Mansour et al. [3, 4], and in 1967, Caughey and Thomas [14], the effective inversion layer mobility was modeled by equation (1).

$$\mu_{\text{eff}} = \mu_0 / [1 + (E_{\text{eff}}/E_{\text{crit}})^\alpha]^{1/\alpha} \quad (1)$$

$E_{\text{crit}}$  is the critical transverse electric field,  $E_{\text{crit}} = 6.98 \times 10^3 (T(K)/300)^{1.55}$

V/cm.  $\alpha = 1$ , for holes and  $\alpha = 2$  for electrons. In 1979, Sabins and Clemens [7] measured the effective mobility of electrons in the inverted (100) Si surface over a wide range of temperatures, gate voltages, and back-bias voltages. The measured effective mobility is calculated experimentally as a function of effective electric field at the doping concentration  $2.31 \times 10^{15} \text{ (cm}^{-3}\text{)}$ .

In 1980, Sun and Plummer [15] modeled the measured effective inversion layer electron mobility by an empirical model, described by equation (2).

$$\mu_{\text{eff}} = \mu_{\text{max}} (E_c/E_{\text{eff}})^{c_2} \quad (2)$$

Where  $\mu_{\text{max}} = \mu_0 N_A / (1 + \alpha N_A Q_f)$ ,  $N_A$  is the doping concentration,  $Q_f$  is the oxide charge density in unit of  $10^{11} \text{ cm}^{-2}$ , the constant  $E_c = A e^{BQ_f}$ ,  $A = 2.79 \times 10^4 \text{ V/cm}$ ,  $B = 8.96 \times 10^{-2}$  for dry oxidation,  $c_1 = 0.313 - 6.05 \times 10^{-3} \log N_A$ ,  $\mu_0 =$

3490-164log $N_A$ , and  $\alpha = -1.04 \times 10^{-1} + 1.39 \times 10^{-2} \log N_A$ . Variation of effective mobility with effective electric field is shown in figure 5, for  $N_A = 1.22 \times 10^{15} \text{ cm}^{-3}$ .

In 1983, Schwarz and Russek [16, 17] the inversion layer electron mobility at room temperature is modeled by semi-empirical relations given in equation (3).

$$\frac{1}{\mu_{eff}} = \frac{1}{1160T^{-0.5}} + \frac{3.2 \times 10^{-21}/z}{z} \quad (3)$$

Where

$$\square = 0.09^{3/2} + 1.5 \times 10^{-8} (N_I/z)^{-1/4} \square^{-1} N_f$$

$$z = 0.039/E_{eff} + 1.24 \times 10^{-8}/E_{eff}^{1/3}$$

$\square = T/300 \text{ K}$ ,  $N_f$  is the interface fixed charge density.

Variation of effective electron mobility with normal electric field is shown in figure 5, for  $N_A = 1.22 \times 10^{15} \text{ cm}^{-3}$ .

In 1988, Walker and Woerlee [19] modeled the effective mobility of carriers by semi-empirical relations using Matthiessen's rule. This model is described by a set of equations (4-7).

$$\mu_{eff}^{-1} = \mu_{ph}^{-1} + \mu_{sr}^{-1} + \mu_c^{-1} \quad (4)$$

$$\text{Where } \mu_c = \left( \frac{CT^{1.5}}{\ln(1+b) - b/(1+b)} \right) \frac{1}{N_I} \quad (5)$$

$b = (24m_{si}^*k^2T^2)/(\hbar^2q^2N_A)$ ,  $N_I$  is charged impurity density,  $N_A$  is the mobile carrier density,  $C$  is constant,  $\epsilon_{si}$  is the dielectric constant of silicon and other symbols have their usual meanings.

$$\mu_{ph} = k_{ph} T^{-1} E_{eff}^{-1/3}, \quad (6)$$

$$\mu_{sr} = k_{sr} E_{eff}^{-2} \quad (7)$$

Constants  $C$ ,  $k_{ph}$ ,  $k_{sr}$  are the fitting parameters. The value of these

parameters for electrons and holes is given in the model [20]. Variation of effective electron mobility with effective electric field for  $N_A = 2.1 \times 10^{16} \text{ cm}^{-3}$

is shown in figure 5. Variation of effective hole mobility with effective electric

field is shown in figure 6.

In 1989, Jeon and Burk [21] the effective inversion layer mobility at room temperature is modeled by a set of equations (8-11).

$$\mu_{\text{eff}}^{-1} = \mu_{\text{ph}}^{-1} + \mu_{\text{sr}}^{-1} + \mu_{\text{c}}^{-1} \quad (8)$$

$$\text{Where } \mu_{\text{c}} = a_1^{-1} T^{-1} E_{\text{eff}}^{\alpha} \quad (9)$$

$$\mu_{\text{ph}} = a_2^{-1} T^{-n} E_{\text{eff}}^{-1/\gamma} \quad (10)$$

$$\mu_{\text{sr}} = a_3^{-1} E_{\text{eff}}^{-2} \quad (11)$$

and  $a_1$ ,  $a_2$ ,  $a_3$ ,  $n$  and  $\alpha$  are the fitting parameters are extracted at room temperature. Variation of effective mobility with effective electric field is shown in figure-5.

In 1991, Shin et. al. [22] the electron inversion layer mobility is modeled by physically-based semi-empirical relations using Matthiessen's rule described by set of equations (12-15).

$$\mu_{\text{eff}}^{-1} = \mu_{\text{ph}}^{-1} + \mu_{\text{sr}}^{-1} + \mu_{\text{c}}^{-1} \quad (12)$$

$$\text{Where } \mu_{\text{ph}} = \left[ (1150 T_n^{-2.8})^{-1} + \left( \frac{0.0388 T_n E_{\text{eff}}^{-2} + 1.78 \times 10^{-6} E_{\text{eff}}^{-1/\gamma}}{3.2 \times 10^{-9} p T_n^{-0.82}} \right)^{-1} \right]^{-1} \quad (13)$$

$$\text{Where } p = 0.09 T_n^{1.75} + 4.53 \times 10^{-8} \left( \frac{N_I}{N} \right)^{-0.25} T_n^{-1} N_f$$

$N_f = 3 \times 10^{10} \text{ cm}^{-2}$ ,  $N_I = (2d/q)(E_{\text{eff}} + E_0)$ ,  $T_n = T/300$ ,  $E_0$  is the transverse electric field at the inversion layer edge.

$$\mu_{\text{sr}} = 6 \times 10^{14} E_{\text{eff}}^{-2} \quad (14)$$

$$\mu_{\text{c}} = \left( \frac{K_c T_n^{2.8}}{\ln(1 + Y_{\text{BH}}^2) - (Y_{\text{BH}}^2 / (1 + Y_{\text{BH}}^2))} \right) \frac{1}{N_A} \quad (15)$$

Where  $Y_{\text{BH}}^2 = \frac{2 \times 10^{20}}{N_I / Z} T_n^2$ ,  $Z$  is the average inversion layer width. Variation of

effective electron mobility with electric field is shown in figure 5.

In 1993, Yue et al. [25] developed an improved comprehensive universal model for effective electron mobility in inversion layers of n-channel MOSFETs for circuit simulation. This model expresses the effective electron mobility at room temperature as a function of effective vertical field. The effective mobility after fitting experimental data taken at room temperature [7, 15, 23, 24] is given by equation (16).

$$\mu_{eff} = \frac{1481}{1 + 0.0738E_{eff}^{0.25} + 2.69 \times 10^{-12}E_{eff}^2} \quad (16)$$

Variation of effective electron mobility with effective electric field is shown in figure 5.

In 1994, Huang and Arora [26] the effective inversion layer mobility at room temperature is modeled by equation (17)

$$\mu_{eff} = \mu_0 / [1 + aE_{eff}^{1/3} + bE_{eff}^2 + c(1 + Q_t/Q_b)^2] \quad (17)$$

Where the 1/3 power dependence models the acoustic phonon scattering and 2 power dependence is the surface roughness scattering, where  $\mu_0$ , a, b and c are

the fitting parameters. Variation of effective electron mobility with electric field is shown in figure 5.

In 1996, Cheng and Woo [27] developed a semi empirical model for electron and hole mobilities of MOSFET inversion layer. This model takes in to account the dependence of different scattering mechanism on temperature and transverse electric field over a wide range. The effective mobility is modeled by equation (18) by the same formula as proposed by Watt, et al. [28].

$$\mu_{eff}^{-1} = \mu_{ph}^{-1} + \mu_{sr}^{-1} + \mu_c^{-1}$$

$$\frac{1}{\mu_{eff}} = \frac{1}{\mu_s} \left( \frac{10^6}{E_{eff}} \right)^{\alpha_s} + \frac{1}{\mu_s} \left( \frac{10^6}{E_{eff}} \right)^{\alpha_s} + \frac{1}{\mu_s} \left( \frac{10^{18}}{N_b + Q_f/z} \right)^{-1} \left( \frac{10^{12}}{Q_{inv}} \right)^{\alpha_s} \quad (18)$$

Where  $N_b$  is substrate doping concentration,  $Q_f$  is the interface state density, and  $z$  is the width of inversion layer given by [17]. Variation of hole mobility with effective electric field is shown in figure 6.

In 1996, Chen, et al. [29] modeled the measured effective inversion layer electron mobility at room temperature by an empirical model as given in equation (19).

$$\mu_{eff} = 540 / [1 + (E_{eff}/0.9)^{1.85}] \quad (19)$$

In 1997 Chain, et al. [30] modeled the effective inversion layer mobility of MOSFET based on physics of scattering mechanism. In this model the four major scattering are: 1. phonon scattering due to lattice vibration. 2. surface roughness scattering due to the microscopic roughness of the Si-SiO<sub>2</sub> interface; 3. coulomb scattering from ionized bulk impurities; and 4. coulomb scattering from the Si-SiO<sub>2</sub> interface charges. The effective mobility is given by equation (20).

$$\mu_{\text{eff}} = \mu_0 \left\{ \left( 1 + U_a E_{\text{eff}}^{1/3} \right) \left( \frac{T}{300} \right)^{3/2} + U_b \left( \frac{T}{300} \right)^{-1/2} \times E_{\text{eff}}^2 (1 + U_{k1} V_{bs}) + \left( U_{c0} T E_{\text{eff}}^{-2} + U_{c1} \left( \frac{300}{T} \right) E_{\text{eff}}^{-1} \right) (1 + U_{k2} V_{bs}) \right\}^{-1} \quad (20)$$

Where  $U_a, U_b, U_{c0}, U_{c1}, U_{k1}$  and  $U_{k2}$ , are the parameters given in [30].

In 2001, Lim and Zhou [34] modeled the measured effective inversion layer mobility at room temperature by a physically based semi-empirical model given by equation (21).

$$\mu_{\text{eff}}^{-1} = \mu_{\text{ph}}^{-1} + \mu_{\text{sr}}^{-1} + \mu_c^{-1}$$

$$\mu_{\text{eff}} = \frac{\mu_c}{1 + (\mu_2/\mu_1) E_{\text{eff}}^{2/3} + (\mu_3/\mu_1) E_{\text{eff}}^2} \quad (21)$$

Where  $\mu_1 = \mu_c$ , coulombic mobility can be assumed as a constant,

$$\mu_2 = BN^{1/3}/T \text{ and } \mu_3 = CN^{2/3}.$$

In 2002, Remashan, et al. [35] modeled the inversion layer carrier mobility in all regions of MOSFET operation with the help of a 2-D drift and diffusion device simulator. In this empirical model two expressions are employed for mobility, one for the universal part, [36] and other for the non universal part [37]. Using the Matthiessen-rule, the resulting mobility is given by equation (22)

$$\mu_{\text{eff}}^{-1} = \mu_{\text{uni}}^{-1} + \mu_{\text{non-uni}}^{-1} \quad (22)$$

Where, for electron

$$\mu_{\text{uni}} = \frac{600 \left( 1 - \frac{V_{bs}}{20} \right)}{1 + \left( \frac{E_{\text{eff}}}{0.8} \right)^2} \text{ cm}^2/\text{Vs}, \text{ and}$$

$$\mu_{\text{non-uni}} = [250 - (100V_{bs})] + 300 \left( \frac{N_i}{10^{21}} \right) \text{ cm}^2/\text{Vs}$$

For holes

$$\mu_{\text{uni}} = \frac{192 \left( 1 + \frac{V_{bs}}{20} \right)}{1 + \left( \frac{E_{\text{eff}}}{0.45} \right)^2} \text{ cm}^2/\text{Vs}, \text{ and}$$

$$\mu_{\text{non-uni}} = [100 + (40V_{bs})] + 70 \left( \frac{N_i}{10^{21}} \right) \text{ cm}^2/\text{Vs},$$

$N_i$  is the electron/hole concentration per unit area in the inversion layer. Variation of hole mobility with effective electric is shown in figure 6.

In nano scale devices the mobility of carriers can be increased by using the strained silicon technology. When oxide thickness is below approximately 10nm, carrier mobility is degraded [38]. In 2005, Lauer and Antoniadis [39] the effective mobility was extracted using equation (23).

$$\mu_{eff} = LI_D / WQ_i V_{DS} \tag{23}$$

Where  $Q_i$  is determined by integration of high frequency C-V characteristic and  $V_{DS}$  is the effective source/drain voltage. Mobility enhancement with applied strain for different inversion layer densities is given in the model [39]. All devices show a linear mobility enhancement with strain. The thick devices shows similar enhancement at low and high fields. The thin device shows more enhancement than the thick device dose, especially at low field.

In 2004, Wei Zhao, et al. [40] the effect of tensile uniaxial strain on the DC performance of partially-depleted silicon on insulator n and p-channel MOSFETs are reported. The drain current of the n-MOSFETs increases for both longitudinal and transverse strain orientations with respect to the current flow direction. In n-MOSFET, longitudinal strain provides greater enhancement than transverse strain. For p-MOSFETs, longitudinal strain decreases the current while transverse strain increases the drain current. A change in drain current with strain can result from changes in the effective mobility. The effective mobility is given by the equation (24)

$$\mu_{eff} = LI_D / [WC_{ox} V_{DS} (V_{GS} - V_T)] \tag{24}$$

The threshold voltage  $V_T$ , is linearly extrapolated from the point of maximum transconductance [41]. The effective vertical field is calculated from using  $E_{eff} = (V_{GS} + V_T) / 6T_{ox}$  for n-channel, and  $E_{eff} = (V_{GS} + 1.5V_T) / 7.5T_{ox}$  for

p-channel devices [42]. The extracted electron and hole effective mobility versus effective vertical effective electric field is shown in figure 4.

Several empirical and semi-empirical mobility models are compared with the experimental results of model [6]. Sun and Plummer [15], Schwarz and Russek [16, 17] and Shin et. al. [22] models give the excellent match with experimental results for electron mobility. All other models are also gives good match with experimental results. For hole mobility Chen, et al. [29] model gives the excellent match with the experimental results.

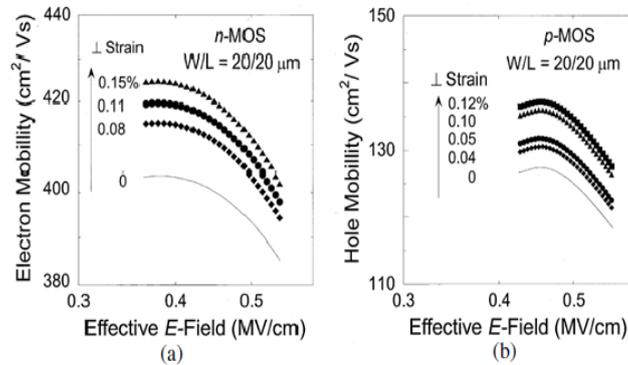


Figure 4: (a) Extracted electron and (b) hole effective mobility versus effective vertical field for n-MOSFET under transverse strain [40].

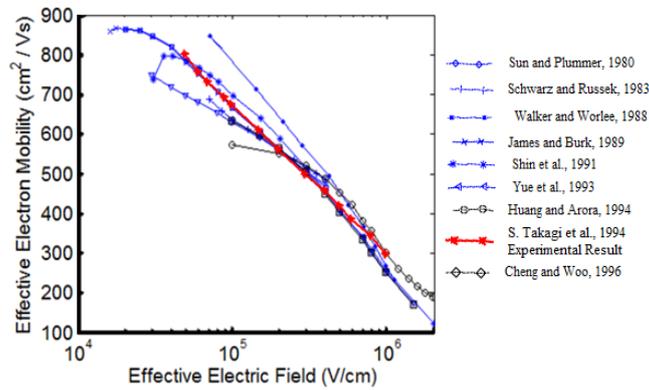


Figure 5: Variation of effective electron mobility in the inversion layer of MOSFET with effective electric field including experimental results of [6] for empirical and semi-empirical models is shown.

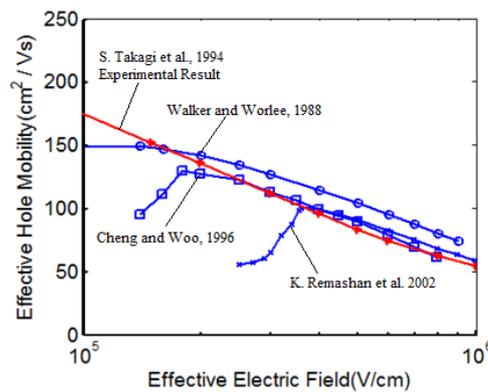


Figure 6: Variation of effective hole mobility in the inversion layer of MOSFET with effective electric field including experimental results of [6], for various empirical and semi-empirical models is shown.

## 4 Analytical Mobility Models

Carrier mobility in the inversion layers of MOSFET have been described analytically. Several analytical models for the mobility of carriers in the inversion layers of MOSFET have been reported.

In 1992, D.B.M. Klaassen [43], represented the first physics-based analytical model that unifies the description of majority and minority carrier mobility and that includes screening of impurities by charge carriers, electron-hole scattering and clustering of impurities. In this model the electron and hole mobility are given as analytical functions of local variables: ionized donor, ionized acceptor, electron and hole concentrations.

In 1998, S. Villa, et al. [44] the effective inversion layer mobility is modeled by analytical relations for heavily-doped ULSI MOSFETs using Matthiessen's rule as described by equations (25-28).

$$\mu_{eff}^{-1} = \mu_{ph}^{-1} + \mu_{sr}^{-1} + \mu_e^{-1} \quad (25)$$

$$\mu_{ph} = \mu_{phb} \left[ \left( \frac{T}{300K} \right)^n + \left( \frac{T}{300K} \right)^r \left( \frac{E_{eff}}{E_p} \right)^{\alpha(T)} \right]^{-1} \quad (26)$$

Where  $\mu_{phb}(300K) = 1470 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$  is the phonon limited bulk mobility,  $n=2.109$ ,  $r=1.7$ ,  $E_0 = 7 \times 10^4 \text{ V/cm}$  and  $\alpha(T) = 0.2(T/300)^{-0.1}$

$$\mu_e = F^2 \mu_0 \frac{F}{N_A L_{th}} \left( 1 + \frac{L_{th}}{F L_s} \right)^2 \quad (27)$$

Where  $L_{th}$  is called the thermal length,  $L_s$  is called the screening length and F is the ratio between Debye-Huckel and effective screening length.

$$\mu_{sr} = \frac{\delta \exp(-(T/T_0)^2)}{E_{eff}^2} \quad (28)$$

Where  $\delta$  and  $T_0$  are the fitting parameters. The effective mobility as a function of effective electric field is shown in figure 7.

In 2007, N.Rodriguez, et al. [45] the analytical expression for the mobility carried out by using the Matthiessen's rule:

$$\mu_{eff}^{-1} = \mu_{ph}^{-1} + \mu_{sr}^{-1} + \mu_n^{-1} \quad (29)$$

The remote scattering mobility component ( $\mu_n$ ) given in equation (2) in the model, the Si-SiO<sub>2</sub> surface roughness mobility component ( $\mu_{sr}$ ) given in equation (5) by an analytical expression in the model and the phonon scattering mobility

component given in equation (6) in the model. Variation of electron mobility including phonon, interface roughness, coulomb scattering, remote roughness and coulomb scattering due to polysilicon depletion charge with electric field at room temperature for doping concentration  $=3 \times 10^{16} \text{ cm}^{-3}$  is shown in figure 7.

In 2011, M.H.Bhuyan and Q.D.M. Khorsu [46] modeled the inversion layer mobility for pocket implanted nano scale n-MOSFETs. The effective mobility for nano MOSFET is given by equation (30) using the Matthiessen's rule:

$$\mu_{\text{eff}}^{-1} = \mu_{\text{equ}}^{-1} + \mu_{\text{bal}}^{-1} \quad (30)$$

Where,  $\mu_{\text{bal}} = 2qL/\pi m_{\text{n,eff}} v_{\text{th}}$ , is the ballistic mobility, and

$\mu_{\text{equ}}^{-1} = \mu_{\text{ph}}^{-1} + \mu_{\text{sr}}^{-1} + \mu_{\text{c}}^{-1}$  is the total mobility that consider the effect of

all scattering mechanisms. The effective mobility in (nano scaled) MOSFETs must be much smaller than the electron mobility in long channel devices. This reduction was predicted for ballistic devices in [47-49]. Variation of effective electron mobility with the effective electric field is shown in figure 7.

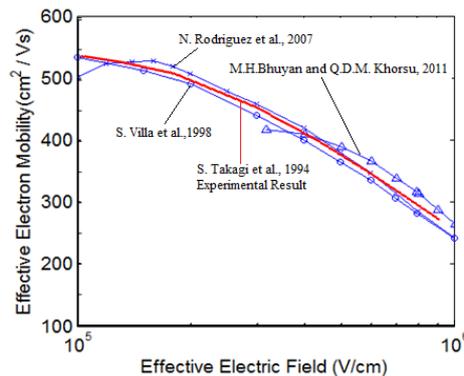


Figure 7: Variation of effective electron mobility with effective electric field for various analytical models including experimental results of [6] for analytical models is shown.

To enhance the mobility in the inversion layer of MOSFETs, strained-Si is regarded as possible alternatives to conventional Si devices mostly for deep submicron devices. In 2003, J.B.Roldan, et al. [50] mobility is modeled for strained-Si MOSFETs accounting for various scattering mechanisms such as surface roughness scattering, phonon scattering and coulomb scattering. Phonon scattering due to the lattice vibrations is given by equation (31).

$$\frac{1}{\mu_{\text{ph}}} = \frac{1}{\mu_{\text{ph}0} \beta(x)} \left[ 1 + \left( \frac{E_{\text{EFF}}}{E_0} \right)^{0.2\alpha(x)} \right] \quad (31)$$

$$\mu_{\text{ph}0} = 1470 \text{ cm}^2/\text{Vs}$$

$$E_0 = 2 \cdot 10^4 \text{ V/cm}$$

$$\alpha(x) = \frac{a_{\alpha} x}{\left(1 + \left(\frac{a_{\alpha} x}{b_{\alpha}}\right)^{b_{\alpha}}\right)^{1/b_{\alpha}}} + 1$$

$$\beta(x) = \frac{a_{\beta} x}{\left(1 + \left(\frac{a_{\beta} x}{b_{\beta}}\right)^{b_{\beta}}\right)^{1/b_{\beta}}} + 1$$

$$a_{\alpha} = 3, b_{\alpha} = 25, a_{\beta} = 11, b_{\beta} = 15$$

Surface roughness scattering due to the microscopic roughness of the Si-SiO<sub>2</sub> interface is given by equation (32).

$$\mu_{sr} = \frac{\delta(x) 10^{24}}{E_{eff}} \quad (32)$$

$$\delta(x) = \frac{a_{\delta} x}{\left(1 + \left(\frac{a_{\delta} x}{b_{\delta}}\right)^{b_{\delta}}\right)^{1/b_{\delta}}} + \delta_0$$

$$a_{\delta} = 7, b_{\delta} = 15, \Delta_{\delta} = 2.1, \delta_0 = 3.8 \quad \text{Coulomb scattering due to impurity}$$

scattering is given by equation (33).

$$\frac{1}{\mu_c} = C \frac{N_A}{N_D} \quad (33)$$

$$C = 45.45 \cdot 10^{-9} \text{ Vcm}^{-1}$$

$$N_A = 5 \cdot 10^{23}, N_D = \frac{Q_{inv}}{q}$$

$Q_{inv}$  is the inversion charge density

$N_A$  = Doping concentration (cm<sup>-3</sup>).

The total mobility of the electrons is calculated as follows from the mathiessen's rule:

$$\mu_{eff}^{-1} = \mu_{ph}^{-1} + \mu_{sr}^{-1} + \mu_c^{-1} \quad (34)$$

The electron mobility versus effective field for strained-Si on Si<sub>1-x</sub>Ge<sub>x</sub> MOSFETs at room temperature for x=0 and x=0.2 is shown in figure 8.

In 2004, Z.Yang, et al. [51] developed an electron and hole mobility analytical model for strained silicon MOSFETs. The mobility enhancement for both electrons and holes in strained silicon layers is studied by analytical expressions. It can be used in simulations to study the ultra-small strained silicon devices.

Several analytical models are compared with the experimental results of model [6] in figure 7. S. Villa, et al. [44], model gives the excellent match with the experimental results.

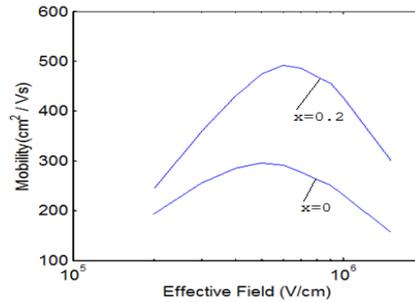


Figure 8: variation of electron mobility with effective field for strained-Si on  $\text{Si}_{1-x}\text{Ge}_x$  MOSFETs at room temperature.

## 5 Summary of Models

The below given table 1 summarizes all the mobility models described in earlier sections of the paper.

Table:1

Year	Model given by	Type of Model	Mobility for Electron/Hole ( $\text{cm}^2 / \text{Vs}$ )	Effective Electric Field (V/cm)
1968	Fang and Fowler	Experimental (unstrained)	955 (electrons)	$10^5$
1979	Ken Yamaguchi	Numerical (unstrained)	930 (electrons)	$10^5$
1987	Nishida and Sha	Numerical (unstrained)	1035 (electrons)	$10^5$
1979	Sabins and Clemens	Experimental (unstrained)	800(electrons)	$10^5$
1980	Sun and Plummer	Empirical (unstrained)	665 (electrons)	$10^5$
1983	Schwarz and russek	Semi-empirical (unstrained)	666 (electrons)	$10^5$
1988	Walker and Woerlee	Semi-empirical (unstrained)	730 (electrons)	$10^5$
1989	Jeon and Burk	Semi-empirical (unstrained)	642 (electrons)	$10^5$
1991	H.Shin et al.	Semi-empirical (unstrained)	700 (electrons)	$10^5$
1993	C.Yue et al.	Empirical (unstrained)	630 (electrons)	$10^5$
1994	Huang and Arora	Empirical (unstrained)	570 (electrons)	$10^5$
1994	S. Takagi et al.	Experimental (unstrained)	776 (electrons)	$10^5$
1996	B.Cheng and J.Woo	Semi-empirical (unstrained)	635 (electrons)	$10^5$
2001	Lim and Zhou	Semi-empirical (unstrained)	340 (electron)	$10^5$
2002	K.Remashan et al.	Empirical (unstrained)	170 (electron)	$10^5$
1998	S.Villa et al.	Analytical (unstrained)	534 (electron)	$10^5$
2007	N.Rodriguez et al.	Analytical (unstrained)	502 (electron)	$10^5$
2011	Bhuyan and Khorsu	Analytical (unstrained)	345 (electron)	$10^5$
2003	J.B.Roldan et al.	Analytical (biaxial for $x=0.2$ )	430 (electron)	$4 \times 10^5$
2003	J.B.Roldan et al.	Analytical (biaxial for $x=0$ )	285 (electron)	$4 \times 10^5$
2004	Z.Yang, et al.	Analytical (biaxial for $x=0.3$ )	720 (electrons)	$4 \times 10^5$
2004	Z.Yang, et al.	Analytical (biaxial for $x=0.2$ )	120 (holes)	$4 \times 10^5$
1979	Ken Yamaguchi	Numerical (unstrained)	180 (holes)	$10^5$
2009	Luca Donetti et al.	Numerical (unstrained)	160 (holes)	$10^5$

2009	Luca Donetti et al.	Numerical (uniaxial for 1GPa)	390 (holes)	$10^5$
2009	Luca Donetti et al.	Numerical (biaxial for 30% Ge)	328 (holes)	$10^5$
1988	Walker and Woerlee	Semi-empirical (unstrained)	127 (holes)	$4 \times 10^5$
1994	S. Takagi et al.	Experimental (unstrained)	97 (holes)	$4 \times 10^5$
1996	B.Cheng and J.Woo	Semi-empirical (unstrained)	100 (holes)	$4 \times 10^5$
2002	K.Remashan et al.	Empirical (unstrained)	98.5 (holes)	$4 \times 10^5$

*Discussion:*

From this summary of models we can select an accurate model for device simulation and analysis for various technology nodes. Out of numerical models, Ken Yamaguchi [9], model gives the excellent match with the experimental model. Luca Donetti et al. [13] shows the enhancement of hole mobility due to uniaxial and biaxial strain. In empirical models, Sun and Plummer [15], Schwarz and Russek [16, 17] and Shin et. al. [22] models give the excellent match with experimental results for electron mobility. For hole mobility Chen, et al. [29] model gives the excellent match with the experimental results. Wei Zhao, et al. [40] model shows the enhancement in mobility due to strain. Out of analytical models, S. Villa, et al. [44] model, gives the excellent match with experimental model. J.B.Roldan, et al. [50] shows the enhancement of mobility due to biaxial strain silicon.

**6 Conclusion**

A critical review of analytical, empirical and numerical models for the effective mobility of charge carriers in MOSFET inversion layer (p and n type) has been done. It has been found that the carrier mobility in strained silicon MOSFETs is higher than the unstrained silicon MOSFET. Certain models have been identified to be applicable for mobility determination in strained and unstrained MOSFETs.

**Acknowledgment**

The authors thank the Director, UIET, Panjab University, Chandigarh, India for allowing to carry out the work. The authors would like to thank to Panjab University, Chandigarh, India for providing excellent research environment to complete this work.

## References

- [1] Schaller.R.R, "MOORE'S LAW: Past, Present and Future", IEEE Spectrum, vol. 34, Issue 6, pp.52-59, June, 1997.
- [2] Moore.G. E, "Cramming more components onto integrated circuits", *Proceedings of the IEEE*, Vol. 86, No. 1, pp. 82-85, 1998.
- [3] I.R.M. Mansour, E.A. Talkhan, and A.I. Barboor, "Investigations on the Effect of Drift-field-dependent Mobility on MOST-Part I:  $Q_B$  Constant", *IEEE Tran Electron Devices*, Vol. ED 19 , No. 8 ,pp. 899-907, (1972).
- [4] E.A. Talkhan, I.R.M. Mansour, and A.I. Barbour, "Investigations on the Effect of Drift-field-dependent Mobility on MOST-Part II:  $Q_B$  Field Dependent", *IEEE Tran Electron Devices*, Vol ED 19, No. 8,pp. 908-916, (1972).
- [5] F. F. Fang and A. B. Fowler, "Transport properties of electrons in inverted silicon surface," *Phys. Rev.*, vol. 169, pp. 619-631, 1968.
- [6] Shin-ichi Takagi, Akira Toriumi, Masao Iwase, and Hiroyuki Tango, "On the Universality of Inversion Layer Mobility in Si MOSFET's: Part I-Effects of Substrate Impurity Concentration", *IEEE Tran Electron Devices*, VOL. 41, NO. 12, pp 2357-2362 December 1994.
- [7] A. G. Sabnis and J. T. Clemens, "Characterization of the electron mobility in the Inverted (100) Si substrate", in *IEEE IEDM Tech. Dig.*, vol. 25, pp. 18–21, 1979.
- [8] G.S.Gildenblat, C.-L.Huang, "Engineering Model of inversion Channel Mobility for 60-300 K Temperature Range", *Electronic Letters*, vol 25, No. 10, pp. 634-636, 11<sup>th</sup> May, 1989.
- [9] Ken Yamaguchi, "Field-Dependent Mobility Model for Two-Dimensional Numerical Analysis; of MOSFET's", *IEEE Tran Electron Devices* VOL. ED-26, NO. 7, pp. 1068-1074, JULY 1979.
- [10] Tashikazu Nishida and Chih-Tang Sah, "Physically Based Mobility Model for MOSFET Numerical Simulation", *IEEE Tran Electron Devices*, VOL. ED-34, NO. 2, pp. 310-320, FEBRUARY, 1987.
- [11] Shin-ichi Takagi, Akira Toriumi, Masao Iwase, and Hiroyuki Tango," On the Universality of Inversion Layer Mobility in Si MOSFET's: Part I I -Effects of Surface Orientation", *IEEE Tran Electron Devices*, Vol. 41, No. 12, December 1994
- [12] S.Takagi, J.L.Hoyt, J.J.Welser, and J.F.Gibbons, "Comparative study of phonon-limited mobility of two-dimensional electrons in strained and unstrained Si metal-oxide-semiconductor field effect transistors", *J. Appl. Phys.* vol. 80, no.3, pp.1567-1577, Aug 1996.
- [13] Luca Donetti, Francisco Gamiz and Noel Rodriguez, "Simulation of hole mobility in two-dimensional systems", *Semicond. Sci. Technol.* 24, 035016 (7pp), 2009.

- [14] D.M. Caughey, R.E. Thomas, "Carrier mobilities in silicon empirically related to doping and field", *Proc. IEEE*, Vol. 55, Issue 12, pp 2192-2193, Dec. 1967.
- [15] S . C. Sun and J. D. Plummer, "Electron mobility in inversion and accumulation layers on thermally oxidized silicon surfaces," *IEEE Trans. Electron Devices*, vol. ED-29, pp. 1497-1508, 1980.
- [16] S. A. Schwarz and S. E. Russek, "Semi-empirical equations for electron velocity in silicon: Part 1-MOS inversion layer", *IEEE Trans. Electron Devices*, vol. ED-29, pp. 1629-1633, 1983.
- [17] S. A. Schwarz and S. E. Russek, "Semi-empirical equations for electron velocity in silicon: Part 11-MOS inversion layer", *IEEE Trans. Electron Devices*, vol. ED-29, pp. 1634-1639, 1983.
- [18] Ken Yamguchi, "A Mobility Model for Carriers in the MOS Inversion Layers", *IEEE Trans. Electron Devices*, VOL. ED-30, NO. 6, pp. 658-663, JUNE 1983.
- [19] A. J. Walker and P. H. Woerlee, "A mobility model for MOSFET device simulation", in *Proc. Europ. Solid-State Dev. Res. Conf. (ESSDERC)*, p. 265, 1988
- [20] Claudio Lombardi, Stefano Manzini, Antonio Saporito, and Massimo Vanzi, "A Physically Based Mobility Model for Numerica Simulation of Nonplanar Devices", *IEEE Transaction on Computer-aided Design*, Vol. 7, No. 11, pp. 1164-1171, November 1988.
- [21] D. S. Jeon and D. E. Burk," MOSFET, "Electron Inversion Layer Mobilities -A Physically Based Semi-Empirical Model for a Wide Temperature Range". *IEEE Trans. Electron Devices*, VOL 36, NO. 8, pp 1456-1463, AUGUST 1989.
- [22] H. Shin, G.M. Yeric, A.F. Tasch, C.M. Maziar, "Physically-based models for effective mobility and local-field mobility of electrons in MOS inversion layers", *Solid-State Electronics*, Volume 34, Issue 6, Pages 545-552,June 1991.
- [23] C. L. Huang and G. S. Gildenblat, "Measurements and modeling of the n-channel MOSFET inversion layer mobility and device characteristics in the temperature range 60-300K," *IEEE Trans. Electron Devices*, vol. 37, pp. 1289-1300, May 1990.
- [24] J. T. Watt and J. D. Plummer, "Universal mobility-field curves for electrons and holes in MOS inversion layers," *Symp. on VLSI Tech. Dig.*, pp. 81-82, 1987.
- [25] C. Yue, V. Martin Agostinelli, Gregory M. Yerk, A and F. Tasch , "Improved Universal MOSFET Electron Mobility Degradation Models for Circuit Simulation", *IEEE Transactions on computer-aided design of integrated circuits and systems*, VOL. 12, NO. 10, pp. 1542-1546, October 1993.
- [26] Cheng-Liang Huang, Narain D. Arora, "Characterization and modeling of the n- and p-channel MOSFETs inversion-layer mobility in the

- range 25–125°C”, *Solid-State Electronics*, Volume 37, Issue 1, Pages 97-103, January 1994.
- [27] Cheng and J. Woo, “ Measurement and Modeling of the n-channel and p-channel MOSFET's Inversion Layer Mobility at Room and Low Temperature Operation”, Colloque 3, suppl Bment au *Journal de Physique* 111, Volume 6, April 1996.
- [28] J. T. Watt, "Modeling the performance of liquid-nitrogen cooled CMOS VLSI," *Stanford Elect. Lab.Tech. Rep.*, No. G725-3, May 1989.
- [29] Kai Chen, H. Clement Wann, Jon Dunster, Ping K. KO and C. Hu, Makato Yashida, “MOSFET Carrier Mobility Model Based on Oxide Thickness, Threshold and Gate Voltage”, *Solid State Electronics*, Vol. 39, No. 10, pp 1515-1518, 1996.
- [30] Kenneth Chain, Jian-hui Huang, Jon Duster, Ping K Ko and Chenming Hu, “ A MOSFET electron mobility model of wide temperature range (77–400 K) for IC simulation”, *Semicond. Sci. Technol.* 12, pp 355–358. Printed in the UK, 1997.
- [31] Mohamed N. Darwish, Janet L. Lentz, Mark R. Pinto, Peter M. Zeitzoff, Thomas J. Krutsick, and Hong Ha Vuong, “ An Improved Electron and Hole Mobility Model for General Purpose Device Simulation”, *IEEE Transactions on Electron Devices*, VOL. 44, NO. 9, September 1997.
- [32] R A Abdel Rassoul, “Mobility degradation model for electron in inversion layer of silicon”, *J.King Saud Univ.*, Vol. 11, Eng.Sci.(1), pp 71-83 (A.H 1419/1999), 1998.
- [33] S. Villa, L. Lacaita, L. M. Perron, and R. Bez, “A physically-based model of the effective mobility in heavily-doped n-MOSFETs”, *IEEE Trans. Electron Devices*, vol. 45, pp. 110–115, Jan. 1998.
- [34] K.Y. Lim, X. Zhou, “A physically-based semi-empirical effective mobility model for MOSFET compact I±V modeling”, *Solid-State Electronics* 45 pp 193-197, 2001.
- [35] K. Remashan, N.A.Wong, K.Chan, S.P.Sim, C.Y.Yang, “Modeling inversion layer carrier mobilities in all regions of MOSFET operations,” *Solid-State Electronics* 46 pp 153-156, 2002.
- [36] Chen K, Wann HC, Ko PK, Hu C, “ The impact of device scaling and power change on CMOS gate performance”, *IEEE Electron Device Lett.*, VOL. 17, NO. 5, pp. 202-204, May 1996.
- [37] M. Suetake, K. Suematsu, H. Nagakura, M. Miura-Mattausch, H. J. Mattausch, “HiSIM: A Drift-Diffusion-Based Advanced MOSFET Model for Circuit Simulation with Easy Parameter Extraction,” *SISPAD*, PP 261-4, 2000.
- [38] J. H. Choi, Y. J. Park, and H. S. Min, “Electron mobility behavior in extremely thin SOI MOSFET’s”, *IEEE Electron Device Lett.*, vol. 16, no. 11, pp. 527–529, Nov. 1995.

- [39] Isaac Lauer and D. A. Antoniadis, "Enhancement of Electron Mobility in Ultrathin-Body Silicon-on-Insulator MOSFETs With Uniaxial Strain", *IEEE Electron Device Letters*, vol. 26, no. 5, may 2005.
- [40] Wei Zhao, Jianli He, Rona E. Belford, Lars-Erik Wernersson, and Alan Seabaugh, "Partially Depleted SOI MOSFETs Under Uniaxial Tensile Strain", *IEEE Trans. Electron Devices*, VOL. 51, NO. 3, pages 317-323, MARCH 2004.
- [41] Y. Cheng and C. Hu, *MOSFET Modeling & BSIM3 User's Guide*. Norwell, MA: Kluwer, p. 93. 1999.
- [42] K. Chen, H. C. Wann, J. Dunster, P. K. Ko, and C. Hu, "MOSFET carrier mobility model based on gate oxide thickness, threshold voltage, and gate voltages", *Solid State Electron.*, vol. 39, pp. 1515–1518, 1996.
- [43] D.B.M. Klaassen, "A Unified Mobility Model for Device Simulation-1. Model Equations and Concentration Dependence", *Solid-State Electronics* Vol. 15, No. 7, pp. 953-959, 1992.
- [44] S. Villa, L. Lacaita, L. M. Perron, and R. Bez, "A physically-based model of the effective mobility in heavily-doped n-MOSFETs", *IEEE Trans. Electron Devices*, vol. 45, pp. 110–115, Jan. 1998.
- [45] N. Rodriguez, J.B. Rolden and F. Gamiz, "An electron mobility model for ultra-thin gate-oxide MOSFETs including the effect of remote scattering mechanism", *Semicond. Sci. Technol.* 22, pp. 348-353, 2007.
- [46] Muhibul Haque Bhuyan, and Quazi D. M. Khosru, "Inversion Layer Effective Mobility Model for Pocket Implanted Nano Scale n-MOSFET", *International Journal of Electrical and Electronics Engineering* 5:1 2011.
- [47] A. A. Kastalsky and M. S. Shur, "Conductance of small semiconductor devices", *Solid-State Commun.*, vol. 39, no. 6, p. 715, 1981.
- [48] K. Lee and M. S. Shur, "Impedance of thin semiconductor films", *J. Appl. Phys.*, vol. 54, no. 7, pp. 4028-4034, July 1983.
- [49] M. Dyakonov and M. S. Shur, "Ballistic transport in high mobility semiconductor", *The Physics of Semiconductors*, M. Scheffler and R. Zimmermann, Eds. Singapore: World Scientific, pp. 145-148, 1996.
- [50] J. B. Roldán, F. Gámiz, P. Cartujo-Cassinello, P. Cartujo, J. E. Carceller, and A. Roldan, "Strained-Si on Si<sub>1-x</sub>Ge<sub>x</sub> MOSFET Mobility Model". *IEEE Tran Electron Devices*, VOL. 50, NO. 5, pp. 1408-1411, MAY 2003.
- [51] Zhao Yang, Zhang Dawei and Tain Lilin, "An improved Strained-Si on Si<sub>1-x</sub>Ge<sub>x</sub> MOSFET Mobility Model", *IEEE*, Vol. 2, pp. 1216-1219, 2004.

Received: April, 2011