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Calculation of Newtonian Component of Mercury's Perihelion Advance by Euler's Algorithm

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Abstract

A new numerical evaluation of the Newtonian component of Mercury's perihelion advance over more than two centuries starting from about the year 2000 is made using the Euler's algorithm. Results are given for about the last 30 years of this interval. Comparison with previous results shows that the advance is at least 2 arc sec/cy more than a previous computation by the author.

Keywords: Celestial Mechanics; Numerical Solution of ODE; Mercury

1. Introduction

Recently [1] I had shown that the numerical computation of the Newtonian component of Mercury's perihelion advance using ODE113 of MATLAB yields a rate close to 533 arc sec/cy. One must remember that MATLAB uses double precision computing accuracy in all its inbuilt ordinary differential equation solvers. In order to estimate if there is any loss of accuracy due to this limitation we are making a new numerical computation of this perihelion advance using the Euler's algorithm which is one of the most basic ordinary differential equation solvers. It is well known that this method is extremely inefficient as far as the computational time is concerned but the accuracy of the computed perihelion and its advance (here the Newtonian component only) by this method will be higher than other methods in a double precision computing system. This is precisely because the algorithm uses corrections which are almost linearly proportional to the time step and hence all attractive planetary forces which are of the order of 10^{-5} times that of solar gravitational pull are considered on an equal footing with that of the Sun. In other words the planetary and solar contributions are considered in their actual proportions in each time integration step. However the same cannot be said of the more sophisticated integration methods as they are supposed to use larger time step to quickly and accurately arrive at the desired result. In doing so these methods neglect the small contribution like those of the planets compared to the large attractive force of the Sun in a double precision computing system. This will lead to erroneous estimates (usually less than the actual value) of perihelion advance since the actual advance is a result of only the contribution of the planetary terms in the equation of motion of Mercury. One cannot guarantee that the Euler's algorithm is completely free of this defect however at small time steps provided that the contributions of planets do not become smaller than the accuracy limit of the computing system these contributions will be more or less correctly estimated in the equations of motion. However to arrive at a correct result one needs to use very small time steps and as our experience shows it may even be a fraction of a second. This decrease in time step was suggested by one of the referees (Prof. Anatoli Andrei Vankov) of my previous article [1] where the time step on the average was near to 0.5 of a day. In this paper we use the Richardson extrapolation technique [2] which is applied to the results of the computation of perihelion advance using different time steps (and not to the integration process itself as for example is done in the Bulirsch Stoer algorithm) to arrive at a more accurate result.

2. Equations of motion, the Euler's algorithm and the Richardson extrapolation technique

We repeat the equation of motion in the heliocentric reference frame as given by us earlier following Arminjon [3] (see Eq. no. (1) of reference [1])

$$\ddot{\vec{r}}_i = -G(m_N + m_i)\frac{\vec{r}_i}{R_i^3} + \sum_{\substack{j=1 \\ j\neq i}}^{N-1} G m_j \left(\frac{\vec{r}_j - \vec{r}_i}{\Delta_{ij}^3} - \frac{\vec{r}_j}{R_j^3} \right), \qquad (i = 1.......N-1) \tag{1}$$

where the same notations have been used. The solution at time t in terms of a small time step $(t - t_0)$ for the $N = 9$ bodies including the Sun and excluding the planet Pluto is

$$\vec{r}_i(t) = \vec{r}_i(t_0) + \left.\frac{d\vec{r}_i}{dt}\right|_{t_0} (t - t_0) + \frac{1}{2!}\left.\frac{d^2\vec{r}_i}{dt^2}\right|_{t_0} (t - t_0)^2 + ... \tag{2a}$$

$$\frac{d\vec{r}_i}{dt} = \left.\frac{d\vec{r}_i}{dt}\right|_{t_0} + \left.\frac{d^2\vec{r}_i}{dt^2}\right|_{t_0} (t - t_0) + ... \tag{2b}$$

In the Euler algorithm these formulas (2) are iteratively applied starting from any initial time to the final time in steps of $\Delta t = (t - t_0)$. The reason for neglecting Pluto is its very small mass compared to the other planets. The initial position coordinates and the velocities are assumed to be given and here we have taken these from the same reference quoted in our earlier paper [1]. The masses used are the same ones used previously. The Euler algorithm as we have noted above takes a long computation time to give acceptable results. In our system it took almost 38 days for computing the final coordinates which is 220 years in advance from the starting ephemeris date JJ=2451600.5 for a time step of 10^{-5} day.

The computations we have made here are using time steps of $1/5000$, $1/10000$, $1/50000$, $1/100000$ of a day successively and since these had not reached a steady value of perihelion precession rate we have used the Richardson extrapolation technique to obtain values of the rate when the time step is theoretically zero. Of course such extrapolation will give erroneous result if the number of iterations is not large enough still it gives an indication where the computations are heading towards. The Richardson extrapolation method is described in the book by Beutler that is reference [2] (see p. 275) to obtain integration algorithms of ordinary differential equations. However it can be used to extrapolate any quantity which is obtained in successive iterations in terms of a smallness parameter. We now describe the application to the perihelion advance of Mercury. For example we have successive time steps of integrations Δt_1, Δt_2, Δt_3 and Δt_4 yielding perihelion precession rates x_1, x_2, x_3 and x_4 then we can write in terms of the constants c_1, c_2, c_3 and c_4 the following

$$\begin{aligned}
x_1 &= c_1 + c_2\Delta t_1 + c_3\Delta t_1^{\,2} + c_4\Delta t_1^{\,3} \\
x_2 &= c_1 + c_2\Delta t_2 + c_3\Delta t_2^{\,2} + c_4\Delta t_2^{\,3} \\
x_3 &= c_1 + c_2\Delta t_3 + c_3\Delta t_3^{\,2} + c_4\Delta t_3^{\,3} \\
x_4 &= c_1 + c_2\Delta t_4 + c_3\Delta t_4^{\,2} + c_4\Delta t_4^{\,3}
\end{aligned} \qquad \ldots. \quad (3)$$

The four quantities c_1 to c_4 are determined by solving these equations simultaneously. If now the time step theoretically tends to zero the perihelion advance rate will tend to c_1.

3. Results of our computation

The time steps and the respective computed average perihelion advance per century is shown in table I for a period of about 30 years ending at the final date of computation. This final date is as already stated above to be 220 years in the future from the starting date of computation which is JJ=2451600.5.

TABLE – I

Time steps in multiples of a day	Perihelion advance in arc-sec/cy
2×10^{-4}	563
1×10^{-4}	598
2×10^{-5}	538
1×10^{-5}	535.5

These averages in column 2 are nothing but were obtained by plotting the advance as a MATLAB figure and subsequently using the data statistics calculator to determine the mean (the value of which is being presented here in this column). The actual advance from which the third row value of 538 arc-sec/cy is obtained is shown in Figure 1. This figure which is similar to Fig. 1 of reference [1] being the same quantity plotted for similar time periods but with different methods of computation show significant differences.

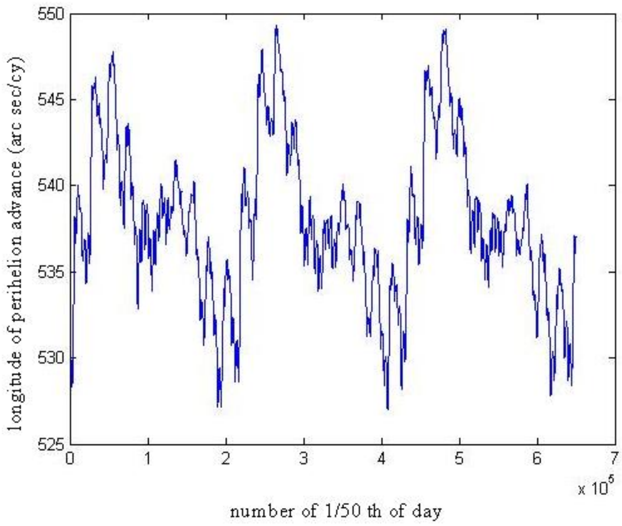

Figure 1. The advance in the longitude of perihelion of Mercury as obtained from a numerical solution of coupled system of N (actually N-1 of the type of Eq. no. (1)) ordinary differential equations. The initial time is JJ=2451600.5 and the total period of computation is 220 years. The plot is that for a period of about the last 30 years.

Finally by solving the simultaneous equations (3) we get an extrapolated rate of perihelion advance which is more than 535 arc-sec/cy. Thus in comparison to my earlier work I have shown that a better estimate of perihelion precession is at least 2 arc-sec/cy more than what I obtained previously.

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