Flow over a Streaming Stretching Surface with Slip Boundary Conditions and Suction/Injection Effects

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Abstract

This study is concerned with the 2-D steady laminar boundary layer flow over a stretching surface with slip boundary conditions near the wall embedded in a suction/ injection .The plate is maintained free stream velocity in the form $U_w(x) = c_1x + c_2x^2$. To find the similar solutions we introduce the modified stream functions. Appropriate similar transformations are used to reduce the arising PDE's into ODE's and solved through the use of Maple-13 [1] . Expressions for stream function and velocity component are obtained .

Nomenclatures

x	Dimensional coordinate along and normal to the plate
y	Dimensional coordinate along and normal to the plate
U_w	Surface velocity along the wall
η	Similarity parameter
f	Stream function
g	Stream function
f'	Velocity profile
g'	Velocity profile

f''	Missing slope
g''	Missing slope
N_1	The slip co-efficient having dimension of length
a	Velocity slip parameter
h	Suction/injection
v_0	The velocity of suction / injection depending on > 0 or < 0 respectively

1. Governing Equations and Mathematical analysis [2]

Let us consider the steady ,incompressible, viscous, laminar flow over a stretching plate with connective boundary conditions as shown in figure .The equal and opposite forces are acted along the x-axis .So that the surface is stretched by keeping the origin fixed .That is $U_w(x) = c_1x + c_2x^2$, c_1 and c_2 are arbitrary constants .No significant gradient of pressure in the x-direction .Surface slip flow condition is used because of rarefied flow .The simplified equation of motion based on above assumptions are known as the boundary layer equations are given as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots\dots\dots(1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + U_w \frac{dU}{dx} \quad \dots\dots\dots(2)$$

Subject to the boundary conditions on the problem are given by:

$$u = U_w + \nu N_1 \frac{\partial u}{\partial y}, \quad v = \frac{1}{x} U_w v_0 \quad \text{at } y = 0 \quad \dots\dots\dots(3)$$

$$u \rightarrow U_w(x), \quad \text{at } y \rightarrow \infty$$

1.1 Scaling analysis:

$$U_w(x) = c_1x + c_2x^2 \quad \dots\dots\dots(4)$$

where $c_1 \wedge c_2$ are constants.

To find a solution of equations (1) and (2), we introduce the modified stream functions $f(\eta)$ and $g(\eta)$, which for the present problem are given by

$$\eta = \sqrt{\frac{c_1}{\nu}} y$$

$$u = c_1 x f'(\eta) + c_2 x^2 g'(\eta) \dots \dots \dots (5)$$

$$v = -\sqrt{c_1 \nu} f(\eta) - \frac{2c_2 x}{\sqrt{c_1/\nu}} g(\eta)$$

Where a prime denotes differentiation with respect to the similarity variable η .

Using relations (5) we find that Equations (1) and (2) can be transformed into the corresponding ordinary differential equations.

1.2 Mathematical formulation:

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta} \frac{\partial \eta}{\partial y}$$

$$= (c_1 x f''(\eta) + c_2 x^2 g''(\eta)) \sqrt{\frac{c_1}{\nu}}$$

$$\frac{\partial v}{\partial y} = -\sqrt{c_1 \nu} f'(\eta) \sqrt{\frac{c_1}{\nu}} - \frac{2c_2 x}{\sqrt{c_1/\nu}} g'(\eta) \sqrt{\frac{c_1}{\nu}}$$

$$= (-c_1 f'(\eta) - 2c_2 x g'(\eta))$$

$$\frac{\partial^2 u}{\partial y^2} = (c_1 x f'''(\eta) + c_2 x^2 g'''(\eta)) \frac{c_1}{\nu}$$

$$\frac{\partial u}{\partial x} = c_1 f'(\eta) + c_2 x f''(\eta).0 + c_2 (2x) g'(\eta) + c_2 x^2 g''(\eta).0$$

$$= c_1 f'(\eta) + 2c_2 x g'(\eta)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = c_1 f'(\eta) + 2c_2 x g'(\eta) - c_1 f'(\eta) - 2c_2 x g'(\eta) = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = v \frac{\partial^2 u}{\partial x^2} + U_w \frac{dU_w}{dx}$$

$$\begin{aligned} & \Rightarrow (c_1 x f'(\eta) + c_2 x^2 g'(\eta)) (c_1 f'(\eta) + 2c_2 x g'(\eta)) \left(-\sqrt{c_1 v} f - \frac{2c_2 x}{\sqrt{\frac{c_1}{v}}} g \right) \\ & + (c_1 x f''(\eta) + c_2 x^2 g''(\eta)) \sqrt{\frac{c_1}{\eta}} = (c_1 x f'''(\eta) + c_2 x^2 g'''(\eta)) \frac{c_1}{\eta} + U_w \frac{dU_w}{dx} \\ & \Rightarrow c_1 x f'^2(\eta) + 3f'^{(\eta)} g'(\eta) + 2c_2 x^3 g'^2(\eta) - c_1^2 x f''^{(\eta)} f(\eta) - 2c_1 c_2 x^2 f''^{(\eta)} g(\eta) - \\ & 2c_2^2 x^3 g''(\eta) g(\eta) - c_1 c_2 x^2 g''(\eta) f(\eta) = c_1^2 x f''' + c_1 c_2 x^2 g''' + U_w \frac{dU_w}{dx} \\ & \Rightarrow c_1^2 x (f'^2 - f f'' - f''') + c_1 c_2 x^2 (3f' g' - 2f'' g - g''' - g'' f) + 2c_2^2 x^3 (g'^2 - g'' g) = \\ & (c_1 x + c_2 x^2)(c_1 + 2c_2 x) \\ & \Rightarrow c_1^2 x (f'^2 - f f'' - f''') + c_1 c_2 x^2 (3f' g' - 2f'' g - g''' - g'' f) + 2c_2^2 x^3 (g'^2 - g'' g) = \\ & (c_1^2 x + 3c_1 c_2 x^2 + 2c_2^2 x^3) \end{aligned}$$

Equating both sides,

$$f''' + f'' - f'^2 + 1 = 0 \quad \dots\dots\dots(6)$$

$$g''' + f g'' - 3f' g' + 2f'' g + 3 = 0 \quad \dots\dots\dots(7)$$

Subject to the boundary conditions:

$$f'(0) = 1 + a f'' \quad , \quad f(0) = h_1$$

$$g'(0) = 1 + a g'' \quad , \quad g(0) = h_1 / 2$$

$$f'(\infty) = 0 \quad , \quad g'(\infty) = 0$$

Where $a = N_1 \sqrt{c_1 v} = \text{velocity slip parameter}$

$h = v_0 \sqrt{\frac{c_1}{\eta}} = \text{Suction (h>0) / injection (h<0) parameter.}$

2. Results and discussion

The solution of the transformed governing equation and boundary conditions (6-8) are accomplished through the use of Maple-13[12] software over a range of velocity slip parameter $a = 0.0, 1.0, 2.0$; suction/injection number $h = -0.5, -1.0, 0, 0.5, 1.0$. The velocity profile f' and g' have been shown in figure (1-6) for different values of the governing parameters.

The effects of suction / injection parameter h on stream function profile $f(\eta)$ which displayed in figure-1. It is observed that the presence of h causes higher induction to the fluid which enhanced the velocity.

Figure-1: $f(\eta)$ for different values of h .

Figure-2 shows the variation in the stream function profile of $g(\eta)$ with an increase in h . From this figure we see that the $g(\eta)$ increase with the increasing values of suction/injection parameter.

Figure-2: $g(\eta)$ for different values of h .

Figure-3 illustrates the effects of suction/injection parameter h on velocity profile of the flow $f'(\eta)$. It is observed that the profile decreases with increasing h .

Figure-3: $f'(\eta)$ for different values of h .

Figure-4 illustrates that presence of h causes a huge restriction to the fluid velocity profile of the flow $g'(\eta)$ which induces less flow motion in momentum boundary layer.

Figure-4: $g'(\eta)$ for different values of h .

The stream function profiles $f(\eta)$ for the various values of the velocity slip parameter a presented in figure-5 shows that $f(\eta)$ decreases as a function η for $h=0.5$.

Figure-5: $f(\eta)$ for different values of a .

The stream function profiles of the flow $g(\eta)$ for the various values of the velocity slip parameter a presented in figure-6 shows that $g(\eta)$ increases with η for $h=0.5$. It can be seen that the flow profiles increase with increasing a .

Figure-6: $g(\eta)$ for different values of a .

In figure-7 shows the effect of velocity slip parameter a on the velocity profiles of the flow $f'(\eta)$ for $h=0.5$. The flow becomes more pronounced near the wall and becomes constant with a larger distance.

Figure-7: $f'(\eta)$ for different values of a .

Figure-8 displays that for an velocity slip parameter the velocity profiles of the flow $g'(\eta)$ increases as a increases.

Figure-8: $g'(\eta)$ for different values of a .

Conclusions

The equation for a laminar boundary layer can be solved nondimensionally in the presence of slip flow conditions.

The similarity solution indicate that:

- (1) Stream function increases as suction / injection parameter increases.
- (2) Velocity profiles decreases as suction / injection parameter increases.

(3) If velocity slip parameter increases f' decreases but g' increases.

(4) If velocity slip parameter increases f' increases but g' decreases.

References

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[2] A. Raptis, H.S. Takhar, Rarefied gas flow near an asymmetric plane stagnation point, Astrophysics and Space Science, Vol. 136, 187-190, 1987.

Received: January 7, 2014