Electromagneto-Mechanical Behavior of Giant Magnetostrictive/Piezoelectric Laminates in Three-Point Bending under Electric Fields

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Abstract

This work deals with the nonlinear electromagneto-mechanical behavior of giant magnetostrictive/piezoelectric laminates in three-point bending under electric fields both numerically and experimentally. The laminates are fabricated using thin Terfenol-D and PZT layers. Three-point bending test was performed on the Terfenol-D/PZT laminates under electric fields, and the deflection and induced magnetic field were measured. Three dimensional finite element analysis was also employed, and the electromagneto-mechanical fields in the laminates were predicted by introducing a second-order magnetoelastic constant of Terfenol-D. Results produced by the model were then compared with experimental values.

Keywords: Magnetostrictive/Piezoelectric Mechanics, Finite Element Method, Material Testing, Electronic Laminates, Detection and Response Characteristics, Smart Materials and Structures

1 Introduction

Magnetostrictive and piezoelectric materials play a significant role as active electronic components in many areas of science and technology, such as smart structures and devices [2]. Among magnetostrictive materials, Terbium Dysprosium Iron alloy (Terfenol-D) is the most attractive one because of its high saturation magnetostrain, good magnetomechanical coupling coefficient, and high Young’s modulus [1,7]. On the other hand, piezoelectric lead zirconate titanate (PZT) ceramics are widely used in sensors and actuators due to their
high output force, compact size, and high power density. The magnetoelectric (ME) effect is defined as an induced polarization in response to an applied magnetic field, or as an induced magnetization in response to an applied electric field. ME composites require magnetostrictive and piezoelectric materials, with a strong coupling between them, and many applications of these composites such as magnetic field sensors [9,13], gyrators and microwave devices [8] are currently under study. One limitation on the practical use of ME composite is the sharp bimaterial interfaces, and extensive work for predicting the intensified electromagneto-mechanical fields is necessary. Additionally, the tools available for modeling and design of magnetostrictive/piezoelectric laminates have not been sufficiently developed. In order to optimize the performance of the magnetostrictive/piezoelectric laminates, it is also important to understand the electromagneto-mechanical behavior of the laminates. Though some researches on the direct ME effect of the magnetostrictive/piezoelectric laminates are found in the literature, the investigations on the characterization of the converse ME effect are very few.

In this work, we study the electromagneto-mechanical behavior of Terfenol-D/PZT laminates in three-point bending under electric fields in a combined numerical and experimental approach. The deflection and induced magnetic field of the Terfenol-D/PZT laminates were measured by using a three-point bending test. The deflection and induced magnetic field in the laminates were also calculated using nonlinear finite element method. Comparison was then made between simulation and experiment. In addition, the effect of electric field on the internal stress for Terfenol-D/PZT laminates under concentrated load is examined.

2 Experimental Procedures

A two-layered Terfenol-D/PZT laminate is shown in Figure 1. Terfenol-D (Etrema Products, Inc., USA) of length \( l_m = 15 \text{ mm} \), width \( w_m = 5 \text{ mm} \) and thickness \( h_m = 1, 3 \text{ mm} \) and PZT C-91 (Fuji Ceramics, Co. Ltd., Japan) of \( l_p = 15 \text{ mm} \), \( w_p = 5 \text{ mm} \) and \( h_p = 1 \text{ mm} \) are used to make Terfenol-D/PZT laminates by epoxy bonding (EP-34B; Kyowa Electronic Instruments Co. Ltd., Japan). Subscripts \( m \) and \( p \) correspond to Terfenol-D and PZT layers, respectively. A rectangular Cartesian coordinate system \( O-xyz \) is employed, and the origin of the coordinate system is located at the center of the bottom left side of Terfenol-D layer. Easy axis of the magnetization of Terfenol-D layer is the \( z \)-direction, while the polarization of PZT layer is the \( x \)-direction. The PZT layer has silver paste electrodes on both sides.

Specimen was loaded by a concentrated load \( P = 1, 3 \text{ N} \), as shown in Figure 2, using a micro-testing machine. The span between the bottom two supports is \( S = 13 \text{ mm} \). A voltage \( V_0 \) was also applied. The applied electric field \( E_0 \) is
calculated by $E_0 = -V_0/h_p$. First, the deflection versus applied electric field curve of the Terfenol-D/PZT laminate was measured with a digital microscope camera at ×1000 magnification (see Figure 3(a)). Next, the average induced magnetic field over the total area on $z = 15$ mm plane of Terfenol-D layer was measured using a Tesla meter (see Figure 3(b)). The Hall probe was touched on the edge of Terfenol-D layer, and this set-up allowed a precision of induced magnetic field measurement of ± 0.01 mT.

![Figure 1. Illustration of Terfenol-D/PZT laminate configuration.](image1)

![Figure 2. Schematic of a central mechanical loaded Terfenol-D/PZT laminate.](image2)
under electric field.

Figure 3. Experimental setup for measuring (a) the deflection and (b) the induced magnetic field.
3 Analysis

3.1 Basic equations

Consider the rectangular Cartesian coordinate system O-x₁x₂x₃. The equilibrium equations are given by

\[ \sigma_{ji,j} = 0 \] (1)
\[ B_{i,i} = 0 \] (2)
\[ D_{i,i} = 0 \] (3)

where \( \sigma_{ij} \) is the stress tensor, \( B_{i} \) is the magnetic induction vector, \( D_{i} \) is the electric displacement vector, a comma followed by an index denotes partial differentiation with respect to the space coordinate \( x_i \), and the summation convention for repeated tensor indices is applied. The constitutive laws are given as follows:

\[ \varepsilon_{ij} = s_{ijkl}^H \sigma_{kl} + d'_{kij} H_k \] (4)
\[ B_{i} = d''_{ijkl} \sigma_{kl} + \mu_{ik} H_k \] (5)

for the magnetostrictive layer, and

\[ \varepsilon_{ij} = s_{ijkl}^E \sigma_{kl} + d_{kij} E_k \] (6)
\[ D_{i} = d_{ijkl} \sigma_{kl} + \varepsilon_{ik}^T E_k \] (7)

for the piezoelectric layer. Here, \( \varepsilon_{ij} \) is the strain tensor, \( H_{i} \) is the magnetic field intensity vector, \( E_{i} \) is the electric field intensity vector, \( s_{ijkl}^H, d''_{kij}, \mu_{ij} \) are the constant magnetic field elastic compliance, magnetoelastic constant and magnetic permittivity of magnetostrictive layer, and \( s_{ijkl}^E, d_{kij}, \varepsilon_{ij}^T \) are the constant electric field elastic compliance, direct piezoelectric constant and dielectric permittivity of piezoelectric layer. Valid symmetry conditions for the material constants are

\[ s_{ijkl}^H = s_{jikl}^H = s_{ijlk}^H = s_{klij}^H, \quad d''_{kij} = d''_{kji}, \quad \mu_{ij} = \mu_{ji} \] (8)
\[ s_{ijkl}^E = s_{jikl}^E = s_{ijlk}^E = s_{klij}^E, \quad d_{kij} = d_{kji}, \quad \varepsilon_{ij}^T = \varepsilon_{ji}^T \] (9)

The relation between the strain tensor and the displacement vector \( u_{i} \) is given by

\[ \varepsilon_{ij} = \frac{1}{2} (u_{j,i} + u_{i,j}) \] (10)

The magnetic and electric field intensities are written as

\[ H_{i} = \varphi_{,i} \] (11)
\[ E_{i} = -\phi_{,i} \] (12)

where \( \varphi \) and \( \phi \) are the magnetic and electric potentials, respectively.
3.2 Model

The three-dimensional finite element model is created in order to calculate the deflection, stress and induced magnetic field for the laminates. Figure 4 shows the finite element mesh and boundary conditions. Let the coordinate axes \( y = x_2 \) and \( z = x_3 \) be chosen such that they coincide with the interface plane of the laminate and the \( x = x_1 \) axis is perpendicular to this plane. Easy axis of the magnetization of Terfenol-D layer is the \( z \)-direction, while the polarization of PZT layer is the \( x \)-direction. The constitutive relations for Terfenol-D and PZT layers are given by Appendix A.

As we know, nonlinearity of magnetostriction versus magnetic field curves arises from the rotation of magnetic domains [12]. A magnetic domain switching gives rise to the changes of the magnetoelastic constants, and the constants \( d'_{15}, d'_{31} \) and \( d'_{33} \) for Terfenol-D layer in the \( z \)-direction magnetic field are

\[
\begin{align*}
  d'_{15} &= d_{15}^m \\
  d'_{31} &= d_{31}^m + m_{31} H_z \\
  d'_{33} &= d_{33}^m + m_{33} H_z
\end{align*}
\]

(13)

where \( d_{15}^m, d_{31}^m, d_{33}^m \) are the piezo-magnetic constants, and \( m_{31} \) and \( m_{33} \) are the second-order magnetoelastic constants. When the length of Terfenol-D is much longer than other two sizes (width and thickness) and a magnetic field is along the length direction (easy axis), the longitudinal (33) magnetostrictive deformation mode is dominant. So it is assumed that only the constant \( d'_{33} \) varies with magnetic field \( H_z \), and the constant \( m_{31} \) equals to zero. The constant \( m_{33} \) can predict well the nonlinearity, without complex computation and more parameters [14].

More realistic material models are much more complicated, see e.g. [6], but the present formulation is able to capture the nonlinear effect, which is a remarkable feature of the presented model. Coupled-field solid elements in ANSYS were used in the analysis. A mechanical load \( P \) was produced by the application of the uniformly distributed load per unit width at \( z = l_m/2 \) \((x = -h_m, -w_m/2 \leq y \leq w_m/2)\). The electric potential on the \( x = h_p \) plane of the PZT layer equals the applied voltage, \( \phi = V_0 \), whereas the \( x = 0 \) plane was grounded, so that \( \phi = 0 \). The average induced magnetic field \( B_{in} \) in the \( z \)-direction at the side surface (at \( z = l_m \) plane) was calculated as

\[
B_{in} = \frac{1}{A} \int_A B_z(x, y, l_m) dA
\]

(14)

where the integration is over the surface area, \( A = w_m h_m \), of the Terfenol-D layer. The model was meshed using an eight-node element. Silver paste electrodes on both sides of the PZT layer were not incorporated into the model. This is because the thickness, 5 \( \mu \)m, of the electrode layer is much smaller than
the thickness, 1 mm, of the PZT layer. In total, 12000, 21600 elements and 12810, 24339 nodes were used for $h_m = 1, 3$ mm, respectively. It should be noted that before carrying

![Figure 4. Finite element mesh of the Terfenol-D/PZT laminate: (a) top view and (b) side view.](image)

out simulations, a mesh sensitivity study was performed to ensure that the mesh was fine enough. The finite element computations were provided by using the program with routines developed by our previous work [10]. Material properties of Terfenol-D [3,5] and C-91 [4] are listed in Tables 1 and 2, and
the constants $m_{33}$ of Terfenol-D layer with $h_m = 1, 3$ mm are $5.0 \times 10^{-12}$, $3.3 \times 10^{-12}$ m$^2$/A$^2$, respectively [11].
4 Results and Discussion

Figure 5 shows the load-point deflection $w_{\text{max}}$ with the applied electric field $E_0$ for Terfenol-D/PZT laminates under concentrated load $P = 1$ N. The lines and plots denote the results of nonlinear finite element analysis (FEA) and test, respectively. As the magnitude of the positive (negative) electric field increases, load-point deflection increases (decreases). Under high electric fields, the load-point deflection also increases as the thickness of Terfenol-D layer decreases. The results show that the applied electric field is capable of controlling the deflection. Similar phenomena for the load-point deflection for $P = 3$ N is also observed in Figure 6. The comparison between the FEA and test is reasonable. Figure 7 shows the induced magnetic field $B$.
Figure 6. Load-point deflection versus applied electric field for Terfenol-D/PZT laminates ($P = 3$ N).

$B_{in}$ versus applied electric field $E_0$ for the Terfenol-D/PZT laminates under $P = 1$ N, obtained from the FEA and test. As the electric field increases, induced magnetic field increases. The magnitude of induced magnetic field also increases as the thickness of the Terfenol-D layer decreases. Similar phenomena for the induced magnetic field for $P = 3$ N is also observed in Figure 8. It can be seen that the trend is sufficiently similar between analysis and test.

Figure 7. Induced magnetic field versus applied electric field for Terfenol-D/PZT laminates ($P = 1$ N).
The variations of normal stress $\sigma_{zz}$ along the thickness direction are calculated at the center ($y = 0$ mm and $z = 7.5$ mm) of the Terfenol-D/PZT laminate under $P = 3$ N and $E_0 = 0, \pm 0.1, \pm 0.2$ MV/m for $h_m = 1$ mm, and the results are shown in Figure 9. The magnitudes of induced magnetic field of the laminate under $P = 3$ N for $h_m = 1$ mm are about 0.51 mT for $-0.2$ MV/m, 0.27 mT for $-0.1$ MV/m, 0.016 mT for 0 MV/m, 0.27 mT for 0.1 MV/m and 0.59 mT for 0.2 MV/m, respectively. There are some stress gaps at the interface between Terfenol-D and PZT layers as is expected. When the electric field magnitude increases, the stress gap increases. The induced magnetic field magnitude also increases as the electric field magnitude increases. Figure 10 illustrates the normal stress distribution $\sigma_{zz}$ for $h_m = 3$ mm.
Figure 9. Normal stress distribution along the thickness direction for $h_m = 1\ mm$

at $y = 0\ mm$ and $z = 7.5\ mm$ for Terfenol-D/PZT laminate ($P = 3\ N$).

Figure 10. Normal stress distribution along the thickness direction for $h_m = 3\ mm$

at $y = 0\ mm$ and $z = 7.5\ mm$ for Terfenol-D/PZT laminate ($P = 3\ N$).

corresponding to Figure 9. The induced magnetic field magnitudes of the laminate under $P = 3\ N$ for $h_m = 3\ mm$ are about $0.40\ mT$ for $-0.2\ MV/m$, $0.20\ mT$ for $-0.1\ MV/m$, $0.0064\ mT$ for $0\ MV/m$, $0.23\ mT$ for $0.1\ MV/m$ and $0.48\ mT$ for $0.2\ MV/m$, respectively. As the electric field magnitude increases, the stress gap and induced magnetic field magnitude increase. The results show that the induced magnetic field is capable of self-monitoring of internal stresses. Figure 11 shows the variations of shear stress $\sigma_{xz}$ along the length direction at $x = y = 0\ mm$ of the Terfenol-D/PZT laminate under $P = 3\ N$ and $E_0 = 0$, $\pm 0.1$, $\pm 0.2\ MV/m$ for $h_m = 1\ mm$. Owing to symmetry, only a half of the specimen was presented. Under the electric fields of $\pm 0.1$ and $\pm 0.2\ MV/m$, the shear stress exhibits a peak at $z \approx 14.5\ mm$. Figure 12
Electromagneto-mechanical behavior

Figure 11. Shear stress distribution along the length direction for \( h_m = 1 \) mm at \( x = 0 \) mm and \( y = 0 \) mm for Terfenol-D/PZT laminate (\( P = 3 \) N).

Figure 12. Shear stress distribution along the length direction for \( h_m = 3 \) mm at \( x = 0 \) mm and \( y = 0 \) mm for Terfenol-D/PZT laminate (\( P = 3 \) N).

shows the shear stress distribution \( \sigma_{xz} \) for \( h_m = 3 \) mm corresponding to Figure 11. High shear stress is noted for large Terfenol-D layer thickness. This situation may be applicable to the other magnetostrictive/piezoelectric laminates with different size and shape. The results for the evaluation of stresses may help magnetostrictive/piezoelectric laminates designers to estimate the fracture risk and to optimize in-service loading conditions.
5 Conclusion

A numerical and experimental investigation of the magnetostrictive/piezoelectric laminates in three-point bending under electric fields was conducted. The deflection and induced magnetic field are predicted using finite element simulations, and comparison with the measured data shows that current predictions are reasonable. It was found that the induced magnetic field depends on the concentrated load and applied electric field, and correlates with the internal stresses. This study may be useful in designing advanced magnetostrictive/piezoelectric laminates with monitoring capabilities.

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References


Appendix A

For Terfenol-D layer, the constitutive relations can be written in the following form:

\[
\begin{align*}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\varepsilon_{yx} \\
\varepsilon_{xy}
\end{bmatrix}
&= 
\begin{bmatrix}
S_{11}^H & S_{12}^H & S_{13}^H & 0 & 0 & 0 \\
S_{12}^H & S_{11}^H & S_{13}^H & 0 & 0 & 0 \\
S_{13}^H & S_{13}^H & S_{33}^H & 0 & 0 & 0 \\
0 & 0 & 0 & S_{44}^H/2 & 0 & 0 \\
0 & 0 & 0 & 0 & S_{44}^H/2 & 0 \\
0 & 0 & 0 & 0 & 0 & S_{66}^H/2
\end{bmatrix}
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yx} \\
\sigma_{xy}
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & d_{31}' \\
0 & 0 & 0 & 0 & 0 & d_{31}' \\
0 & 0 & 0 & 0 & 0 & d_{33}' \\
0 & 0 & 0 & d'_{15}/2 & 0 & 0 \\
d'_{15}/2 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
H_x \\
H_y \\
H_z
\end{bmatrix}
\end{align*}
\]  
(A.1)
\[
\begin{bmatrix}
B_x \\
B_y \\
B_z
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 & d'_{15} & 0 \\
0 & 0 & 0 & d'_{15} & 0 & 0 \\
d''_{31} & d''_{31} & d''_{33} & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yz} \\
\sigma_{zx} \\
\sigma_{xy}
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
\mu_{11} & 0 & 0 \\
0 & \mu_{11} & 0 \\
0 & 0 & \mu_{33}
\end{bmatrix}
\begin{bmatrix}
H_x \\
H_y \\
H_z
\end{bmatrix}
\]

(A.2)

where

\[
s_{11}^H = s_{1111}^H = s_{2222}^H, s_{12}^H = s_{1122}^H, s_{13}^H = s_{1133}^H = s_{2233}^H, s_{33}^H = s_{3333}^H
\]

\[
s_{44}^H = 4s_{2323}^H = 4s_{3131}^H, s_{66}^H = 4s_{1212}^H = 2(s_{11}^H - s_{12}^H)
\]

(A.3)

\[
d'_{15} = 2d'_{131}, d''_{131} = d''_{322}, d''_{33} = d''_{333}
\]

(A.4)

The constitutive relations for PZT (hexagonal crystal of class 6mm) layer are

\[
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\varepsilon_{yz} \\
\varepsilon_{zx} \\
\varepsilon_{xy}
\end{bmatrix} =
\begin{bmatrix}
s_{33}^E & s_{13}^E & s_{13}^E & 0 & 0 & 0 \\
0 & 0 & 0 & s_{66}^E/2 & 0 & 0 \\
0 & 0 & 0 & 0 & s_{44}^E/2 & 0 \\
0 & 0 & 0 & 0 & 0 & s_{44}^E/2
\end{bmatrix}
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yz} \\
\sigma_{zx} \\
\sigma_{xy}
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
d_{33} & 0 & 0 \\
d_{31} & 0 & 0 \\
d_{31} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & d_{15}/2 \\
0 & d_{15}/2 & 0
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix}
\]

(A.5)

\[
\begin{bmatrix}
D_x \\
D_y \\
D_z
\end{bmatrix} =
\begin{bmatrix}
d_{33} & d_{31} & d_{31} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & d_{15} \\
0 & 0 & 0 & 0 & d_{15} & 0
\end{bmatrix}
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yz} \\
\sigma_{zx} \\
\sigma_{xy}
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
\epsilon_{T33}^T & 0 & 0 \\
0 & \epsilon_{11}^T & 0 \\
0 & 0 & \epsilon_{11}^T
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix}
\]

(A.6)
where

\[
\begin{align*}
    s_{11}^E &= s_{2222}^E = s_{3333}^E, \\
    s_{12}^E &= s_{2233}^E, \\
    s_{13}^E &= s_{1122}^E = s_{1133}^E, \\
    s_{33}^E &= s_{1111}^E, \\
    s_{11}^E &= s_{1212}^E = s_{1313}^E, \\
    s_{23}^E &= 2(s_{11}^E - s_{12}^E)
\end{align*}
\]  

\tag{A.7}

\[
\begin{align*}
    d_{15} &= 2d_{313} = 2d_{212}, \\
    d_{31} &= d_{122} = d_{133}, \\
    d_{33} &= d_{111}
\end{align*}
\]  

\tag{A.8}

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