

**The Correction Factor for Rate of Energy  
Generated in the Friction Clutches  
under Uniform Pressure Condition**

**Oday I. Abdullah**

Laser and System Technologies (AmP)  
Technical University Hamburg-Harburg, Germany  
oday.abdullah@tu-harburg.de

**Josef Schlattmann**

Laser and System Technologies (AmP)  
Technical University Hamburg-Harburg, Germany  
j.schlattmann@tu-harburg.de

**Abstract**

Most of failures in friction clutches occur due to excessive heat generated between contact surfaces during the slipping; for that reason the equation of rate of energy generated is considered the essential element in the design process for friction clutches to obtain temperature field and thermal stresses, and then to estimate the lifecycle. High temperatures produce high thermal stresses at contact area. When the clutch continues working under these conditions, this brings about several disadvantages such as: surface cracks and permanent distortions, and likely this might lead to failure before expected lifetime of the clutch. Therefore, if there is an error, however small might appear to be, in the equation of thermal load due to the assumption, the final results will be greatly affected. The accumulative errors during repeated engagements and the value of error produced by equation of thermal load will be unacceptable for design. This paper present the correction factor for the

thermal load equation derived from the equation of motion for two-inertia system when the pressure is assume uniform on the contact surfaces.

**Keywords:** Friction clutch slipping, Frictional heating, Temperature distribution

## 1 Introduction

The friction clutch is considered an essential component in the process of power transmission; therefore all designers want to obtain the best possible performance increasing lifecycle of the friction clutches. The heat generated during the sliding is one of the biggest obstacles designers ever faced. Apart of the reason, there are many variables that affect on this process such as pressure distribution, coefficient of friction, materials properties, and sliding velocity ...etc. For that reason, the equation of the frictional heat generated with assumption is considered the key to obtain acceptable results with low values of errors.

The equation of energy rate generated during the slipping derived from the equation of motion for two-inertia system have been extensively used in early studies and in rarely researched nowadays. These studies assumed when applied uniform pressure, that the effect of the heat flux variation (directly with radius) on temperature is small and can be neglected, because most clutches are designed with ratio of outer diameter to inner diameter not much greater than 1.3. Furthermore, these studies assume full contact between friction surfaces (between flywheel, clutch and pressure plate). Important is that the heat flow is one-dimensional and the maximum heat generated is assumed to occur at the start of the slipping .e.g. [1-6].

Many researchers investigated the heat generated phenomenon between contact surfaces in automotive clutches and brakes to predict the temperature distribution and especially the maximum temperature during the clutch engagement and braking to avoid the failure before estimated lifecycle. This process is very complex because of the following characteristics (pressure, coefficient of friction and sliding speed). The researchers used different numerical techniques (e.g. finite element method and finite difference) to compute the sliding surface temperature [7-17].The equation of rate of energy generated ( $q$ ) was used in these studies is the function of the coefficient of friction ( $\mu$ ), pressure between contact surfaces ( $p$ ) and sliding velocity ( $V_r$ ),

$$q = \mu p V_r \quad (1)$$

The heat generates when two bodies are in contact and are sliding relatively, and the high temperature causes thermoelastic distortion. Consequently, the contact

Pressure distribution changes. It has been found out that the system will be unstable when the sliding speed exceeds a certain critical value “critical speed”. This phenomenon was first identified and explained by Barber [18 & 19] and was called “frictionally excited thermoelastic instability” or TEI. A microscopic disturbance in the contact pressure can grow resulting in areas of high-pressure concentrations and subsequently creating areas of high heat generations or ‘hot spots’. The hot spots have been appeared in a number of mechanical systems such as mechanical seals, aircraft brakes, railways and automotive clutch and brake systems. These researchers make clear the importance to knowing the values of heat generated, to obtain accurate result of temperature distribution and then the values of deformations to find the new pressure distribution [20-27]. Besides, in these studies, the rate of energy generated ( $q$ ) was used.

It was found out from the results of a research adopted the equation of heat generated ( $q$ ) that the equation proved to be reliable because it provided good results with acceptable values of error. Therefore, the rate of energy generated ( $q$ ) is called in this paper “the reliable rate of energy generated ( $q_r$ )”.

The aim of this work is to present the correction factor for the equation of heat generated between contact surfaces for the friction clutch (takes the effect of variation of heat flux with radius in the consideration), which derived from the equation of motion for two-inertia system when the pressure is assume uniform to obtain more accurate results for the temperature distribution for dry friction clutches.

## 2 Mathematical Models

There are two basic methods designed of friction clutch, namely the uniform pressure and uniform rate of wear. In this paper, these methods are used to compare between the rate of energy generated derived from the equation of motion for two-inertia system ( $q_{I.S}$ ) during the slipping and the reliable of rate of energy generated ( $q_r$ ), and presents the correction factor  $C_f$  for ( $q_{I.S}$ ) relative to ( $q_r$ ) in case of uniform pressure.

$$q_{I.S} = \frac{T \omega_r}{A_t}; \quad 0 \leq t \leq t_s \quad (2)$$

$$q_r = \mu p V_r = \mu p \omega_r r; \quad 0 \leq t \leq t_s \quad (3)$$

Where  $T$ ,  $\omega_r$ ,  $A_t$ ,  $r$  and  $t_s$  are the transmission torque, sliding angular velocity, the total area of contact of friction clutch, disc radius and slipping time respectively.

### a Uniform Pressure

The assumption of a uniform pressure distribution at the interface between mating surfaces is valid for an unworn accurately manufactured clutch with rigid outer discs. The total frictional torque for multiple-disc is [28],

$$T = \frac{2}{3} n \mu \pi p (r_o^3 - r_i^3) \quad (4)$$

Where  $n$ ,  $r_i$  and  $r_o$  are the number of friction surfaces in clutch, clutch inner radius and clutch outer radius respectively.

Substitute eq. (4) into eq. (2) and rearranged yield:-

$$q_{I.S} = q_r \left( \frac{2(r_o^2 + r_i r_o + r_i^2)}{3r(r_i + r_o)} \right); \quad 0 \leq t \leq t_s \quad (5)$$

The modified heat generated at any instant per unit area is,

$$q_{I.S.M} = q_r = q_{I.S} C_f = \frac{T \omega_r C_f}{A_t}; \quad 0 \leq t \leq t_s \quad (6)$$

Where,  $C_f = \frac{3r(r_i + r_o)}{2(r_o^2 + r_i r_o + r_i^2)}$

It's clear from eq. (6) that the  $q_{I.S.M}$  increases linearly with disc radius ( $r$ ), and the values of the heat flux start from minimum value at inner radius to maximum value at outer radius. Then, it can shape the correction factor  $C_f$  at inner and outer radius,

$$C_{f r_i} = \frac{3(R^2 + R)}{2(R^2 + R + 1)} \quad (7)$$

$$C_{f r_o} = \frac{3(R + 1)}{2(R^2 + R + 1)} \quad (8)$$

Where  $R = \frac{r_i}{r_o}$

### b Uniform Wear

The wear rate is assumed to be proportional to the product of the pressure and velocity. The frictional torque, functioning as a maximum pressure for multiple-disc clutch is,

$$T = n p_{max} \mu \pi r_i (r_o^2 - r_i^2) \quad (9)$$

The pressure (p) as a function of disc radius (r):

$$p_{max} r_i = p r = c \quad (10)$$

Where, c is constant.

Substitute eq. (9) and eq. (10) into eq. (2) and rearrange yield the rate of energy generated,

$$q_{I.S.W} = \mu p \omega_r r = q_r; \quad 0 \leq t \leq t_s \quad (11)$$

It's clear that the rate of heat energy generated is a function of time only, and the equation of rate of energy generated during slipping derived from the equation of motion for two-inertia system ( $q_{I.S.W}$ ) and reliable equation ( $q_r$ ) are identical in case of uniform wear.

### 3 Finite Element Formulations

The transient heat conduction equation for an axisymmetric problem described in the cylindrical coordinate system is given as [29],

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r K \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left( K \frac{\partial T}{\partial z} \right) + Q \quad (12)$$

And the thermal load (heat flux) on the clutch side is,

$$q_{I.S.M} = \frac{f_c T \omega_r C_f}{A_t}; \quad 0 \leq t \leq t_s \quad (13)$$

Assuming the sliding angular velocity decreases linearly with time as,

$$\omega_s(t) = \omega_o \left(1 - \frac{t}{t_s}\right), \quad 0 \leq t \leq t_s \quad (14)$$

Then, the heat flux on the clutch sides at any time of slipping when applied uniform pressure is,

$$q_{I.S.M} = \frac{f_c T \omega_o C_f}{A_t} \left(1 - \frac{t}{t_s}\right); \quad 0 \leq t \leq t_s \quad (15)$$

Where  $f_c$  is the heat partition ratio which imposes division of heat entering the clutch, pressure plate and flywheel (assume the same material properties for the flywheel and pressure plate), and is given as follows [30].

$$f_c = \frac{\sqrt{K_c \rho_c c_c}}{\sqrt{K_c \rho_c c_c} + \sqrt{K_f \rho_f c_f}} = \frac{\sqrt{K_c \rho_c c_c}}{\sqrt{K_c \rho_c c_c} + \sqrt{K_p \rho_p c_p}} \quad (14)$$

Where  $k$  is the thermal conductivity,  $\rho$  is the density and  $c$  is the specific heat. All values and parameters, referred to the axial cushion, friction material, flywheel and pressure plate in the following considerations, will have bottom indexes  $c_u$ ,  $c$ ,  $f$  and  $p$  respectively.

Figs. 1 & 2 show the axisymmetric model of friction clutch with boundary conditions and finite element models with load types (with and without correction factor). The slipping time is 0.4 sec and angular sliding velocity  $\omega_r$  is assumed to be linearly decays and finally reaches zero at 0.4 sec. The heat transfer coefficient has been taken as 40.89 W/m<sup>2</sup> K [31] and is assumed to be constant over all exposed surfaces. The eight-noded thermal element (PLANE77) was used in this analysis. The element has one temperature degree of freedom at each node as the temperature is scalar. Since the gradient of temperature is localized near the contact interfaces at the small values of time, the mesh refinement is needed at this region. A mesh sensitivity study was done to choose the optimum mesh from computational accuracy point of view. In all computations for the friction clutch model, it has been assumed a homogeneous and isotropic material and all parameters and materials properties are listed in Table 1.

The normal operation of friction clutch makes repeated engagements and the maximum temperature during this operation is very important, because of the temperature will increases with increases the number of engagements due to the kinetic energy is absorbed during slippage. Temperature calculation has been made for repeated engagements made at regular intervals of time for the same energy dissipations. The time between engagements is taken 5 seconds.

## 4 Results and Discussions

In order to understand the error exists in the rate of energy generated  $q_{I.S}$  during the slipping for the clutches due to assumed the heat generated is constant with radius and assume  $q_{I.S}$  is function of time only, and it's affect on the results of temperature distributed on the friction surfaces during the slipping for single and repeated engagement, this analysis has been done using ANSYS 13 software.

Fig. 3 shows the variation of the heat flux on the clutch with clutch radius ( $r$ ) at the beginning of a slip ( $t_s=0$ ), it can be seen from this figure, the values of heat flux ( $q_{I.S}$ ) are constant with disc radius, and the values of heat flux ( $q_{I.S.M}$ ) increases linearly with disc radius and the maximum value of heat flux occurs at the outer radius.

Figs. 4 and 5 show the variation of temperature with time and disc radius for the  $q_{I.S}$  and  $q_{I.S.M}$  (1<sup>st</sup> engagement). From these figures, it can be noted, that the temperature values is approximately constant with radius (very small effect of heat convection on the temperatures near  $r_i$  and  $r_o$ ) at certain time when applied  $q_{I.S}$  and the temperature increases linearly with disc radius at any certain time when applied  $q_{I.S.M}$ . Also, it can be seen for both cases of load ( $q_{I.S}$  &  $q_{I.S.M}$ ), that the temperature starts from initial value ( $T_i$ ) at beginning of slipping ( $t_s=0$ ) and increases to maximum value ( $T_{max}$ ) approximately at half time of slipping ( $t_s=0.2$  s), and then it's gradually decreases from  $T_{max}$  to final temperature ( $T_f$ ) at end of slipping ( $t_s=0.4$ ). The ratio of decreases in deference temperature  $(T_{max}-T_i)/(T_f-T_i)$  is approximately 1.4 for all cases when the slipping time change from (0.2 to 0.4 s).

The distribution state of temperature for the friction clutch disc for both cases of load ( $q_{I.S.M}$  &  $q_{I.S}$ ) during the 10<sup>th</sup> engagement is as shown in fig. (6). from this figure, it can be seen that the maximum temperatures when applied  $q_{I.S}$  are less than the values of maximum temperature when applied  $q_{I.S.M}$  during this engagements, and for all engagements, this because of the maximum values of  $q_{I.S.M}$  is greater than the values of  $q_{I.S}$  at  $r_o$ , and this makes always the values of maximum temperatures when applied  $q_{I.S.M}$  is greater than the maximum values of temperature when applied  $q_{I.S}$  under the same boundary conditions.

Figs 7, 8 and 9 show the variation of temperature with time at three locations on the friction clutch surface ( $r_i$ ,  $r_m$  and  $r_o$ ) when applied  $q_{I.S}$  and  $q_{I.S.M}$ . During all engagements, it is observed that the values of temperatures when applied  $q_{I.S}$  are greater than the values of temperature when applied  $q_{I.S.M}$  at  $r_i$  and the temperature is almost equal for both cases at  $r_m$ . Also it can be seen, that the values of temperatures when applied  $q_{I.S}$  are less than the values of temperature when applied  $q_{I.S.M}$  at  $r_o$ . Table. (2) shows the difference in the maximum temperature for both load cases ( $q_{I.S.M}$  &  $q_{I.S}$ ) at different locations ( $r_i$ ,  $r_m$  and  $r_o$ ), it's clear that the difference increases

with number of engagements at all locations, this is because the values of error in equation  $q_{I,S}$  will accumulate with increase number of engagements.

## 5 Concluding Remarks

To conclude, the mathematical correction for equation of rate of heat generated  $q_{I,S}$  assuming the pressure is uniform between contact surfaces, is achieved. The error exists in the eq. (2) is due to the assumption that the heat flux is constant with disc radius ( $r$ ) at any time. The transient thermal analysis of friction clutch disc during single and repeated engagements is performed comparing the results of temperature distribution when applied  $q_{I,S}$  and  $q_{I,S,M}$  (heat flux before and after correction).

The result derived from eq. (2) indicates the values of temperature at  $r_m$ , but the results when used eq. (6) show that the values of temperature increase linearly with disc radius and the maximum temperature will occur at  $r_o$ . Besides, under the same boundary conditions, the maximum values of temperature when using eq. (6) are greater than those generated by eq. (2). The difference in the maximum temperature ( $T_{\max.(q_{I,S,M})} - T_{\max.(q_{I,S})}$ ) grow with number of engagements due to accumulative errors during the repeated engagements. The highest temperature occurs approximately at half slipping time for all engagements.

The outcomes obtained in this work show the effect of neglecting the variation of heat flux with radius in case of uniform pressure on the temperature distribution, maximum temperature. Last but not the least, the error existing in the temperature distribution will lead the designers to obtain inaccurate estimation for lifecycles of friction clutches.

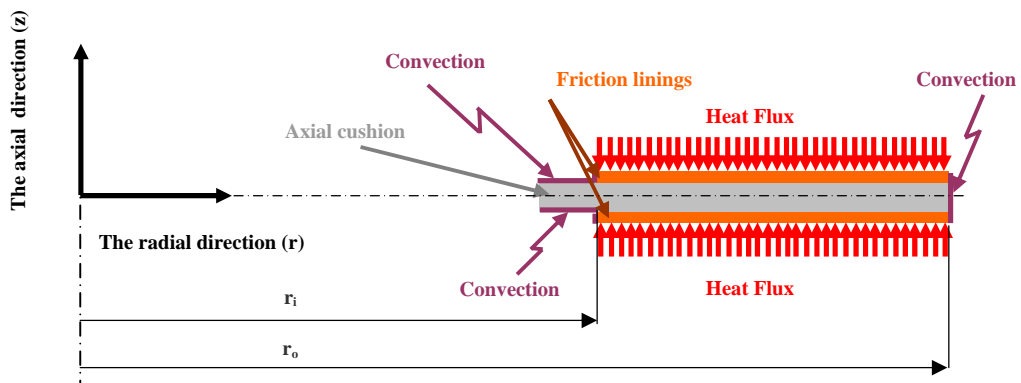
**Table 1** The model parameters and materials properties

Inner radius, $r_i$ [m]	0.085
Outer radius, $r_o$ [m]	0.135
Torque, $T$ [Nm]	580
Maximum pressure, $p_{\max}$ (MN/m <sup>2</sup> )	0.25
Coefficient of friction, $\mu$	0.3
Number of friction surfaces, $n$	2
Maximum angular slipping speed, $\omega_r$ (rad/sec)	220
Conductivity for friction material, $K_c$ (W/mK)	0.75
Conductivity for pressure plate & flywheel, $K_p$ & $K_f$ (W/mK)	56
Density for friction material, $\rho_c$ (kg/m <sup>3</sup> )	1300
Density for pressure plate & flywheel, $\rho_p$ & $\rho_f$ (kg/m <sup>3</sup> )	7200
Specific heat for friction material, $c_c$ (J/kgK)	1400
Specific heat for pressure plate & flywheel, $c_p$ & $c_f$ (J/kgK)	450
Thickness of friction material, $t_c$ (m)	0.002
Thickness of cushion, $t$ (m)	0.001
Time step, $\Delta t$ (s)	0.002



**Table 2** The difference in the maximum temperature between  $q_{I.S.M}$  and  $q_{I.S}$  during repeated engagements

No. of engagements	$T_{\max.(q I.S.M)} - T_{\max.(q I.S)}$		
	$r_i$	$r_m$	$r_o$
1	-8.41	-0.6	7.222
2	-9.36	-0.69	8.121
3	-10	-0.75	8.762
4	-10.6	-0.8	9.296
5	-11.1	-0.83	9.749
6	-11.5	-0.85	10.14
7	-11.9	-0.87	10.47
8	-12.3	-0.88	10.76
9	-12.5	-0.88	11.01
10	-12.8	-0.88	11.22



**Fig. 1** boundary conditions for friction clutch

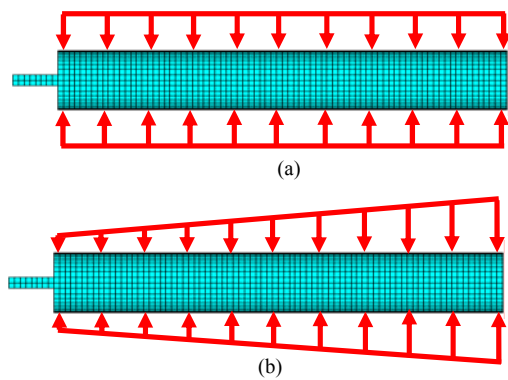


Fig. 2 Finite element models of friction clutch with load types (a)  $q_{I,S}$  (b)  $q_{I,S,M}$ .

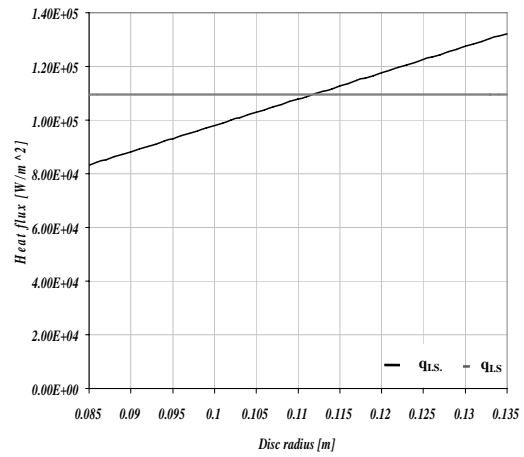


Fig. 3 Variation of  $q_{I,S}$  &  $q_{I,S,M}$  on the clutch surface with disc radius ( $r$ ) at  $t_s=0$

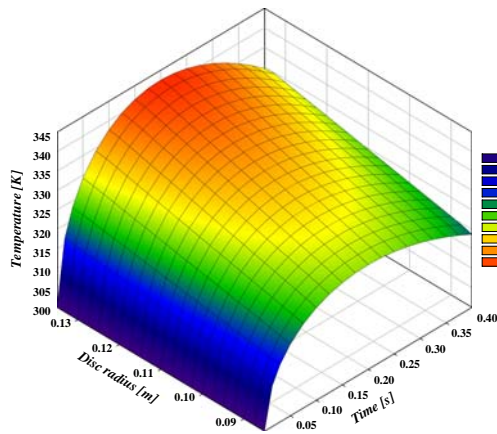


Fig. 4 Variation of temperature on the clutch surfaces with time and disc radius (load- $q_{I,S,M}$ )

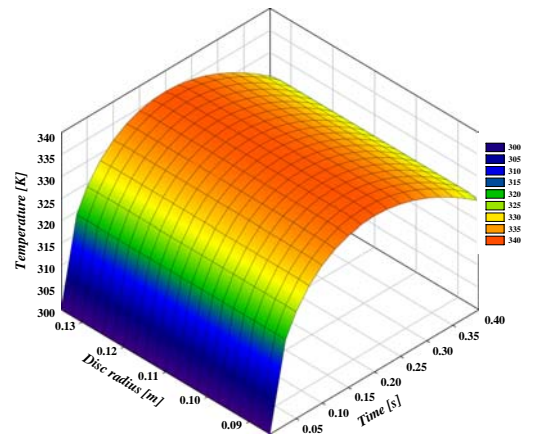


Fig. 5 Variation of temperature on the clutch surfaces with time and disc radius (load- $q_{I,S}$ )

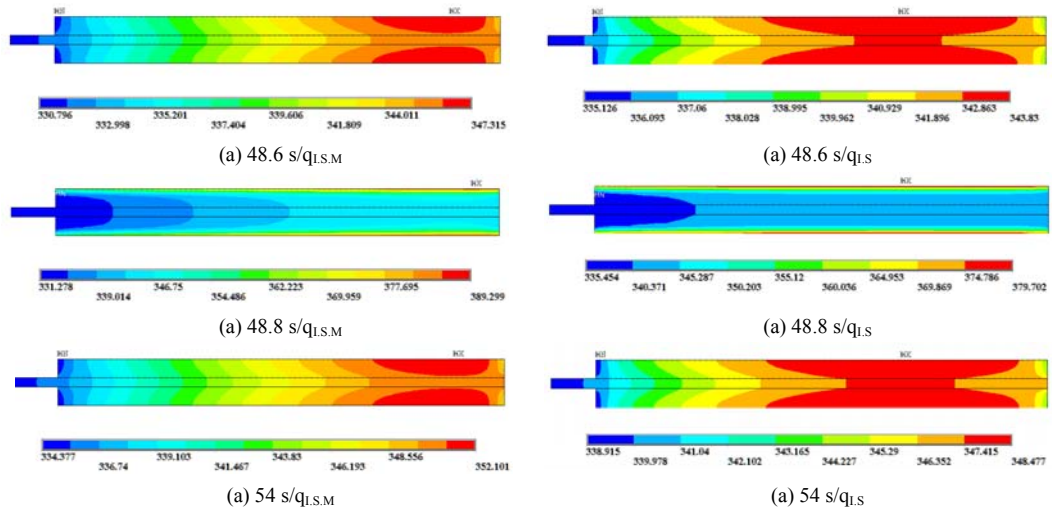


Fig. (6) Temperature distribution for the friction clutch disc when applied  $q_{I,S,M}$  and  $q_{I,S}$  (Engagement no. 10)

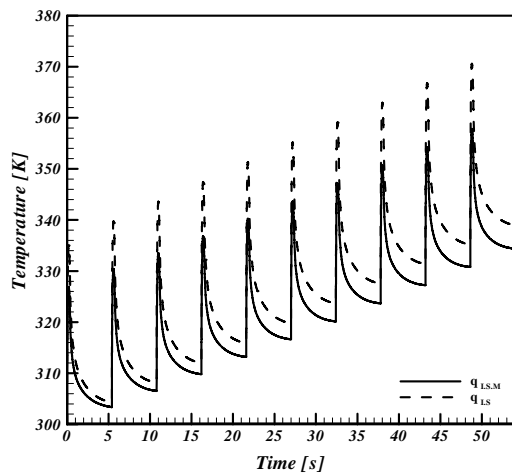


Fig. 7 Variation of temperature with time at  $r_i$

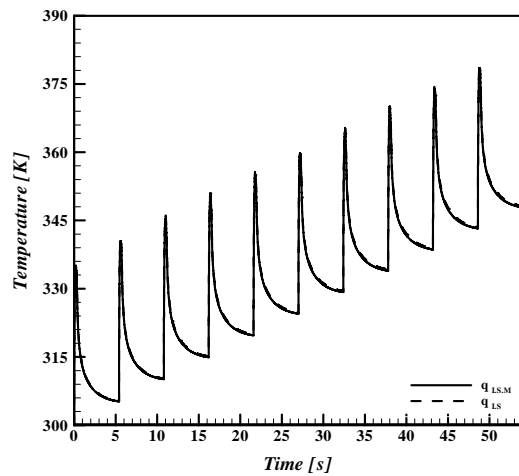


Fig. 8 Variation of temperature with time at  $r_m$

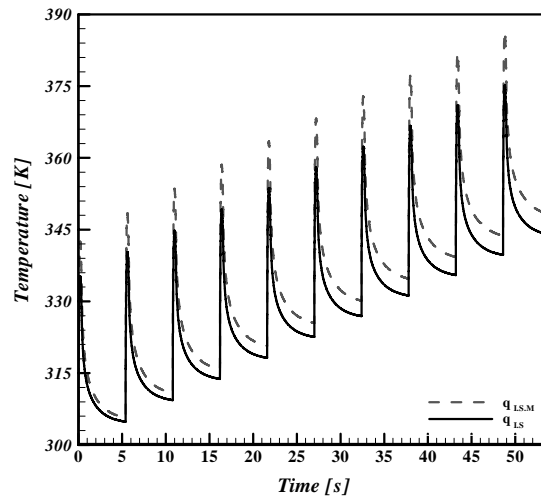


Fig. 9 Variation of temperature with time at  $r_o$

## References

- [1] JANIA, Z. J, 'Friction-clutch Transmissions', part (3), Machine Design, Vol. 30, No. 25, 1958.
- [2] NEWCOMB, T. P, 'Temperature Reached in Friction Clutch Transmissions', J. Mech. Engng. Sci., Vol. 2, No. 4, pp.273-287, 1960.
- [3] NEWCOMB, T. P, 'Calculation of Surface Temperatures Reached in Clutches When the Torque Varies with Time', J. Mech. Engng. Sci., Vol. 3, No. 4, pp. 340-347, 1961.
- [4] V. A. Kulev, 'heating calculations in friction clutch design', J. Russian Eng., Vol. LII, No. 9, pp.24-27, 1972.
- [5] Harold Rothbart, 'Mechanical Design Handbook', chapter 17 Friction Clutches, 2nd edition 2006.
- [6] Borozan I. Silviu, Maniu Inocentiu, Argesanu Veronica and Kulcsar Miklos, 'the energetic balance of the friction clutches used in automotive', World Scientific and Engineering Academy and Society (WSEAS) 2011, pp.252-256.
- [7] Josef Voldrich, Stefan Moravka and Josef Student, 'transient temperature field in intermittent sliding contact at temperature dependent coefficient of friction', 22nd conference with International participation-Computational Mechanics, 2006.

- [8] J. Voldřich, 'analysis of heat partition and temperature distribution in sliding systems', *Applied and Computational Mechanics*, Vol. 1, pp.357 – 362, 1, 2007.
- [9] F. Talati and S. Jalalifar, 'investigation of heat transfer phenomena in a ventilated brake rotor with straight radial rounded vanes', *J. Applied Sciences*, Vol. 8, No. 20, pp.3583-3592, 2008.
- [10] Balázs Czéla, Károly Váradi, Albert Albersb and Michael Mitariub, 'Fe Thermal Analysis Of Ceramic Clutch', *Tribology International*, Vol. 42, No. 5, pp.714–723, 2009.
- [11] Jin-le Zhang , Biao Ma , Ying-Feng Zhang and He-Yan Li, ' simulation and experimental studies on the temperature field of a wet shift clutch during one engagement', *International Conference of Computational Intelligence and Software Engineering*, pp.1-5, 2009.
- [12] Piotr GRZEŚ, 'finite element analysis of disc temperature during braking process', *acta mechanica et automatica*, vol.3, No.4, pp.36-42, 2009.
- [13] Pyung Hwang and Xuan Wu, 'Investigation of temperature and thermal stress in ventilated disc brake based on 3D thermo-mechanical coupling model', *J. Mechanical Science And Technology*, Vol. 24, No. 1, pp. 81-84 , 2010.
- [14] Piotr Grzes, 'use of the finite element method for simulation of heat generation in disc brakes', *Zeszyty Naukowe Wsinf*, Vol. 9, No. 3, pp. 124-140, 2010.
- [15] H. Mazidi, S. Jalalifar, S. Jalalifar and J. Chakhoo, 'mathematical modeling of heat conduction in a disk brake system during braking', *Asian Journal of Applied Sciences*, Vol. 4: pp.119-136, 2011.
- [16] Michaá Kuciej and Piotr Grzes, 'The Comparable Analysis Of Temperature Distriubuteds Assessment In Disc Brake Obtained Using Analytical Method And Fe Model', *J. KONES Powertrain and Transport*, Vol. 18, No. 2, pp. 235-250, 2011.
- [17] Adam Adamowicz and Piotr Grześ, 'three dimensional fe model of frictional heat generation and convective cooling in disc brake', *CMM-2011–Computer Methods in Mechanics*, 2011.
- [18] Barber, J.R. (1967), The influence of thermal expansion on the friction and wear process, *Wear*, Vol., 10, pp. 155-159.
- [19] Barber, J.R. (1969), Thermoelastic instabilities in the sliding of conforming solids, *Proc. Roy. Soc.*, Vol., A312, pp. 381-394.
- [20] Abdullah M. Al-Shabibi, 'The Thermo-Mechanical Behavior in Automotive Brake and Clutch Systems' In *New Trends and Developments in Automotive System Engineering*, Rijeka, Croatia, Intech, 2011, pp. 207 - 230.
- [21] Kwangjin Lee and J. R. Barber, 'frictionally excited thermoelastic instability in automotive disk brakes', *J. Teratology*, Vol. 115, pp. 607-614, 1993.

- [22] H. W. Sonn, C. G. Kim and C. S. Hong, 'transient thermoelastic analysis of composite brake disks', *J. Reinforced Plastics and Composites*, Vol. 14, No. 12, pp. 1337-1361, 1995.
- [23] Jose R. Ruiz Ayala, Kwangjin Lee, Mujibur Rahman and J. R. Barber, 'effect of intermittent contact on the stability of thermoelastic sliding contact', *Transactions of the ASME*, Vol. 118, pp.102-108, 1996.
- [24] Shuqin Du, P. Zagrodzki, J. R. Barber & G. M. Hulbert, 'finite element analysis of frictionally excited thermoelastic instability', *J. Thermal Stresses*, Vol. 20, pp.185-201, 1997.
- [25] Abdullah M. Al-Shabibi and J.R. Barber, 'transient solution of a thermoelastic instability problem using a reduced order model', *International Journal of Mechanical Sciences*, Vol. 44, No. 3, pp. 451-464, 2002.
- [26] Yi, J R Barber, P. Zagrodzki, 'eigenvalue solution of thermo-elastic instability problems using fourier reduction', *Proc. R. Soc. Lond.*, Vol. 456, pp.2799-2821, 2003.
- [27] Josef Voldřich, 'frictionally excited thermoelastic instability in disc brakes-transient problem in the full contact regime' *International Journal of Mechanical Sciences*, Vol. 49, No. 2, pp.129-137, 2007.
- [28] A C Ugural. 'Mechanical Design'. McGraw-Hill Professional, 2004.
- [29] O. C. Zienkiewicz, *The Finite Element Method*, McGraw-Hill, New York, 1977.
- [30] H. Blok, *Fundamental Mechanical Aspects in Boundary Lubrication*, SAE Trans., vol. 46, pp. 54-68, 1940.
- [31] Oday Ibraheem Abdullah and Josef Schlattmann, 'The Effect of Disc Radius on Heat Flux and Temperature Distribution in Friction Clutches', *J. Advanced Materials Research*, Vol. 505, pp. 154-164, 2012.

**Received: May, 2012**