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Chiral Trigintaduonion Emanation Leads to the Standard Model of Particle Physics and to Quantum Matter

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Abstract

The chiral trigintaduonions T, with right product operation ((T x T) x T)…, used previously for maximum information transmission, are here shown to be H×O when arranged for achiral emanation. When considering a sum over chiral emanations to obtain an achiral emanator, with T as phase factor, we have the exponentiation operation exp(iT), which leads to a theory that is $C \times H \times O$. As such, we have the foundation for the associative operator algebra of the Standard Model: $U(1) \times SU(2)_L \times SU(3)$. A complication with T products is you can have zero divisors. A framework is adopted to remove the zero divisors by requirement of maximum domain of analyticity on the log trigintaduonion multiplication, resulting in a description for the meromorphic precipitation of matter. In this process a fundamental quantum is indicated from the zero-divisor residue terms. Analyticity in the form of a Wick rotation also provides a mechanism whereby we can transition to a dimensionful action and quantum and arrive at an explanation for the critical 'smallness' of Planck's constant. A review of emanation theory will be given first, including the origin of α, the fine-structure constant, followed by showing that the form of the emanator is $T_{em} \cong H \times O$, followed by a description of the possible meromorphic origin of point-like matter.

Keywords: Trigintaduonion, emanator theory, emanation, the standard model, U(1), SU(2), SU(3), alpha, fine-structure constant, Feigenbaum constant, Planck's constant, quantum matter, path integral, propagation, Cayley algebra

Introduction

The theory of trigintaduonion emanation is partly based on chains of trigintaduonion operators. As with prior efforts based on chains of octonion operators [1-3], the hope is to provide the basis for the standard model of particle physics, among other things. For chains of octonions, seven right octonion products make a single left octonion product [4,5], or visa versa, just as three rights make a left when driving (a familiar result from chains of quaternions). The 'most complex' chain of right octonion multiplications thus occurs with six products. So the chain can be described as being isomorphic to the 6-dimensional Clifford algebra Cl(0,6) [3]. In parallel with the vector formations above, there are multivector formulations, e.g., geometric algebra and space-time algebra descriptions [6-10].

Getting an associative algebra from the repeated operation of a non-associative algebra is first described in [1] in the context of repeated octonion products: ((OxO)xO)…, where the algebra SU(3) can result. This is found to be equivalent to fixing one of the octonion imaginary components in such a repeated-product operation [1,2]. Dixon shows in [2] that the C × H x O product algebra lays the foundation for the associative operator algebra of the Standard Model: U(1) × $SU(2)_L$ × SU(3). In later work this is explored in the form of ideals [3]. Emanator theory, described in a brief background to follow, is based on unit-norm propagation at maximal dimension. It turns out this maximal dimension is not octonion-based but trigintaduonion-based [11], although it does have an octonion sub-algebra: $T_{em} \cong H\times O$, as will be shown here.

Background on recent material is given next. Further background on critical topics referenced [12-19] is given in an a lengthy appendix in the preprint version of this paper [20]. These topics include complex mappings (for fractal limits) [12], trigintaduonion zero divisor properties (yes, trigintaduonions have zero divisors) [13], the theory of residues (to remove those zero divisors) [14], and the theory of integrals with large parameter [15-19]. Further background on Trigintaduonions (and Sedenions) can be found elsewhere [42].

In prior work [11,20-23] we hypothesized maximal algebraic information flow, where the "emanation" of information is represented as multiplication by an element of an algebra in two steps: (i) take the maximal current-state element that is a unit-norm trigintaduonion; and (ii) perform the emanator step that consists of an achiral sum of multiplications with chiral trigintaduonion emanators. In prior efforts this was considered without the complication of zero divisors. We will see that zero divisors act as “sources”, so the prior work was effectively analysis of sourceless information flow.

In this paper we show $T_{em} \cong H \times O$ (which will lead to the standard model) and we consider zero divisors and their impact on the maximal information flow and in doing so see a mechanism for meromorphic precipitation of quantum matter with dimensionful action.

Background

Existence of generalized unit-norm propagation structure [11]

Unit-norm right product propagation is trivial for the division algebras since norm(XY) = norm(X) × norm(Y). From this it is apparent that we have an automorphism group given by the norm itself (A(XY)=A(X)A(Y)), and in the case of the octonions this automorphism group is G2 [1]. It can be shown that SU(3) is in G2 [1]. Let’s now consider the situation with a higher-order Cayley algebra, the Sedenions, ‘S’. We obviously don’t have norm(S_1S_2) = norm(S_1) × norm(S_2) in general, as this would then allow S to join the ranks of the division algebras, and it is proven that such don’t exist above the Octonions [24]. Can we still have a propagation structure? Is it possible to have a ‘base’ sedenion for which norm(S_{base})=1, and to have a right propagator (product) sedenion also norm(S_{right})=1, such that norm(S_{base} x S_{right}) =1? The answer is yes (see appendix of [20] and [11]), when the sedenion has the (chiral) form of an octonion crossed with a real octonion: S_{chiral} = (O,O_{real}) or S_{chiral} = (O_{real},O). Can we continue this to arrive at a propagation structure on the Trigintaduonions? Again the answer is yes, with the chiral form generalizing off the chiral Sedenion as might be expected: T_{chiral} = (S_{chiral}, S_{real}) or (S_{real}, S_{chiral}) [11]. It is proven that this extension process will go no further [11]. What happens is that due to the chiral form we are still able to re-express all T products (or S) as collections of terms involving tri-octonionic products (which have nice properties as described in [11]), and this can no longer occur above the (chiral) trigintaduonion level.

Chiral Trigintaduonion emanation involves 137 independent octonionic terms

The derivation below follows [2], but with a more succinct accounting of the independent terms.

Consider a general norm=1 bisedenion in list notation: (A,B), where A and B are sedenions. Consider a propagator bisedenion (C,β), C = (c,α), where c is an octonion and α is shorthand for the real octonion (α,0,0,0,0,0,0,0), where α is a real number, and β is shorthand for the real sedenion (β,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0), where β is a real number. Using A=(a,b), B=(u,v), and the multiplication rule from Sec. 2, we have:

(A,B)(C,β) = ([AC-β*B], [BC*+βA]), where

AC = (a,b)(c,α) = ([ac-α*b],[bc*+αa]); BC*=(u,v)(c*,-α) = ([uc*+α*v],[vc*-αu]).

Thus, we have:

(A,B)(C,β) = ([(ac-α*b , bc*+αa)-β*(u,v)] , [(uc*+α*v , vc-αu)+β(a,b)]), so,

(A,B)(C,β) = ([ac-α*b-β*u , bc*+αa-β*v] , [uc*+α*v+βa , vc-αu+βb]).

Now consider another propagator bisedenion (C′,β′), C′ = (c′,α′), and form the product corresponding to the next multiplicative step:

((A,B)(C,β)) (C′,β′) = ([(ac)c′ - α*bc′ - β*uc′ - α′*(bc*+αa-β*v) , …] , [… , …]), where only the first expression at octonionic level ($T=(O_1,O_2,O_3,O_4)$) is shown:

O_1 = (ac)c′ - α*bc′ - β*uc′ - α′*(bc*+αa-β*v).

At octonionic-level there are 10x9x8/3x2=120 independent terms for 8 octonionic components (labeled a, b, c) plus a separate octonion component (α) and one sedenion component (β), e.g., have 10 choose 3. Also have telescoping terms with repeated real octonion factors, such as with the aαα′* term (think $a\alpha(\alpha'^*)^n$), which gives an additional 8 independent terms. Also have telescoping terms with alternating real octonion factors and real sedenion factors, such as with the vβ*α′* term (think $v(\beta^*\alpha'^*)^n$), which gives another 8 independent terms. There is one other ‘telescoping’ term due to repeated octonion right products seen in (ac)c′ (now think ((ac)c′)c′…..c′)). The change in this term corresponds to an element of the automorphism group on octonions, G2, and as such provides one last independent term, for a total of 137 independent terms at octonion level.

All of the octonion products involve octonions with norms at most unity, and by the normed division algebra rules on octonions, their norm is simply the norm of the individual octonions multiplied together, all of which are bounded by unity, thus their product is bounded by unity. The overall bound for the expression, each individual term being bounded by unity, is therefore simply the counting on the independent terms.

The maximum magnitude of each component of the octonion in the product term is given with a ‘channel multiplier’ of 137. Also, in seeking the maximum information propagation we require that the real chiral component never cross zero (e.g., stay in its connected {α,β} quadrant), thus the strictest condition on evaluating evolution might be intuited to be when the imaginary components combine to have real component contribution that is antiphase, e.g., the total imaginary angle is π. The choice of antiphase will used in what follows and will be justified when “C ×” allows the antiphase to be understood in the context of the Universal Mandelbrot set [25] position on the negative real axis that gives the maximal magnitude of displacement from the origin: C_∞. We limit the maximum perturbation allowable by the antiphase worst case. At octonionic-level there is thus the channel multiplier: $137 + i\pi$.

For what follows, it helps to recall some important properties of the exponential, particularly its well-defined properties with hypercomplex numbers [2]. Important map relations:

(1) exponential map on Im(T) gives unit norm object: $\exp(\mathrm{Im}(T)\theta) = \cos\theta + \mathrm{Im}(T)\sin\theta$.

(2) exponential map on iT gives $C \times T$:
$\exp(iT) = \exp(i\mathrm{Re}(T)) \times \exp(i\mathrm{Im}(T)) = (\cos\theta + i\mathrm{Re}(T)\sin\theta) \times (\cos\varphi + i\mathrm{Im}(T)\sin\varphi) = C \times T$

Use (1) to focus on fluctuations in imaginary parameters free of normalization concerns.

Use (2) to get complex structure C × (object). Note that exponentiation into phase terms is precisely what occurs in the path integral propagator formalism, and will occur here as well for the emanator formalism, thus the "C ×" complex factor. When drawn upon in the emanator formalism, this method of achieving additional "C ×" complex structure will be forced by the zero-divisor handling (that will give rise to point-like matter with very small phase coupling, thus a highly oscillatory integral, and ties over to foundational aspects of the path integral formalism).

Achiral Trigintaduonion unit-norm propagation has maximum perturbation α [21,22]

The maximum perturbation, referred to as maximum noise in what follows, is first evaluated for a chiral emanation where we take a norm=1 $T_{base}=(A,B)$ and take the right product with T_{chiral} in the form $T_{chiral}=(C,\beta)$, with product $(A,B)(C,\beta)$ proven to be unit norm above [11]. In the prior section we saw that there are 137 independent octonion terms at the octonion sub-level of the new unit norm trigintaduonion that results, which leads to 137 independent terms at component level. In order to use the map rules mentioned in the previous section, it is necessary to move from the trigintaduonion, T, space, to the C × T space. This is done in later sections anyway where we consider sums on exp(iT). The exponential function (map) provides a well-defined 'lift' of a hypercomplex (Cayley) algebra from T to C × T. The exponential map also provides a very useful maneuver when working with unit-norm hypercomplex numbers via the generalized deMoivre theorem $\exp(\mathrm{Im}(T))=\cos(\theta)+\mathrm{Im}(T)\sin(\theta)$, with the real part recoverable from $\cos(\theta)$. More details on this follow later but for now, in evaluating the maximum noise allowed we have three structures to adopt: (1) the noise is generalized to be complex (as will be the case for the components themselves once the T→C × T structure is adopted). (2) At component-level, the noise (for maximum noise) is equipartitioned in both real and imaginary parts. (3) Total imaginary noise magnitude is π for maximal antiphase (to be justified later).

(I) Chiral emanation noise: have 137 terms with max unit norm each, for the real part, and for the imaginary part have a "phase angle" β such that 137β=π (here referred to as a phase angle in the sense that the exp(Im(T)) map is being used). The noise magnitude at octonionic-component level is then given by the right triangle with real part = 137 and angle β=π/137, thus maximum chiral emanation noise magnitude is:

$$H = 137/\cos(\pi/137)$$

(II) Achiral emanation noise: now have 29 "free" components, each with 137 independent terms. For maximum achiral emanation we thus have 137 x 29 independent terms that are built from the aforementioned chiral emanation terms (to make achiral). If we equipartition as before, with noise magnitude Hc, we have a "noise triangle" with magnitude (hypotenuse) Hc and with angle $\theta = \pi/137x29$. The imaginary part is then (Hc)sin(θ). As regards the H magnitude separated form (separating out the 'H' factor for now), we have for the imaginary part sin(θ)c. As before, we take maximal noise transmission when all the imaginary parts add to maximal antiphase. Given the equipartitioning assumption, we then simply have the factor 137x29:

sin(θ) c (137x29) = π → $c = \theta/\sin\theta$.

The maximum real noise perturbation that the system can have is then α, where:

$$\alpha^{-1} = \frac{137}{\cos\beta}\cos\theta\,\frac{\theta}{\sin\theta}, \qquad where\ \beta = \frac{\pi}{137}\ and\ \theta = \frac{\pi}{137x29}$$

$$\alpha^{-1} = 137.03599978669910\,,$$

where the evaluation was done at WolframAlpha to high precision [26] (e.g., higher precision than that reported in earlier work [21]). This matches the experimentally observed value to all 11 decimal places currently known. As of 2002 [27], the measured value of α is:

$$\alpha^{-1} = 137.03599976(50).$$

Note that in quantum field theory the parameters are renormalized at a particular energy scale. Thus choice of energy scale impacts the value of α (as a coupling constant in the classical theory or a perturbation expansion factor in the quantum theory). At 0K we have the extreme low-energy end of the renormalization group (with the largest α value). We are at the 2.7K CMBR, so we have the max α to very high precision. (In studies at high energy scale at LEP, at the energy scale of the Z-boson (91GeV), we get the renormalized value to be [27,28]: $\alpha^{-1}[M_Z] \cong 127.5$. Note that 91GeV is way above the energy scale of the familiar Hagedorn temperature at ~pion mass=150MeV or 1.7x10^12 K) [29], where hadronic matter 'evaporates' into quark matter.)

Achiral emanation using a 72-card deck

There are 4 chiralities, so to get an achiral emanator candidate, minimally need a "4-card deck" to emanate in the four chiralities, with emanator equal to normalized sum. The actual deck appears to require a normalized sum over sub-chiralities, as will be explicitly enumerated in what follows. Regardless of the form of the achiral sum over chiral variants, since each chiral emanation has 29 free components, their norm sum will again have 29 free components. Thus, the form for α^{-1}shown above is the complete, non-approximating, result since it only need rely on 29 free component number. Let's now consider the emanation deck in more detail.

Here are the four chiralities with real fluctuation noise shown:

$$(((O[0] \pm \delta, \ldots), \alpha \pm \delta), \beta \pm \delta)$$
$$((\alpha \pm \delta, (O[0] \pm \delta, \ldots)), \beta \pm \delta)$$
$$(\beta \pm \delta, ((O[0] \pm \delta, \ldots), \alpha \pm \delta))$$
$$(\beta \pm \delta, (\alpha \pm \delta, (O[0] \pm \delta, \ldots)))$$

where α is a real octonion and β is a real sedenion, and Tem is an equal weight sum of the action of each of the sub-chiral propagations on the base T, with the fluctuations indicated each done separately. We have the constraints $\alpha \neq 0$, $\beta \neq 0$ (discussed further in the Methods), and common octonion O not pure real (discussed further in the Results).

Each of the δ's is an independent fluctuation corresponding to its own sub-chiral emanation, but no subscripting on δ's is used or shown. There are thus 9x2x4=72 independent *imaginary* noise fluctuations to consider in the exp(Im(Tem)) evaluation (that automatically provides unit-norm). The real noise fluctuations in the real (first) component are, thus, not counted. If our definition for Tem entails only one card being dealt, then the sum over those possibilities is the sum

$$\mathbf{T} \bullet \mathrm{T}_{\mathrm{em}} \equiv \mathrm{Emanation}(\mathbf{T}) = \frac{1}{72} \sum_{k \in \{72\}} \mathbf{T} \bullet \boldsymbol{T}_{\boldsymbol{chiral}}^{(\boldsymbol{k})}$$

For one-card, or a one-step, emanation, with real components and real noise, this makes sense from the counting shown, and it's what we use going forward. Using this will allow an entirely separate method for evaluating α (here at the one-card hand approximation). This will be done by determining the effective dimension 29*>29 of maximal information propagation (or maximal noise fluctuation). Before moving on, however, let's examine what happens when we allow complex noise fluctuations as this will trivially be allowed when we consider C × T via exp(iT) in later discussion anyway.

Maximum information transmission involves a complex extension to the T components and their noise fluctuations, but in doing this it must retain emanation structures such as the octonionic triple that occurs in previous expressions (starting with the proof of the T_{chiral} solution itself), which leads to the counting that gives 137 independent terms, etc. Thus, the maximal complex extension on the noise is that it remain real in the octonion components:

$$\begin{gathered} (\,(\,(\mathrm{O}[0] \pm \delta, \dots\,), \alpha \pm i\delta), \beta \pm i\delta) \\ (\,(\,\alpha \pm i\delta, (\mathrm{O}[0] \pm \delta, \dots\,)), \beta \pm i\delta) \\ (\,\beta \pm i\delta, (\,(\mathrm{O}[0] \pm \delta, \dots\,), \alpha \pm i\delta)\,) \\ (\,\beta \pm i\delta, (\,\alpha \pm i\delta, (\mathrm{O}[0] \pm \delta, \dots\,)\,)\,) \end{gathered}$$

The first chiral T component is where new imaginary terms might arise (the others are already counted since in imaginary components). We see there are six more, so the deck is now78.

When to use the 72-deck and when to use the 78-deck isn't clear yet, this will eventually be something that can be determined by how the theory converges on the non-approximate α^{-1} given above. At one card, or the first card, emanation is just a sum over chiral T's, so still a T product (acting on T_{base}) without the complication of zero divisors (to be described in later sections), so this is a convenient dividing line between the α estimation, or theory value, based on T, and that based on C × T. The C × T description, trivially allowing complex noise fluctuations, may go best in the multi-card emanation description where the exponential map exp(iT) = C × T will be indicated anyway. Further consideration of emanation with a multi-card hand (or multi-step path) will be discussed at a later time.

All noise terms will be treated additively, including terms in different imaginary components as well as imaginary noise terms in the real component. The criterion for max noise (in-phase constructive interference) gives the extreme of linear additivity. (Not like Gaussian statistical noise that adds in quadrature.) Also note that the discussion in terms of “noise transmission” and “information transmission” will be used almost interchangeably, whenever one description or the other best suits the analysis it will be used. Note that with this kind of noise analysis we can effectively shift around T noise terms associatively. Also note that application of the Kato-Rellich theorem [22,23] is related to the noise budget analysis done here focusing on first order terms.

There are 137 independent tri-octonionic terms in each of 29 free components indicated by a particular chirality (within the 32 components of a general trigintaduonion). This is a nontrivial result since ($T_{chiral} \bullet T_{chiral}$) is no longer T_{chiral} type (but still T_{norm1} type), so direct expansions are needed to identify the number of independent terms and this is briefly described below, with more detail in [21].

Single-step achiral 72-deck emanation has noise propagation dimension 29* [30]
Obtaining an achiral emanation from a collection of chiral emanations requires that all chiralities be summed over (there are four) as well as sub-chiralities (there are 72). Noise analysis requires collecting of first-order terms. Analysis of noise transmission indicates 29* dimensions, where:

$$29^{*} = 29 + \left(\frac{4\pi}{72}\right)\left[1 + \left(\frac{\pi}{137 \cdot 29}\right)\left(\left(\frac{\pi}{72}\right) + \left(\frac{3}{72}\right)\right)\right]$$

The above result was obtained in [23] to describe the 72-card chiral ‘deck’ of chiral emanation products for a single-step emanation. In the Methods to follow this is reviewed and elaborated further.

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‘Edge of chaos’ maximal perturbation hypothesis [23]
Consider the ‘edge of chaos’ maximal perturbation in each of the 29* dimensions to be at position C_∞ (see Appendix for background on Mandelbrot Set), which is on the negative real axis, i.e., at π rotation to have −1 factor, ***thus at maximal antiphase***. This results in the relation for maximal perturbation at maximal antiphase (maximum reference angle with sign chosen positive by convention) has a lower bound on α given by:

$$\alpha_0^{-1} = \left(\sqrt{C_\infty}\right)^{29^{*}}.$$

where

$$C_\infty = 1.4011551890920506004\ldots$$

This ties $1/\alpha$ to the second Feigenbaum constant C_∞in the context of the Mandelbrot set. It is well known that the Feigenbaum constants are universal, and part of a description of a universal transition to chaos regime. The Mandelbrot set is also universal [25], and maximal in that its fractal boundary has maximal fractal dimension of 2 [25], a detail that will be important in the meromorphic matter description given later.

For C_∞, most references only provide $C_\infty = 1.401155189 \ldots$, and a higher precision tabulation is not readily found, so use is made of the relation

$$C_n = a_n(a_n - 2)/4,$$

together with the tabulation on a_∞ [31]:

$$a_\infty = 3.56994567187094490 18 \ldots$$

The resulting C_∞ is:

$$C_\infty = 1.40115518909205060 04 \ldots$$

The resulting α_0^{-1} is:

$$\alpha_0^{-1} = 137.03599933370198263 \ldots$$

For the multi-card analysis we have:

$$\alpha^{-1} = \alpha_0^{-1} + \alpha_1^{-1} + \cdots$$

where α_0^{-1} involves the sum over emanation by one-card. For sum on two-chiral products we have further 'noise' contribution α_1^{-1}. With the multi-card modifications, albeit small, there is the complication of shift from 72-deck to 78-deck, and whether there is a chiral step ('card') type that can be exactly repeated (i.e., are cards from the deck played with replacement when considering a multi-card flop sequence). There may be a reason why the sums must be done without card-replacement. This might be because card replacement would allow degenerate tri-octonionic product terms, again throwing off the 137 braid term total, perhaps, leading to non-optimality. This is being explored in further work and will not be discussed further at this time.

Methods

The effective noise transmission dimension

For emanation we produce an achiral sum from the four primary chiral emanations. To achieve an emanator with counting that provides agreement with the $\{C_\infty, \alpha, \pi\}$ relation, we see the need for sub-achirality as well, necessitating a sum over the 'full deck' of specific subtypes of chiral emanation. One of the chiralities is written $T = ((0, \alpha), \beta)$ where the normalization factor to achieve unit norm is suppressed, where

O is an octonion, α is a real octonion, and β is a real sedenion, or a real octonion in the pair S=(β,0), depending on notational convenience. Also, depending on notational convenience, often α and β will be treated as simple real numbers (equal to the real component of the respective octonion).

Let's consider the generative process of arriving at an achiral emanation, and the counting results that are obtained:

(1) There are four chiral forms, let's start an emanation by choosing a common octonion element for all four chiralities. Now let's choose $[+\alpha, +\beta]$. We can't have $\alpha = 0$ or $\beta = 0$, so we have a splitting into four disconnected regions in the $[\alpha, \beta]$ plane (the four quadrants). Let's choose positive $[\alpha, \beta]$ then generate four cases (respective to the four quadrants) by having cases with $[\pm\alpha, \pm\beta]$. (In this way the four principle chiralities will have the same normalization constant).

(2) Each chiral form has 9 imaginary components, Let's consider two maximal perturbations for each (via α perturbation limit), one with the imaginary component plus the max-α fluctuation, one with minus the max-α fluctuation. The normalization constants will now differ.

The number of Cases = (4 chiralities) x (4 $[\pm\alpha, \pm\beta]$ subchiralities) x (9 imag perturbations) x (2 max pert. at ±max-α) = 4 x 72. Consider the 4 x 72 cases summed over all but the 4 chiralities. We may have 'merging' at the core four chirality level:

$$\text{Emanation}(\mathbf{T}) = \frac{1}{4} \sum_{k \in \{4\}} \mathbf{T}_{\boldsymbol{base}} \bullet \boldsymbol{T}^{(\boldsymbol{k})}_{\boldsymbol{chiral}},$$

Going further, and using the notation of a simple product relation will be adopted for the "1 card" emanation. It may not exist, from the norm sum form, but in the noise analysis, with the simple rules on counting noise terms additively, it helps to arrange terms for that counting, Thus, let's consider the noise in:

$$\mathbf{T}_{\boldsymbol{base}} \bullet \mathbf{T}_{\mathbf{em},(\mathbf{1\ card})}$$

which will have groupings:

$$\{real + real\ noise + imag\ noise\} \times \{(real \approx 1) + real\ noise + imag\ noise\}.$$

From the previous analysis of the maximal noise with the achiral emanator we had the "noise triangle" with noise magnitude Hc, noise angle $\theta = \pi/(137 \times 29)$, imaginary noise = $(Hc)\sin(\theta)$, real noise = $(Hc)\cos(\theta)$, where $H = 137/\cos(\pi/137)$

and $c = \theta/\sin\theta$. Let's take $\mathbf{T}_{\boldsymbol{base}}$to have this generic form, associated with maximal information flow α^{-1} = max real noise = (Hc)cos(θ). Also, we don't know the overall scale, so common scale factors can be dropped (H), such that the imaginary noise term is $\delta\theta\boldsymbol{i}$, and there are 29 of them for the 29 unconstrained dimensions (due to choice of chirality and norm=1):

$$\{1, \ldots .29\delta\theta\boldsymbol{i}\} \times \{real + imag + real\ noise + imag\ noise\}.$$

For the emanator T on the right-hand-side, we know the imaginary terms will be the same, thus a $29\delta\boldsymbol{i}$ factor. Note, the analysis is done for the first quadrant of the $[\pm\alpha, \pm\beta]$ subspace, the one connected to T=1 with $[+\alpha, +\beta]$ such that its real term is $T_{em}[0] = 1 - \Delta$. (Later, the four $[\pm\alpha, \pm\beta]$ sub-chirality sectors will give an overall factor of four.) Aside from the $29\delta\boldsymbol{i}$ factor, the imaginary terms will have a max 1 transmission path as well as an equipartition of max-antiphase π over 29 free dimensions and 137 independent terms in each, thus have a factor of $[1 + \theta i]$ (ignoring a common normalization term that factors out in what follows). So far we've described the imaginary noise that might occur 'internal' to a chiral sum, effectively for one card of the deck (or for a minimal 4-suit achiral that expresses the key 29 free dimensions property). For the imaginary noise terms we expect another phase factor from the 'deck sum' (here the phase analysis on the 137 independent terms is separated from the phase analysis, at deck-level, of the 72 independent cards). This factor should involve an equipartition over 72 cards, each with 29 free dimensions: $\pi/(72 \times 29)$. Putting this together:

$$\{1, \ldots .29\delta\theta\boldsymbol{i}\} \times \left\{1 - \Delta, 4 \times 29\delta\boldsymbol{i} \times \frac{\pi}{72 \times 29} \times [1 + \theta\boldsymbol{j}]\right\}.$$

Simplified further:

$$\{1, \ldots .29\delta\theta\boldsymbol{i}\} \times \left\{1 - \Delta, \delta\left(\frac{4\pi}{72}\right)\boldsymbol{i} + \delta\left(\frac{4\pi}{72}\right)\theta\boldsymbol{j}]\right\},$$

where the $\boldsymbol{i}$ and $\boldsymbol{j}$ different imaginary terms are only held separate for clarity, eventually they will be merged as one imaginary term (according to the additivity on terms in the maximum noise analysis). Of the cards in the 72-deck, 3 of them don't match with the choice of T≅1 with $[+\alpha, +\beta]$ subspace. This is why in the noise analysis we have $T_{em}[0] = 1 - \Delta$ and not simply '1' (when ignoring overall normalization factor), the Δ corrects for needing to subtract 3 cards from the emanation deck, and thus their channels of (card-level) noise. Thus, we have:

$$\Delta = \left(\frac{3}{72}\right)\delta\left(\frac{4\pi}{72}\right)\theta\boldsymbol{j}.$$

The maximum noise transmission then occurs with:

$$\{1, \qquad 29\delta\theta\boldsymbol{i}\} \times \left\{1 - \left(\frac{3}{72}\right)\delta\left(\frac{4\pi}{72}\right)\theta\boldsymbol{j}, \qquad \delta\left(\frac{4\pi}{72}\right)\boldsymbol{i} + \delta\left(\frac{4\pi}{72}\right)\theta\boldsymbol{j}]\right\},$$

The imaginary noise terms, at first-order, are:

$$\delta\boldsymbol{i}(29)\left[29 + \left(\frac{4\pi}{72}\right)\{1 + \left(\frac{\pi}{137x29}\right)[\left(\frac{\pi}{72}\right) - \left(\frac{3}{72}\right)\boldsymbol{j}\right]$$

Gathering the imaginary noise terms, *but all additive*, such that maximum noise is achieved with whatever constructive interference on noise phase, we get:

maximum imaginary noise = δ{ 29 + (4π/72)[(1+θ{(π/72)+(3/72)}] }, $\theta = \pi/(137 \times 29)$.
effective noise transmission dimension = { 29 + (4π/72)[(1+θ{(π/72)+(3/72)}] } ≡ 29*.

Trigintaduonion Emanation: achirality from chirality
The Mandelbrot Set (see Appendix) is one of many that encounter the universal constant C_∞. The Mandelbrot set also describes a boundary with 2D fractal dimension [25] at its "edge of chaos" [23]. If driven to similar optimality in approaching a zero-value (a zero-divider issue), we see a two-value zero-crossing specification effectively like a double zero. The parameterization of the zeros of the Emanator at chiral zero-divisor points will thus be as double-zeros.

For what follows we use the simple description of the emanator:

$$T_{chiral}^{(k)} = \begin{cases} ((0,\alpha),\beta) \\ ((\alpha,0),\beta) \\ (\beta,(0,\alpha)) \\ (\beta,(\alpha,0)) \end{cases}.$$

$$\text{Emanation}(\mathbf{T}) = \frac{1}{N}\sum_{k\in\{4x72\}^n} \mathbf{T} \bullet \boldsymbol{T}_{\boldsymbol{chiral}}^{(\boldsymbol{k})} = \frac{1}{N}\sum_{K\in 4\ chiralities} \mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}^{(\boldsymbol{K})}$$

The zero-divisor problem
For the real numbers, xy=0 only if x=0 and/or y=0, i.e., there are no "zero divisors: y=0/x. This is true of all division algebras (the Cayley algebras R, C, H, O). Starting with the sedenions (S), and even more so with the trigintaduonions (T), we have zero divisors (see Appendix). Let's re-examine the emanator product with this complication in mind.

Suppose we add the rule that emanation may not proceed when a particular chirality is zeroed-out in other words:

$$\mathbf{T} \bullet \overline{\boldsymbol{T}}^{(\boldsymbol{K})}_{\boldsymbol{chiral}} \neq \mathbf{0}\,.$$

For sedenions the dimensionality of the zero-divisor event is mostly constrained, while for trigintaduonions it is significant (see Appendix in [20]). If such zeros were eliminated from the emanator description by using analytic extension component-wise (on 29* effective components) we see how a description devoid of matter (pure static field with no source or sink) might acquire matter by way of extending to a maximal domain of analyticity be removing zero-divisor events (a Wick transformation from real dimensionless action to pure imaginary action that is dimensionless but consisting of a dimensionful ratio). Using the notation adopted in [30], let's parameterize the zero-divisors and index them such that:

$$\mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{S}^*{}_{\boldsymbol{i}}) \rightarrow \mathbf{0}\ as\ double\ zero\ \forall \boldsymbol{S}^*{}_{\boldsymbol{i}}\,.$$

Maximal emanator analyticity via removal of zeros (closely follows efforts in [30])

The sum over the zero-divisors means that the part of the emanator requiring analytic 'repair is given by:

$$\sum_{\{\boldsymbol{S}^*{}_{\boldsymbol{i}}\}} \mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{S}^*{}_{\boldsymbol{i}}) \rightarrow \sum_{\{\boldsymbol{S}^*{}_{\boldsymbol{i}}\}} \boldsymbol{e}^{\mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{S}^*{}_{\boldsymbol{i}})} \rightarrow \sum_{\{\boldsymbol{i}\}} \boldsymbol{e}^{\boldsymbol{S}^*{}_{\boldsymbol{i}}/\boldsymbol{h}^*}\,.$$

Where use is made of the fact that approach to zero-divisor (ZD) is purely involving imaginary components. The shift to exponential form will be explained with the choice of analytic continuation or 'repair' described in the next section. The sum on ZD events (for all 'time') can thus be described as a sum on (ZD) paths. We can now see the identification of matter with the zero-divisor 'residues' that occur when imposing maximal analyticity. The dimensionality of possible ZD's (for trigintaduonions) thus indicates a dimensionality on "paths," with result:

$$\sum_{\boldsymbol{zd's}} \boldsymbol{e}^{\boldsymbol{S}^*{}_{\boldsymbol{i}}/\boldsymbol{h}^*} \rightarrow \int_{\boldsymbol{zd\ paths}} \boldsymbol{e}^{\boldsymbol{S}^*(\boldsymbol{i})/\boldsymbol{h}^*}\,.$$

Now do a Wick rotation and go from real dimensionless iteration-number to imaginary dimensionful action, with dimensionful Planck's constant. We then get the highly oscillatory integral that is the basis of quantum field theory and quantum mechanics, with their classical and semiclassical reductions. With matter reified by Wick rotation, we go from an integral on zd paths with large parameter 1/h* to an integral on matter paths with large parameter 1/h. We, thus, maintain the large-parameter form as we go from a Laplace-type integral to a Stokes-type integral, and thus arrive at a path integral formulation:

$$\int_{zd\ paths} \boldsymbol{e}^{\boldsymbol{S}^*(\boldsymbol{i})/\boldsymbol{h}^*} \to \int_{matter\ paths} \boldsymbol{e}^{\boldsymbol{iS}(\boldsymbol{i})/\boldsymbol{h}}, \quad where\ \boldsymbol{S}(\boldsymbol{i}) = \int \boldsymbol{Ldt}\,.$$

In what follows, the shift from emanator projection to discrete-time propagation with (S*/h*) and, most notably, a shift from propagation in terms of trigintaduonion emanation steps comprising trigintaduonion multiplications to the more conventional propagation in terms of complex propagators comprising multiplication of complex functions of a complex variable. The shift from 32D non-associative emanator numbers to 2D (complex, associative) propagator functions necessitated by consistency with the maximal info flow hypothesis and the known constraints of the quantum deFinetti relation to information flow with complex propagators [30].

Zero-divisor removal at component level (closely follows efforts in [30])

In the Results we will require zero removal for analyticity on the log of the trigintaduonion products for a particular chirality of emanation. Let's now calculate the zero removal residue seen as a product of each of the 29* effective dimensions of the analytically-continued real components. Recall that:

$$\oint_C \frac{1}{z}dz = \oint_C d(\ln z) = 2\pi i \quad (simple\ pole).$$

where C is a contour that encloses the pole, which generalizes to:

$$\oint_C d(\ln f(z)) = \sum_{zeros} 2\pi mi \quad (multiple\ zeros),$$

where f has multiple zeros of order m, and where the last result requires that $f(z)$ be analytic throughout the domain, D, with boundary C inside that analytic domain (and D is simply connected). Let $\boldsymbol{S}^*{}_{\boldsymbol{i}}$ be the zeros of $f(z)$ where at lowest order $f(z)$ has a double zero at each of the $\boldsymbol{S}^*{}_{\boldsymbol{i}}$ according to the maximum fractal dimension possible for the boundary condition at the edge-of-chaos (where the dim=2 boundary dimension is actually the case for the Mandelbrot Set [25]). Let's use this information to parameterize the approach to the zeros:

$$\mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{z}) \propto \prod_{\boldsymbol{d}=\boldsymbol{29}*} (\boldsymbol{z} - \boldsymbol{S}^*{}_{\boldsymbol{i}})^{\mathbf{2}},$$

thus, for multiple zeros:

$$\oint_C \frac{d}{dz}(\ln[\mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{z})])dz = \sum_{zeros} \prod_{29*} 4\pi i.$$

Focusing on just one of the zeroes and the line integral dominated by a local, stationary phase, contribution, we need to integrate and set $\boldsymbol{z} = \boldsymbol{S^*}_{\boldsymbol{i}}$:

$$d\left(\ln\left[\mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{z})\right]\right) = 4\pi i^{29*} dz.$$

and, with choice of integration constants (phase factors):

$$\mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{S^*}_{\boldsymbol{i}}) = \boldsymbol{e}^{\boldsymbol{S^*}_i/|\boldsymbol{h}^*|}.$$

Summing on the zeros of the latter expression:

$$\sum_{zeros\, \boldsymbol{S^*}_{\boldsymbol{i}}} \mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{S^*}_{\boldsymbol{i}}) = \sum_{zeros\, \boldsymbol{S^*}_{\boldsymbol{i}}} \boldsymbol{e}^{\boldsymbol{S^*}_i/|\boldsymbol{h}^*|}.$$

Thus, the general form of maximal, analytic, information emanation gives rise to a sum on residue-like terms associated with each of the zero-divisors (zd's), and an 'action' variable is indicated to result from the parameterization of the approach to each of the zero-divisors, with their individual actions additive (phase contributions multiplicative) for parts contributing to a particular zd. The sum over all the zd's will, upon analytic continuation, be associated with a sum over paths. The zd action variable is written in the form of the integral of a functional along a path parameterized by 'time', with the usual definition for Action if the functional is the Lagrangian:

$$\sum_{zeros\, \boldsymbol{S^*}_{\boldsymbol{i}}} \boldsymbol{e}^{\boldsymbol{S^*}_i/|\boldsymbol{h}^*|} \rightarrow \int_{\boldsymbol{matter\ paths}} \boldsymbol{e}^{\boldsymbol{iS}(i)/\boldsymbol{h}}, where\ \boldsymbol{S}(\boldsymbol{i}) = \int \boldsymbol{Ldt},$$

where the definition for action above is kept to the simple form for a point particle trajectory. More complex forms can be written for field descriptions, where we generalize from point particle forms in various ways, but still with point-like coupling terms. Further generalization to actions describing 1-D objects, string not points, or beyond (n-D objects, or branes) and their trajectories is possible at this point but note how the chain of associations is altered, if not broken. Tracing back to a fundamental issue of analyticity when going from emanator form to propagator form, we saw that analyticity requires *isolated* zeros to not make the entire solution trivially zero. Thus, the fundamental meromorphic 'precipitate' for matter might be point and point-based field constructs as we've developed them already, where the role of string theory emerges separately (perhaps on space-time boundaries), although possibly as early as the "matter-precipitating" Wick rotation step above. Note that in going from

$$\frac{\boldsymbol{S^{*}}_{\boldsymbol{i}}}{|\boldsymbol{h^{*}}|} \to \boldsymbol{i}\frac{\boldsymbol{S(i)}}{\boldsymbol{h}}$$

> (1) Both ratios are dimensionless, but the quantities on the left are a ratio of pure numbers themselves dimensionless, while the RHS has a ratio of S, the action, with action dimensions, and h, Planck's constant, also with action unit dimensions.
>
> (2) Both $1/|\boldsymbol{h^{*}}|$ and **"$1/|\boldsymbol{h}|$"**, where the abs value operation on the latter simply drops the dimensionful units, are extremely large numbers and occurring in a phase argument. This *sets up a highly oscillatory integral* (see Appendix in [20]) such that the classical solution $\delta S = 0$ results (if a classical solution exists for the system studied), among other things.

If the emanator is to remain an achiral mix, as well as analytic, then we can't allow the zero divisor events that would drop a chirality mentioned above, where these occurrences are treated as isolated singular events removable from the domain of definition of the emanator by repeated application of the analytic domain 'surgery' (repeated on both events, and for given event, it's different independent components). This analytic 'surgery' occurs for each of the independent component dimensions for a given chiral emanation, and for each of those dimensions it returns a zero-removal 'residue' of $4\pi i$ (with an extra factor of 2 since a double zero at the fractal boundary). We found in [23] that the effective dimension is 29*, thus the remnant of the surgery for each zd removed is:

$$\frac{\boldsymbol{1}}{|\boldsymbol{h^{*}}|} = (\boldsymbol{4\pi})^{\boldsymbol{29^{*}}}.$$

So far we've examined the zd's and their residues on the (C ×)^29* local factorization. If we consider the complex structure in a global sense, any meromorphic function on a sphere, due to its compactness, must be rational. Evaluation of zero divisors happens to occur when we consider the product of two pure unit-norm imaginary trigintaduonions, thus on a 31-sphere. A quantization on the matter has occurred in the sense that the meromorphic function must be rational, so a discrete, countable, number of matter-associated events must occur. Also note that we have:

$$|\boldsymbol{h^{*}}| = \boldsymbol{6.630 \times 10^{-33}} \quad \boldsymbol{vs} \quad \boldsymbol{Plank's\ constant\ \ h = 6.626070 \times 10^{-34} J\ s}$$

where we only need these two ‘h’ numbers to satisfy the same extreme smallness property in order to obtain integrals with large parameter and thus *a highly oscillatory integral with stationary phase domination*.

Achieving Dimensionality (e.g., physical units)

The next issue is how to dress up the key parameters with dimensionful units, to arrive at the standard physics formulations, when the original formalism is purely algebraic, albeit with dimensionless constants already found to exist (α). This will be accomplished by Wick rotation from integration on real terms to integration on imaginary phase contributions. In making this analytic continuation we introduce units via transforming a dimensionless ratio of dimensionless numbers to a dimensionless ratio of *dimensionful* numbers. We also go from summing on zero-divisor associated terms to summing on zero-divisor associated ‘paths’. The summations on path add according to their phase, the latter dependent on the action expressed as

$$\boldsymbol{S} = \int \boldsymbol{L}\boldsymbol{d}\boldsymbol{t}$$

where time emerges as the parameterization of the path and the Lagrangian is that indicated by the Standard Model (plus Hilbert General Relativity term to be discussed further in the Discussion). Analyticity on this integral (and all integrals encountered thus far), in the form of the Wick rotation especially, is encompassed by the description of complex time Euclideanization in later sections. Note, this describes a doubly analytic structure (at level of emanator and at level of propagator). Since the Wick rotation on the trigintaduonion (32D) objects represents use of an analytic complex structure to extend each of the real components to complex components, we have an analytic extension off of the 32D Cayley algebra into the enveloping 64D Cayley algebra. This is best seen in terms of the well-defined exponential map described earlier where $T \rightarrow C \times T$ by means of $\exp(iT)=\cos\theta+iT\sin\theta$. Another way to view the analytic structure is not as an added structure but the residual structure of the hypercomplex selection for maximal transmission settling to the highest order Cayley ‘sub-algebra’ it can manage. As such, the emanation process that arrived at the 32D Cayley algebra did so in a context where an analytic 64D Cayley algebra extension already existed (albeit briefly).

For a zero-divisor to occur with the T_{base} the real component must be zero, but this is possible for the base trigintaduonion in the Emanator. (The number of emanation steps to a zero-crossing event, with random-walk statistics on the real component, is revealed in [22]).

Results

$\mathbf{Emanation(T)} \cong \mathbf{O} \times \mathbf{H}$

The simplest emanation, at the (summed over) chiral level, is a mixture of four forms:

$$T_{chiral}^{(k)} = \begin{cases} ((0,\alpha),\beta) \\ ((\alpha,0),\beta) \\ (\beta,(0,\alpha)) \\ (\beta,(\alpha,0)) \end{cases}.$$

Consider the chiral emanation $((0,\alpha),\beta)$ if repeated in right operation on **T,** with $\alpha = 0$ and $\beta = 0$ (not an allowed degenerate case in emanation sum):

$$\Big(\big(\mathbf{T}\bullet((\mathbf{O},\alpha),\beta)\big)\bullet((\mathbf{O},\alpha),\beta)\bullet\bullet\bullet\Big)$$

becomes

$$\Big(\big(\mathbf{T}\bullet((\mathbf{O},0),(0{,}0))\big)\bullet((\mathbf{O},0),(0{,}0))\bullet\bullet\bullet\Big)$$

where the octonion product with $\mathbf{O}$ is carried directly to component level in $\mathbf{T}$. Let T=((x,y),(u,v)), then get T=((((x$\mathbf{O}$)$\mathbf{O}\ldots\mathbf{O}$), ((y$\mathbf{O}^*$)$\mathbf{O}^*\ldots\mathbf{O}^*$)), (((u$\mathbf{O}^*$)$\mathbf{O}^*\ldots\mathbf{O}^*$), ((v$\mathbf{O}$)$\mathbf{O}\ldots\mathbf{O}$))). Thus, have an octonion right product, repeated in each of the four octonions in the T, giving the action of the T in this case to be like that of a common octonion right product. A sequence of octonion products produces a SU(3) algebra, and SU(3)xU(1) with suitable normalization [1,2]. If we now consider the actual chiral form $((0,\alpha),\beta)$ without $\alpha = 0$ and $\beta = 0$, is it possible to show an additional $SU(2)_L$ (and $SU(2)_R$) algebraic product to arrive at the Standard Model (as well as a possible $SU(2)_R$ extension)? The answer is Yes. And this is easy to show because the main result derives from the property that elements of the algebra C × H x O, when taken as a repeated product, have a associative product group symmetry that is that of the Standard Model: U(1) × $SU(2)_L$ × SU(3) [2].

In the noise perturbation analysis we already generalized to complex noise on the basis that later applications would involve sums over trigintaduonion phase terms consisting of exponentiated iT_{em}. Fortunately the exp map is well-defined for hypercomplex number generalizations [2] and, via the generalized De Moivre relation, expresses a clear 'lift' from a T number to a C × T number. What remains is to show that the particular form of T_{em}chosen is manifestly $\mathbf{O}\,\mathbf{x}\,\mathbf{H}$. As suggested earlier, this is the case when the emanator sum has sub-chiralities generated from a common octonion. This leads to expressing $T_{em} = (O_{em}a, O_{em}b, O_{em}c, O_{em}d) = O_{em} \times H$, where $H = (a,b,c,d)$.

Meromorphic Matter

The success of the 29* factoring argument (getting an estimate of α good to 10 decimal places) suggests that such a factoring of the emanation process is valid in certain applications. Let's consider such a factoring together with addition of complex structure and examine the local analytic domains that result (29* of them). Why would we do this? Well suppose your algebraic formalism has a critical weakness, such as the existence of zero divisors. How might a maximum information theory go about repairing the situation? In a particular complex dimension we could define a fundamental "log T" theory, where the singular zero must be removed, with resulting residue… and this is what is done, with some interesting results in what follows.

Emanation when base trigintaduonion contains Zero Divisors (previously shown in [30])

Consider emanation when the base trigintaduonion is pure imaginary, and thus a potential zero-divisor $\mathbf{T}_{ZD}$:

$$\text{Emanation}(\mathbf{T}_{ZD}) = \frac{1}{N} \sum_{k \in \{4x72\}^n} \mathbf{T}_{ZD} \bullet \boldsymbol{T}_{chiral}^{(k)},$$

and suppose the number n (like the number of cards in 'flop' to make a reading) is such that $\{4x72\}^n$ is large, such that the sum on trigintaduonion products is dominated by stationary phase terms. Such domination by stationary phase is expected with appropriate handling on the normalization, even without zero real component and unit norm, since we have phase addition on a compact space, the 31-sphere. We now have a new mechanism driving the stationary phase solution, however, due to the existence of zero divisors, for which a new type of solution class is indicated. Suppose stationary phase in this context selects such that:

$$\text{Emanation}(\mathbf{T}_{ZD}) = \frac{1}{N} \mathbf{T}_{ZD} \bullet (\mathrm{R} + \mathbf{T}_{ZD*}) = \mathbf{T}_{ZD}, \qquad \Delta\mathbf{T}_{base} = \mathbf{0}$$

where $\mathbf{T}_{ZD*}$ is the zd conjugate to $\mathbf{T}_{ZD}$, i.e. $\mathbf{T}_{ZD} \bullet \mathbf{T}_{ZD*} = \mathbf{0},$ and N is the appropriate normalization constant to arrive at unit norm as before. Since $\Delta\mathbf{T}_{base} = \mathbf{0}$, in the emanation process it is unchanging, thus this is the condition that will relate to the classic equilibrium (or quantum stationarity).

Let's now consider the $\mathbf{T}_{base}$ that consists of a sum over a countable collection of zero divisors with separate weighting factors:

$$\mathbf{T}_{base} = \sum_{i \,\epsilon\, all} a_i \mathbf{T}_{ZD,i}$$

Suppose stationary phase in this context selects such that:

$$\text{Emanation}(\mathbf{T}_{\boldsymbol{base}}) = \frac{1}{N}\mathbf{T}_{\boldsymbol{base}}\bullet \sum_{i\,\epsilon\,all} \mathbf{T}_{\boldsymbol{ZD*,i}} \bullet (\mathbf{T}_{\boldsymbol{ZD*,i}})^{-\mathbf{1}} = \mathbf{T}_{\boldsymbol{base}}$$

where the order of 3-T multiplications is with the inverse last, and where an overall constant is eliminated by the renormalization term to arrive back at the starting base trigintaduonion. This appears to be the general condition for describing the emanation form of equilibrium.

Let’s now consider what happens if the real component is nonzero as well (and assume Z ZD’s):

$$\mathbf{T}_{\boldsymbol{base}} = \boldsymbol{R} + \sum_{i\,\epsilon\,all} a_i \mathbf{T}_{\boldsymbol{ZD,i}}$$

$$\text{Emanation}(\mathbf{T}_{\boldsymbol{base}}) = \frac{1}{N}(\boldsymbol{R} + \sum_{i\,\epsilon\,all} a_i \mathbf{T}_{\boldsymbol{ZD,i}})\bullet \sum_{i\,\epsilon\,all} \mathbf{T}_{\boldsymbol{ZD*,i}} \bullet (\mathbf{T}_{\boldsymbol{ZD*,i}})^{-\mathbf{1}}$$

$$\text{Em}(\mathbf{T}_{\boldsymbol{base}}) = \frac{1}{N}\left(Z\boldsymbol{R} + \sum_{i\,\epsilon\,all} (Z-1)a_i \mathbf{T}_{\boldsymbol{ZD,i}}\right) = \frac{\mathbf{1}}{\mathbf{N}}(\mathbf{R} + (\mathbf{Z}-\mathbf{1})\mathbf{T}_{\boldsymbol{base}}) \cong \mathbf{T}_{\boldsymbol{base}}$$

with a slight overall increase in the real component, and notably retaining all of the ZD’s.

Let’s now consider the general case where ZD’s are indicated as a separate portion (and assume Z ZD’s):

$$\mathbf{T}_{\boldsymbol{base}} = (\boldsymbol{R} + \boldsymbol{T}_{\boldsymbol{imag}}) + \sum_{i\,\epsilon\,all} a_i \mathbf{T}_{\boldsymbol{ZD,i}}$$

and

$$\text{Em}(\mathbf{T}_{\boldsymbol{base}}) = \frac{\mathbf{1}}{\mathbf{N}}((\boldsymbol{R} + \boldsymbol{T}_{\boldsymbol{imag}}) + (\mathbf{Z}-\mathbf{1})\mathbf{T}_{\boldsymbol{base}}) \cong \mathbf{T}_{\boldsymbol{base}}$$

with a slight overall increase in the non-ZD part while still notably retaining all of the ZD’s. There is thus conservation of ZD’s, suggesting association of ZD’s with matter/energy and the conservation of the latter seen in the emanated propagator formalism. The nature of this matter association is still unclear, however, until we consider the next condition on the emanator.

Let's now consider the form of the emanator when it is summed into the 4 chiralities (with 78 or 72 card decks dependent on form):

$$\text{Emanation}(\mathbf{T}_{\boldsymbol{base}}) = \frac{1}{N} \sum_{K \in 4\ chiralities} \mathbf{T}_{\boldsymbol{base}} \bullet \overline{\boldsymbol{T}}^{(\boldsymbol{K})}_{\boldsymbol{chiral}}$$

and, thus

$$\mathbf{T}_{\boldsymbol{base}} \bullet \overline{\boldsymbol{T}}^{(\boldsymbol{K})}_{\boldsymbol{chiral}} \neq \mathbf{0}$$

In this context the zero divisors in the base force an unexpected constraint if we require that no elimination of chirality (thus violation of emanator achirality) can occur. In other words, we hypothesize the emanation is constrained such that it is analytic on the log of the products such that zero's are eliminated from the maximal analytic domain.

On the other hand, suppose the form of the emanator can be written as a sum on achiral groups. Such groups *can* be zeroed-out, which describes a form of wave-collapse or measurement filter for the theory:

$$\text{Emanation}(\mathbf{T}_{\boldsymbol{base}}) = \frac{1}{N} \sum_{K \in achiral\ group} \mathbf{T}_{\boldsymbol{base}} \bullet \overline{\boldsymbol{T}}^{(\boldsymbol{K})}_{\boldsymbol{achiral}}$$

and, thus, we can have:

$$\mathbf{T}_{\boldsymbol{base}} \bullet \overline{\boldsymbol{T}}^{(\boldsymbol{K})}_{\boldsymbol{achiral}} = \mathbf{0}.$$

From the preceding results we then see that we can formulate a hypothesis for the meromorphic precipitation of quantum matter with dimensionful action, where:

(1) The trigintaduonion emanator is doubly analytic, where the first analyticity is in regards to removing the zero-divisors from the domain of the trigintaduonion emanator by means of analytic operations to remove the zero-event for each of the effective dimensions, giving rise to a dimensionless 'action' $\boldsymbol{S}^*$ and a quantum of that action given by:

$$|\boldsymbol{h}^*| = \left(\frac{\mathbf{1}}{\mathbf{2}\boldsymbol{\pi m}}\right)^{\mathbf{29}^*}, where\ \boldsymbol{m} = \mathbf{2}.$$

While the second analyticity is in regards to the resulting sum on associated zero-divisor paths. Upon analytic operation (Wick rotation) we arrive at a sum on paths whose phase is given by a dimensionful action with respect to a dimensionful quantum of action (Planck's constant):

$$\boldsymbol{S}^*/\boldsymbol{h}^* \rightarrow \boldsymbol{S}/\boldsymbol{h}$$

(2) We arrive at large-parameter integral over paths, that is highly oscillatory given $|\boldsymbol{h}^*| \approx |\boldsymbol{h}| \ll \propto < \mathbf{1}$, and it must satisfy the quantum deFinetti relation [32], to give rise to a real action, with:

$$\boldsymbol{S} = \int \boldsymbol{Ldt}$$

where the real-valued Lagrangian is selected to be consistent with the standard model (plus small extension) indicated in the prior section.

Thus, the dimensionless quantum arises from analyticity in the form of a meromorphic function association to each of the 29D in a given chiral propagation, where associated zero divisor (ZD) surgery gives h*<<α<1 since each ZD has 29 real component dimensions (plus a remnant of imaginary dimensionality, thus effective dim=29* [23]), and where a point-like location is given by the location of the cut-out.

The dimensionful quantum arises from Wick rotation from real to pure imaginary (with ZD cut-outs) such that (S*/h*) with discrete time steps 'n' Wick rotates to S/h with dimensionful time 't'. The exact numerical relation h* →|h| may be a truly random emergence that will never be defined further (simply part of the 22 parameters). The main constraint, which is satisfied, is that the quantum be very small, giving rise to an oscillatory integral formalism. A shift in the small constant can't be explained further with the current development of the theory.

Why 22 parameters?

The Trigintaduonion emanation hypothesis strongly indicated the possibility of only 22 parameters in early efforts [22]. Here we see a clearer mathematical argument for this being the case. Consider the emanator for the excluded case of a pure real common octonion δ:

$$T_{chiral}^{(k)} = \begin{cases} ((\delta, \alpha), \beta) \\ ((\alpha, \delta), \beta) \\ (\beta, (\delta, \alpha)) \\ (\beta, (\alpha, \delta)) \end{cases}.$$

We get no sub-octonionic mixing for this type of emanation: $\mathbf{T_{em,}} = \boldsymbol{O} \times \boldsymbol{H}$, where $\boldsymbol{O} = \mathbf{1}$. Such an emanation, acting on T_{base}, in order to remain achiral, will change nothing vis-a-vis future achiral T_{em} products, thus it is associated with a "constant of the emanation". How many such emanations, effecting no change, are there? They correspond to the non-octonion based fluctuations in the evaluation of the 78 deck described previously, of which there are 22.

Discussion

Standard Model with possible $SU(2)_R$ extension is consistent with 22 parameters

The emanator sum is achiral, but is composed of a sum over chiral T emanators. This collection of chiral emanators, if seeded with a common octonion, with positive and negative fluctuations in each component, leads to a trigintaduonion emanator that has the form $T_{em} \cong O \times H$. As seen in the "sum on phase" analysis naturally indicated in the zero-divisor curing, we will soon consider the properties of the mathematical object of $\exp(iT_{em})$, which then explicitly promotes the theory to be $C \times H \times O$. According to Dixon [2], it is then possible to obtain an action on the T_{base} space that is precisely the $U(1)\times SU(2)\times SU(3)$ form desired. This then leads to light matter with maximal species of particles (thus generations), acted on via $U(1)\times SU(2)_L\times SU(3)$, and dark matter with minimal species (sterile neutrinos) acting only via $SU(2)_R$.

From 22-parameter hypothesis, with maximal info transmission, it is apparent that we will get three generation results for the maximal number of interacting particles in one sector (that we will call the "Light' sector accordingly since it has Light, e.g., U(1)), with the remainder left to the "Dark" sector, with only $SU(2)_R$, the sterile neutrino. The reason for light/dark asymmetry is simple, it allows for the maximal complexity of information transmission. Suppose the number of particles in the Light sector is L and that in the Dark is D, then the number of binary interactions is L^2+D^2. Given L+D=constant=C, we find the maxima to occur at $\{L_{min}. R_{max}\}$ or $\{L_{max}, R_{min}\}$. The convention is adopted that what we call "Light" matter is the matter that is most interactive, thus we have the labeling $\{L_{max}, R_{min}\}$.

Trigintaduonion emanation theory indicates 22 free parameters with maximum perturbation amount α in the larger 32D trigintaduonion algebra. In the analysis of the possible emanators *analyticity* is indicated in numerous ways, such that this is a core hypothesis for the maximal information propagating solution. This, in turn, indicates analytic surgery via the residue theorem, on the log of the emanator, to create a maximal analytic region. When we Wick rotate from $\boldsymbol{S}^*/\boldsymbol{h}^* \rightarrow \boldsymbol{S}/\boldsymbol{h}$, there should be 22 independent parameters in the action $\boldsymbol{S}$ [22], with Planck's constant counted separately. Can we fit the parameters of the Standard Model, with a possible extension for the dark matter mentioned (e.g., sterile neutrinos), and the gravitational constant G, all into that 22 count? Yes, if we adopt the Koide relation [33]. Let's show this by first listing the 19 parameters in the Standard Model:

(I) 9 Yukawa coupling constants (masses) for the charged fermions

(II) 5 constants for Weinberg Angle and the CKM matrix (with three mixing angles and CP-violating phase)
(III) 3 Constants for electromagnetic coupling (α), for strong interaction (g3), and strong CP-violating phase ($\theta_3 \approx 0$).
(IV) 2 Higgs parameters: Mass and Vacuum Expectation

If we allow for the neutrinos to have mass, then we get 3 more masses and another 4 constants for the PMNS matrix (three mixing angles and a CP-violating phase):

(V) Extended model: 7 more constants → We, thus, have 26 parameters.

If we add the constant for Gravitation (G) to have all constants for Std. Model + Gravitation, we now have 27 parameters. Note, however, that the α constant is listed above as the EM coupling constant, but isn't really a separate parameter since it is the same for any emergent chiral trigintaduonion emanation. This is all the more apparent if we go with a listing of 19 independent parameters in terms of the g_1 and g_2 coupling constants which share the following relation with α:

$$\alpha = \frac{1}{4\pi}\frac{{g_1}^2{g_2}^2}{{g_1}^2 + {g_2}^2}$$

So, we take the separate (double) count on α away from the count to get to 26.

The Koide relation [33] was first observed for the three massive leptons currently known:

$$\frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = \frac{2}{3}\ almost\ exactly.$$

To a lesser extend this relation is satisfied for the quarks as well, particularly for the three most massive, where the value is 0.6695. The problem with a simple application to the quark masses is that they are dependent on energy scale. A theoretical explanation for the Koide relation describes how this relation might exist for the masses of a given generation (or family group) [34]. Assuming this or some other theoretical explanation can show that the three masses of a given generation aren't truly three independent parameters, but two. With this correction on 4 generation of masses (now counting the sterile neutrino generation), we arrive at 26-4=22 free parameters as desired., and the emanator theory thus indicates a nearly complete theory in that the 22 parameters are almost all known.

Note that the fine-structure constant α and Planck's constant have very different trigintaduonion emanation origins and uniqueness: α derives from T-emanation directly, without reference to zero divisors, is dimensionless, and is precisely defined. Also, α is one of the 22 fundamental parameters of the trigintaduonion emanation process. Planck's constant, on the other hand, is not one of the 22, arising from the meromorphic matter description instead. Also, Planck's constant is only specified to have an essentially small quantum to establish an oscillatory integral with $h^* << \alpha$, and derives from T-emanation when zero divisors are accounted for by way of maximal analyticity.

Matter appears in emanator theory as a meromorphic residue of a zero-divisor

We see matter as meromorphic residue precipitation, in amounts of one quantum given by a precursor to Planck's constant h*. The meromorphic residue winding number is also notable in that it gives an integer that stays constant in the meromorphic region. This raises the possibility that elementary particle attributes might encode by way of different winding numbers (as with some string theories), with reference to their different winding numbers at residues, but this will not be discussed further here.

What is the precise form of emanation with possible multi-card 'hands'?

So far have a deck with 72 or 78 cards when considering 'dealing a hand' of a particular size for a particular order emanation (order=number of em-steps=size of 'hand'). When we sum over the one-step emanations we have a very close match with α. When we sum over multistep emanations this will potentially be improved further. What is not apparent is what the maximal size hand is (if any) that is dealt in the emanator sum variant that considers all sizes of hands? (without replacement we theoretically have 78). The terminology of card and hand (or spread or flop) is not chosen arbitrarily since there is a precise match-up with the size of the tarot deck (78) and its decomposition via four suits (chiralities), etc. What is the maximal card flop in tarot spreads? Generally it is less than 10 or 11. Perhaps this should be taken as a hint of what will eventually be shown mathematically to be the case (for maximal information transmission). It might be that the maximal for size (hand) is dictated by $\alpha_0^{-1} + \alpha_1^{-1} + \cdots$ with upper bound, α^{-1}, given by {α, π} relation. In this way we know the size, composition of the deck, and how many cards are dealt in a (maximal) hand. So … not only does god play dice with the universe, but his wife plays cards…

Where's the geometry?

So far we have an explanation of standard model and h and α, but no clear explanation of G and no better understanding of gravitation. Trigintaduonion emanation shows the standard model as a direct outcome, thus a higher temperature GUT theory for unification is not required (although may still be a fundamental stage, possibly with echoes imprinted on the cosmological data and on boundaries). A theory consistent with avoidance of GUT theory and the need for inflation, is where we get inflation from conformal bounce having the same conditions. The starting point of this theory is noting that the early (Big Bang) universe and the entropy-death universe both lose all matter and thus all matter scale, and become conformally invariant. Roger Penrose has pointed this out on a number of occasions with his conformal bounce hypothesis (Conformal Cyclic Cosmology) for the early Big Bang [35-37]. Penrose makes an excellent point about the oddity of geometry not equilibrated when matter/radiation is equilibrated. This is explained in a cyclic universe when conformal death leads to conformal birth (big bang), because we can get a perfect start like inflation without inflation. Once again, however, geometry and matter/radiation, although influencing each other, appear to enter the formalism in fundamentally different ways.

With the emanation theory we have the standard model Lie algebra acting on a space (possibly creating that 'information-space' Prolog-style merely thru consistent accessing), here the algebra acting on the unit norm trigintaduonion 'base' and, over time, arrives at an 'average' of sorts of all the T_{chiral} shifts that have acted upon it, all at the 'edge of chaos', such that an emergent manifold construct appears, providing geometry (and entropy via neuromanifold constructs [38]). The role of string theory, via holographic hypothesis (ADS-CFT relation [39]), may be critical to evaluation of such complex boundary conditions, such as with black hole thermodynamics and big bounce cosmology 'boundaries'.

The geometry side of emanation theory does not result from the action of the repeated emanator product directly, but from the accumulated product in the T base that results. Geometry is, in effect, emergent (projected) on the T_{base} 'space' of the T_{em} product action. Geometry appears as a manifold construct in both space-time curvature (where it is locally given with the standard model action) and as an intrinsic entropic property, via neuromanifold 'geodesic' motion being equivalent to, and possibly the origin of, the minimization of the relative entropy (and maximization of entropy, the 2nd Law) [38]. Setting aside thermodynamic issues in this discussion (more details in forthcoming book [40]), this puts the Lagrangian formulation with standard model terms, and Hilbert action for GR, into better perspective. The representation of the geometry via the Hilbert action for GR suffices with maximal extension in whatever causally connected domain of interest. So, we've got the existing QFT in CST space-time formulations in the black hole exterior, for example. We have the resolution at the black hole horizon causal boundary via 1st Quantization String Theory on the surface (using Ads/CFT relation and related holographic hypothesis [39,41]). Suppose an evaporating black hole, where horizon curvature becomes high towards end, and along with radiative back-reaction get breakdown in manifold structure at the boundary. At this point we might also consider such a manifold breakdown on both space boundary plus the space itself (in a cosmological setting). In these cases we may arrive at a general associative construct, e.g. a Clifford algebra, and have a theory based on Cl(32) $\times$ T(32), mathematical properties of such a mathematical object are examined in [10].

We describe repeated chiral product action on the trigintaduonion spinor space. The emanation process, consisting of a chain of chiral trigintaduonion products, leads to a Lagrangian variational formalism *with the standard model*. The origins of the parameters of the model are beginning to be understood as well. Apparently state information memory/inertia is carried via the manifold curvature response to the matter density, where 'G' is the linkage for the balance on this 'learning' process. Presumably the G learning rate is set for optimal learning, e.g., maximal information flow, and as such its value may eventually be clarified theoretically, but that must await another paper.

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