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Cosmological Exact Solution of Petrov Type D of a Non-Disrupted Primordial Magnetic Field and a Mixture of Dark Energy and a Nonlinear Fluid that Becomes into Radiation

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Abstract

In this paper, an exact cosmological solution is obtained from Einstein's equations in an anisotropic and homogeneous symmetry of Petrov D. A fluid is studied and is a result of a mixture of dark energy and a nonlinear fluid that tends to transform into a fluid of radiation due to time increment and the presence of a primordial magnetic field. The primordial magnetic field does not induce currents either electric fields. It is stated that the solution tends to be isotropic as time increases and equal to the dark energy solution for FRWL. It is determined that an initial singularity exists when $t = 0$. The temperature of the fluid mixture is studied and it is established that time dependence of such magnitude is because of a nonlinear fluid and a magnetic field presence that occurs due to its relation with it. Parameters of Hubble and deceleration, and their behaviors in time in that solution are also studied.

Keywords: cosmology, exact solution, Einstein, magnetic field, nonlinear, Kretschmann, singularity, fluid mixture

1 Introduction

Interest in cosmology has increased mainly because of the findings related to microwave background radiation obtained by COBE, WMAP, and Planck

satellites, universe acceleration and its possible scenarios (states) that it could have gone through. In this regard, other aspects in cosmology where its literature is based have been discussed in [1].
Another aspect of great interest in cosmology is the problem of the existence of cosmic magnetic fields in the interstellar and intergalactic medium. Regarding this matter, some fluid solutions with equations of state of linear and nonlinear fluids between pressure and energy density have been discussed and analyzed. Some examples with a primordial magnetic field in a Petrov D symmetry are [2] and [3] that include literature related to that concern. A mixture of a dark energy fluid and a nonlinear one that evolves into a fluid of radiation, both in the presence of a primordial magnetic field, are studied and analyzed in this work.

2 Symmetry, Einstein's Tensor, Electromagnetic Field and Dark Energy

2.1 Symmetry and Einstein's Tensor

The anisotropic symmetry of Petrov Type D has been considered in [1] as the form

$$ds^2 = F dt^2 - t^{2/3} K (dx^2 + dy^2) - \frac{t^{2/3}}{K^2} dz^2, \tag{1}$$

where F and K are functions of t.
Einstein's tensor components ($G_\alpha^\beta = R_\alpha^\beta - \delta_\alpha^\beta R/2$) different from zero, of (1) are

$$G_0^0 = \frac{4\,K^2 - 9\,t^2 \dot{K}^2}{12 t^2 K^2 F}, \tag{2}$$

$$G_1^1 = -\frac{3Kt\dot{K}\left(2F - \dot{F}t\right) + 3Ft^2\left(2K\ddot{K} - 5\dot{K}^2\right) + 4K^2\left(\dot{F}t + F\right)}{12t^2K^2F^2}, \tag{3}$$

$$G_2^2 = G_1^1 = -\frac{G_3^3}{2} + \frac{9\,Ft^2\dot{K}^2 - 4\,K^2\dot{F}\,t - 4\,K^2 F}{8t^2K^2F^2}, \tag{4}$$

where the points over these functions represent derivatives by time.

2.2 Magnetic Field

The magnetic field will be considered in a matter where the only tensor components of an electromagnetic field $F_{\mu\nu}$ different from zero are $F_{12} =$

$-F_{21} = B_{0z} = const$ where the invariant $F_{\mu\nu}F^{\mu\nu} = 2B(t)^2$, wherein $B(t) = B_{0z}\pi^{1/2}/(t^{2/3}K)$ is the effective-magnetic-field magnitude. The effective magnetic field does not generate currents either induce electric fields since its flow Φ does not change in relation to time

$$d\Phi = B(t)dA(t) = B(t)\sqrt{g_{11}g_{22}}dxdy = B_{0z}\pi^{1/2}dxdy. \tag{5}$$

The choice of the field, in this given form, allows to meet the field equation criteria $F^{\mu\nu}_{;\mu} = 0$ besides the equality to zero of divergence of the energy-momentum tensor of the magnetic field ${}_{em}T^{\mu\nu}_{;\mu} = 0$. The tensor ${}_{em}T^{\mu\nu}$ is the energy-momentum tensor of the magnetic field of which unique components different from zero for ${}_{em}T^{\mu}_{\nu}$ are

$${}_{em}T^0_0 = -{}_{em}T^1_1 = -{}_{em}T^2_2 ={}_{em} T^3_3 = \frac{{B_{0z}}^2}{8\,t^{4/3}K^2}. \tag{6}$$

2.3 Ideal Fluid Model

The ideal fluid model can be considered as a fluid of which energy-momentum tensor has the form

$${}_{fl}T_{\alpha\beta} = (\mu + P)\,u_\alpha u_\beta - g_{\alpha\beta}P, \tag{7}$$

where ${}_{fl}T_{\alpha\beta}$ is the energy-momentum tensor of a perfect fluid, u_α the tetradimensional speed, $g_{\alpha\beta}$ the metric tensor, μ and P the energy density and the pressure of the fluid respectively.
A fluid with a tetradimensional speed will be considered as $u_\alpha = (u_0, 0, 0, 0)$. Hence, the energy-momentum-tensor components (7) different from zero are ${}_{fl}T^0_0 = \mu$, ${}_{fl}T^1_1 ={}_{fl} T^2_2 ={}_{fl} T^3_3 = -P$. In this work, it is assumed that $\mu = \mu_{DE} + \mu_{nl} = \Lambda + \mu_{nl}$ and $P = P_{DE} + P_{nl} = -\Lambda + P_{nl}$ where Λ is a constant that determines dark energy, so it complies with the following criteria

$${}_{fl}T^{\mu\nu}_{;\mu} = 0,\ and\ {}_{fl}T^{\mu\nu} ={}_{DE} T^{\mu\nu} +_{nl} T^{\mu\nu} \tag{8}$$

3 Einstein's Equations and the Solution of the Magnetic Field Model and the Fluid

Einstein's equations have the form $G^\beta_\alpha = \kappa T^\beta_\alpha$ where $T^\beta_\alpha = {}_{em}T^\beta_\alpha + {}_{fl}T^\beta_\alpha$. From (2-4, 6, 7) the next system of equations mutually independent is obtained

$$\frac{4\,K^2 - 9\,t^2\dot{K}^2}{12t^2K^2F} - \frac{{B_{0z}}^2 + 8\,\mu\ \ t^{4/3}K^2}{8t^{4/3}K^2} = 0, \tag{9}$$

$$G^1_1 + \frac{t^{2/3}{B_{0z}}^2 + 8\,P\ \ t^2K^2}{8K^2\,t^2} = 0, \tag{10}$$

$$G_3^3 + \frac{-t^{2/3}{B_{0z}}^2 + 8\,P\;\; t^2 K^2}{8K^2\,t^2} = 0, \tag{11}$$

From the equation (9), it is determined that

$$F = \frac{2\left(4\,K^2 - 9\,t^2\dot{K}^2\right)}{3t^{2/3}\left({B_{0z}}^2 + 8\,t^{4/3}\mu\;\;K^2\right)}. \tag{12}$$

Bearing in mind (8) and (12) in (10) and (11), both equations are satisfied if

$$\begin{aligned}\left(-27t^3\dot{K}^3 + 48K^2t\dot{K} + 36\ddot{K}K^2t^2 - 16K^3\right){B_{0z}}^2 +\\ +72t^{4/3}K^2\mu\left(-9\,t^3\dot{K}^3 - 4t^2K\dot{K}^2 + 4\ddot{K}\,K^2t^2 + 8\,K^2t\dot{K}\right) +\\ +36K^2t^{10/3}\left(4K^2\dot{K} - 9\dot{K}^3t^2\right)\dot{\mu} = 0.\end{aligned} \tag{13}$$

The fluid mixture of dark energy ($\mu_{DE} = \Lambda$) and a nonlinear fluid μ_{nl} has an energy density $\mu = \Lambda + \mu_{nl}$ (for more information about μ_{DE} see [1]). The equation (13), if $K \neq const$, it can be expressed as the form

$$\begin{aligned}4K^2t^{4/3}\left(2\,\mu\left(W\dot{K}\left(9\,t\dot{K} + 4\,K\right) - 2\,K^2\dot{W}\right) + 9\,W\dot{K}^2t^2\dot{\mu}\right) +\\ +\left(3\,t\dot{K}^2W - 2\,K^2\dot{W}\right){B_{0z}}^2 = 0,\end{aligned} \tag{14}$$

where $W = 4\,K^2 - 9\,\dot{K}^2t^2$. Two particular solutions of that equation can be obtained if $W = 0$ which represent Kasner's vacuum E_{D_1} and flat space-time solutions E_{D_0} (see [1])
The search for the solution of the equation (14) considering a dark energy mixture with a nonlinear fluid that declines as time increases can be conducted taking into account that

$$W = \sum_{n=-\infty}^{\infty} a_n K^{n/2},\ and\ \sum_{n=-\infty}^{\infty} a_n = 4, \tag{15}$$

the particular case when $W = a_{-4}K^{-2} + a_0$ and considering $a_{-4} = -4$ and $a_0 = 8$ results in equations (15), (14) and $\mu = \Lambda + \mu_{nl}$

$$K = t^{-2/3}\sqrt{t^{4/3} + C_{int}}, \tag{16}$$

wherein C_{int} is a constant of integration that will be used similarly as $C_{int} = \frac{{B_{0z}}^2}{4\Lambda}$, so

$$\mu_{nl} = \left(\left(\frac{\pi}{2} - \arctan(2\,\frac{t^{2/3}\sqrt{\Lambda}}{|B_{0z}|})\right)\frac{\Lambda^{3/2}}{|B_{0z}|} - \frac{\Lambda^2 t^{2/3}}{{B_{0z}}^2 + 2\,\Lambda\,t^{4/3}}\right)\left(\frac{{B_{0z}}^2}{2\Lambda t^{2/3}} + t^{2/3}\right). \tag{17}$$

The solution (17) for values of $B_{0z} \to 0$ or $t \to \infty$ approximates to $\mu_{nl} \simeq 5B_{0z}^2/(24t^{4/3})$ which exemplifies the energy density of a fluid of radiation (see [1]).

From (17), (16) and (12), F form is

$$F = \frac{16\left(4\,\Lambda\,t^{4/3} + {B_{0z}}^2\right)^{-1}}{3\left(2\,t^{2/3} + 2\,\sqrt{\Lambda}\left(t^{4/3} + \frac{{B_{0z}}^2}{4\Lambda}\right)\frac{\left(\pi - 2\,\arctan\left(2\,\frac{t^{2/3}\sqrt{\Lambda}}{|B_{0z}|}\right)\right)}{|B_{0z}|}\right)}, \tag{18}$$

for values of $B_{0z} \to 0$ or $t \to \infty$, it is close to $F \simeq \frac{1}{3\Lambda t^2} - \frac{B_{0z}^2}{9\Lambda^2 t^{10/3}}$ where the term $\frac{1}{3\Lambda t^2}$ determines a dark energy fluid for an isotropic and homogeneous solution of FRWL type. The term $-\frac{B_{0z}^2}{9\Lambda^2 t^{10/3}}$ is responsible for the anisotropy of the solution and represents the fluid of radiation.

Pressure has the form $P = -\Lambda + P_{nl}$ where

$$\begin{aligned} P_{nl} = \frac{\Lambda\,\left(7\,{B_{0z}}^2 + 20\,\Lambda\,t^{4/3}\right)}{24\,\Lambda\,t^{4/3} + 6{B_{0z}}^2} + \\ + \frac{\Lambda^{3/2}\left(2\,\arctan\left(2\,\frac{t^{2/3}\sqrt{\Lambda}}{|B_{0z}|} - \pi\right)\right)\left(\frac{{B_{0z}}^2}{2\Lambda} + 5\,t^{4/3}\right)}{6\,|B_{0z}|\,t^{2/3}}, \end{aligned} \tag{19}$$

and for values of $B_{0z} \to 0$ or $t \to \infty$, it approximates to $P_{nl} \simeq 5B_{0z}^2/(72t^{4/3})$ that represents the pressure of a fluid of radiation and that corresponds to what is established for μ_{nl}.

4 Analysis of the Solution

4.1 About μ_{nl}, P_{nl}, F and K

From solutions (17) and (19), the relation $P_{nl}/\mu_{nl} \in [-1/3, 1/3]$, so: a) when $t \to 0$, $P_{nl}/\mu_{nl} \to -1/3$ which represents a fluid of quintessence type, b) when $t \to 0.17037393(\frac{|B_{0z}|}{\sqrt{\Lambda}})^{3/2}$, $P_{nl}/\mu_{nl} \to 0$ that is a fluid of dust type, c) when $t \to \infty$, $P_{nl}/\mu_{nl} \to 1/3$ which typifies a fluid of radiation type, so a nonlinear fluid in different time intervals can represent states of linear fluids that go from a fluid of quintessence type to one of radiation type.

The existence of the magnetic field determines essentially the solutions of metric functions F and K, and the constant of a nonlinear fluid is adjusted to the constant B_{0z} from the magnetic field that is a required condition, so (14) is satisfied. Then, when the magnetic field gets eliminated, the fluid of radiation also disappears, so the solution becomes equal to dark energy for FRWL.

For long-term times ($t \to \infty$) or when $B_{0z} \to 0$, the metric can be written as

$$ds^2 \approx d\eta^2 - e^{2/3\,\sqrt{3\Lambda}\eta}\left(dx^2 + dy^2 + dz^2\right), \tag{20}$$

where $t = t_0 e^{\sqrt{3\Lambda}\eta}$ and it is considered that $t_0 = 1$, (20) is the metric of the solution of dark energy in FRWL space-time.

4.2 Singularities. Kretschmann's Invariant

The analysis of the solution in close proximity to $t = 0$ will be conducted using Kretschmann's invariant. The condition $Krets < \infty$ is required and sufficient for the finitude of all invariants of algebraic curvatures (see [1] and its respective literature). It is defined as $Krets = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$ and has the form for the metric (1),

$$\begin{aligned} Krets = {} & \frac{6\ddot{K}^2}{F^2K^2} + \left(-\frac{18\dot{K}^2}{F^2K^3} + \frac{2(-3\dot{F}t + 4F)\dot{K}}{F^3K^2t}\right)\ddot{K} + \frac{75\dot{K}^4}{4F^2K^4} + \\ & + \frac{(9\dot{F}t - 10F)\dot{K}^3}{F^3K^3t} - \frac{3\dot{F}(-\dot{F}t + 4F)\dot{K}^2}{2K^2F^4t} + \frac{\dot{F}^2}{3F^4t^2} + \frac{8\dot{F}}{9F^3t^3} + \frac{20}{27F^2t^4}, \end{aligned} \tag{21}$$

from (16), (18) and (21), when $t \to 0$,

$$Krets \to \frac{\Lambda\, {B_1}^2 \pi^2}{6t^{4/3}} \to \infty \tag{22}$$

so the solution is singular in $t = 0$. The singularity of (21) is basically due to the presence of a nonlinear fluid (17). When a dark energy fluid and a magnetic field are studied, the singularity is not present in the solution [2].

5 Temperature, Hubble Parameter H and Deceleration q

The temperature for the studied fluid type is defined as [4], so

$$\frac{dP}{\mu + P} = \frac{dT}{T}, \tag{23}$$

where T is the fluid temperature. From the solution (17), (19) and (23) and taking into account that $\mu = \Lambda + \mu_{nl}$ y $P = \Lambda + P_{nl}$, it is known that

$$\begin{aligned} T = {} & 2\frac{T_0\sqrt[3]{t}\Lambda\,\left({B_{0z}}^4 t^{2/3} + 4\,\Lambda\,{B_{0z}}^2 t^2 + 4\,\Lambda^2 t^{10/3}\right)}{3\left({B_{0z}}^2 + 2\,\Lambda\, t^{4/3}\right)\left(4\,\Lambda\, t^{4/3} + {B_{0z}}^2\right)} + \\ & \frac{T_0\sqrt[3]{t}\sqrt{\Lambda}R\,(t)\left(\pi - 2\,\arctan\left(2\,\frac{t^{2/3}\sqrt{\Lambda}}{|B_{0z}|}\right)\right)}{\left({B_{0z}}^2 + 2\,\Lambda\, t^{4/3}\right)^{-1}\left(4\,\Lambda\, t^{4/3} + {B_{0z}}^2\right)(|B_{0z}|)} \end{aligned} \tag{24}$$

wherein T_0 is a constant and $R(t) = 4\,\Lambda\, t^{4/3} {B_{0z}}^4 + {B_{0z}}^6 - 4\,\Lambda^2 t^{8/3} {B_{0z}}^2 - 16\,\Lambda^3 t^4$. From (24), temperature goes to zero when $t \to 0$ and it has the maximum value $T_{max} = 0.269602773 B_{0z}^{3/2} \Lambda^{1/4} T_0$ en $t_{max} = 0.388423425 \left(\frac{B_1}{\sqrt{\Lambda}}\right)^{3/2}$. Temperature for values when $t \to \infty$ or when $B_{0z} \to 0$ tends to $T \to 5 T_0 B_{0z}^2/(18 t^{1/3}) \to 0$. Temperature dependence on time is because of the nonlinear fluid and the magnetic field presence that occurs due to its relation with that fluid.
Parameters of Hubble H and deceleration q are described as [3]

$$H = \frac{\left((g_{11} g_{22} g_{33})^{1/6}\right)^{\cdot}}{\sqrt{g_{00}}(g_{11} g_{22} g_{33})^{1/6}} = \frac{1}{3t\sqrt{F}}. \tag{25}$$

From (18) and (25), Hubble parameter goes to infinity if $t \to 0$ and tends to $H \to \sqrt{\Lambda/3}$ when $t \to \infty$. Then, it corresponds with Hubble parameter for the dark energy fluid. The deceleration parameter q can be defined as

$$q = -(1 + \frac{\dot{H}}{\sqrt{g_{00}} H^2}) = -(1 + \frac{\dot{H}}{\sqrt{F} H^2}) \tag{26}$$

From (25) and (26), the deceleration parameter q goes to $q \to 2$ when $t \to 0$, so in this model, the universe expands initially decelerated at the instant $t_0 = 0.536189935 (B_{0z}/\Lambda^{1/2})^{3/2}$, $q \approx 0$ where there is no acceleration. Then, it transforms continously into a universe in accelerated expansion, so $q \to -1$ when $t \to \infty$.

6 Conclusions

About the solution found in this work, a dark energy fluid and a nonlinear fluid (it starts similarly as a quintessence fluid until it becomes into a radiation type one) have been considered and both under the presence of a primordial magnetic field that does not generate electric currents. It was determined that the solution is singular in $t \to 0$ and that its singularity is due to the presence of a nonlinear fluid. The solution transforms into the solution of dark energy of FRWL as time increases. Also, it was stated that at first the fluid temperature is null and as time increases, it goes to zero and goes through a maximum value T_{max} at a certain time such as t_{max}. Obtained parameters of Hubble (H) and deceleration (q) allow to conclude that H at the beginning when $t \to 0$, ($H \to \infty$) is indefinite and when $t \to \infty$, H tends to a constant value (the same for the dark energy solution). The parameter q initially shows a deceleration process going through a null instant and then, as t, $q \to -1$ increase, the universe accelerates when time increases.

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