A Single Complex Potential for
Gravity and Electromagnetism

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Abstract

The similarity between gravity and static electricity is said to stop at the inverse square law that governs the two, and even that was disputed after the introduction of GR. Some claimed that the form of the law itself is only the result of the thinning of forces over a spherical surface that grows as the square of the radius away from the source. We show here that the similarity is in fact much more fundamental and deep because of the single origin of the two phenomena - namely EM radiation, which must be why gravity too travels at the speed of light. In [1] radiation was visualized as evaporated matter and matter as condensed radiation, since radiation carried electrical as well as mechanical attributes and when condensing in the form of a topological EM soliton, it causes the emergence of the rest of the matter attributes of mass, gravity, charge, intrinsic spin and dipole moment. The inverse square law for both gravity and electricity then appears simply as a direct consequence of momentum conservation in the condensed form [1]. This is a fundamental connection, but nonetheless it leaves one important difference. It is that similar masses attract whereas similar charges repel. However, if we took mass to be an imaginary electric charge, similar masses would repel as for electric charges and a complete similarity is achieved. With gravity fields having such imaginary values, their squares and their force effects remain real. This is then used to construct potential representations for each one of the two phenomena. The usual retarded potential integral is followed to derive a full set of fields for gravity like
Maxwell EM equations- starting from the inverse square law of Newton. These are then combined into a single complex potential. The complex potential idea is old and was first attempted by Einstein himself in his efforts to unify gravity and electromagnetism [5] starting from GR. One possible use of such a step is to help explaining newly observed phenomena in physics like dark matter, dark energy etc. resulting from various observations- especially if we admitted negative masses. The complex representation results in new hitherto unstudied forces that could prove useful in this regard.

Keywords: Maxwell Equations, Gravito-magnetism, Retarded potentials. Gravity and Inertia, unification theories, topological solitons, dark matter, dark energy, negative mass

1 Introduction

After the introduction of GR, physics looked divided into three seemingly unconnected parts. Maxwell equations describing the motion of charges and currents, gravity for the motion of masses with energy, and QM for the small and elementary particles of physics. This wouldn’t be a problem if we didn’t have cases that fall within more than one range of application. Like QM applying to terrestrial bodies, significant EM appearing with masses as a force within huge plasma accumulations in outer space [6], or when some aspects of GR are applied in particle physics at high speeds. For all these reasons and the appearance of unexplained forces like dark matter and dark energy, it becomes essential to have a single frame that relates all these fields. It was probably Einstein (and some of his contemporaries) who first tried this step, and he even attempted a complex version of GR. His argument was that; ‘‘only by going to larger dimensions one can generalize a theory to encompasses another and the complex field is one way to go’’- see also these quotations [5, 9].

‘‘the period 1923 – 1933 Einstein had tried one geometry after the other for the construction of UFT, i.e., Eddington’s affine, Carton’s tele-parallel, Kaluza’s 5 dimensional Riemannian geometry, and finally mixed geometry, a blend of affine geometry and Foerster’s (alias Bach’s) idea of using a metric with a skew-symmetric part. ‘‘

‘‘The physical agent inseparably associated with gravitation is mass, just as the physical entity inseparably associated with electromagnetism is electric charge. If a unified theory of gravity and electromagnetism is desired it becomes necessary to examine these two fundamental entities .... Now the most obvious resemblance is found in the inverse square law of interaction between two masses or two charges. The difference in the manner of their interaction lies in the fact that while two masses necessarily of the same sign attract, two charges of the same sign repel.}
Thus, two charges of the same sign interact in the opposite way as two masses of the same sign. The difference in the manner of interaction and the double sign of the electric charge can be formally brought out if we associated the factor $\sqrt{-1}$ with the electric charge.

... It is seen from (15) and (16) that the vectors $E$ and $H$ are derived from the retarded potentials in the same way as the electric and magnetic vectors of Maxwell’s theory.... In concluding, the author wishes to thank Professors Edington, Einstein, Struik, Vallarta, Z.T. Change and P. Y. Chow for either encouragement or hopeful criticism.’’

In the present contribution we shall start instead with the results in [1] that radiation gives rise to all matter attributes on condensing into matter in the form of EM solitons, which in turn causes the emergence of mass, gravity, charge, and the rest of matter attributes. Since gravity and EM have the same origin, we can subject gravity to the same process that led to the derivation of the phenomenally accurate and immensely useful equations of Maxwell for EM- starting from Coulomb law and the conservation of electric charge [2, 7, 11]. The result of doing this would be a set of equations for gravity that resembles those of electromagnetism (EM) starting from Newton’s law and the conservation of mass. These shall be called the gravity-inertia equations (GI) to identify them from the already known gravito-magnetism (GM) equations derived instead directly from linearized GR [8]. Our path makes use of results from EM and not dependent on the linearized version of GR, see this quote [8];

‘‘A gravito-magnetic field, according to Einstein’s theory of general relativity, arises from moving matter (mass current) just as an ordinary magnetic field arises in Maxwell’s theory from moving charge (electric current). The weak-field linearized theory of general relativity unveils a mathematical structure comparable to the Maxwell equations. This weak-field approximation splits gravitation into components similar to the electric and magnetic fields. In the case of the gravitational field, the source is the mass of the body, whereas in the case of the electromagnetic field, the source is the charge of the particle. Moving the charge particle creates a magnetic field according to Ampere’s law. Analogously, moving the mass creates a mass current which generates a gravito-magnetic field according to Einstein’s general relativity.’’

Well before the introduction of SR, Wiechert [2] in 1900 introduced his retarded potential integral in which he managed to derive all of Maxwell equations for electromagnetism starting from just the static scalar and vector potential in Coulomb’s law, by simply assuming that the forces and potential fields travel at the finite speed of light in vacuum ‘c’. His integral resulted in few additional terms for the force between two bodies when in relative motion. In addition to the usual static forces of Coulomb that depended on the inverse square of the distance, there appeared one term that depended on the inverse square of the distance but also proportional to the velocity normal to the line joining the two interacting charges.
as well. This may be recognized as the usual magnetic field force of a current created by the moving of a charge. In addition- see next section, there appeared a force that depended on the inverse (not inverse squared) of the distance and also on the change in the velocity- that is acceleration. This is the radiation force. Thus, by simply moving the electric charge with charge conservation and assuming a finite speed for the propagation of the forces, a magnetic field appears as well as a radiation field. The appearance of the magnetic field due to motion was later confirmed in Einstein SR in 1905 using a different approach.

Another important observation in this regard is that the static force (Coulomb) term- the first term of (2a) in the next section, while being attenuated in value and taking time to travel at c from the source to the receiver particle (retarded action), the direction of force stayed along the line joining the two interacting particles- That is pointing to the actual/instantaneous position of the charges. This is important in the argument to remove the objections for a finite speed of gravity, as it was known then and now that even a small change in the line of sight of the forces, could throw the stability of planets out of the window- unless the speed of gravity is thousands of times higher than that of light (7x10^6 c, according to Laplace [11]). So, if this finding is applied to gravity, it would amount to us getting energy from the sun for example, fainter, 8 minutes later, but coming from the actual/non-retarded position. This point gives one more incentive to try to treat gravity like electromagnetism- (see quote end of next section). In the next sections we shall first show the working of the retarded potential integral in the Coulomb case then continue to show how we propose to extend it to Newton’s gravity case and discuss the implications.

2 The Retarded Potentials for Coulomb’s Law

The forces between two electrostatic charges are given by Coulomb’s law \( f = K \frac{q_1 q_2}{r^2} \), where \( f \) is force along the separating distance \( r \), \( q \) electric charge, and \( K = 1/4\pi\varepsilon_0 = 8.987 \times 10^9 \text{Nm}^2/\text{C}^2 \) is the electrostatic coupling constant. This and the Newton’s inverse square laws are a direct consequence of momentum conservation as shown in Bertrand theorem- see also [1]- if the velocities involved are small compared to that of light in empty space \( c \). If the speeds become larger, it is necessary to take account of the fact that the forces needed time to travel (retarded) between any two interacting point charges- the source and receiver.

Suppose charge \( q_2 \) is at distance \( R \) from \( q_1 \) and moved with a velocity \( v \) a small distance \( dR \) at time \( dt \) to the new position \( R_2 \) so that the new position is, \( R_2 = R + dR \). The inverse square formula holds for instantaneous action. So, as forces proceed with the limited velocity of light, there will appear a delay in the arrival of the force. This means the distance travelled by the charge would be less than the expected \( vt \) by the factor \( v \) \( dt \), where \( \tau = dR/c \), giving \( dR = dR(1-v/c) \). The new position becomes nearer/farther- depending on the sign/direction of \( v \). Accordingly, the forces need to be computed/adjusted for this new distance [2]. This gives: \( f = K \frac{q_1 q_2}{r (1-v/c)^2} \) along the line joining the two charges in the retarded time.

The forces involved in the above are vectors, making an integration over space difficult. Instead, we revert to the scalar potential, then do a volume a scalar
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introduction over all charges. This gives the well-known Lienard-Wiechert scalar and vector potentials $\varphi$, $A$. When calculated for two particles at the retarded time(ret) at the receiver position we get- see [2, 4, 11];

$$\varphi(r, t) = \frac{Kq}{\delta r (1 - n \cdot \beta)} \text{ (ret)} \quad A(r, t) = \left(\frac{K}{c}\right) \varphi(r, t) \text{ (ret)} \quad (1a, b)$$

Where; $\delta r$ is the distance between a source containing charge $q$ and a receiver at position $r$, with $K=1/4\pi$, being the coupling constant for the Coulomb forces in empty space. $n$ is a unit vector along $r$ and $\beta=v/c$, is the ratio of the speed of charge $q$ to that of light $c$. The scalar potential represents the static part of the force, and the vector potential represents the dynamic effects due to a current $\rho v$ for a moving charge as in Ampere’s law. Bold cases are vectors.

As in the static case, the denominator goes to zero and the potential to infinity at the source position $\delta r = 0$, but also at $v=c$, at $n \cdot \beta = 1$. At this condition all energy is condensed at the receiver position as the speed $c$ is the limit speed. This resembles the static case when the distance is zero- because the distance is rendered effectively zero by the potential reaching the receiver the moment it is produced. It is also like the case of a sonic boom in fluids. Note that in this case, the direction of force becomes normal to the separation distance and all the electric field becomes magnetic in line with the velocity vector and normal to the separation vector- as can be seen in the vector potential formula given below in(2) where $A$ is along $\beta$. We note also that this result is valid even for motion with varying velocity and along any smooth curved line [4,9].

The forces are determined by the electric and magnetic fields defined as usual by; $= -\nabla \varphi - \partial A/\partial t$, and $B = \nabla \times A$. Using the above we get [4, 9];

$$E(r, t) = \frac{Kq}{(1 - n \cdot \beta)^{2}} \left(\frac{(n \cdot \beta)}{\gamma^{2}(\delta r)^{2}} + \frac{n \Lambda (n \cdot \beta) \Lambda \beta}{c \delta r}\right), \quad B(r, t) = \left(\frac{1}{c}\right) n \Lambda E(r, t) \quad (2a, b)$$

Where $\gamma^{2} = 1 - \beta^{2}$. Note the appearance as discussed before, of two terms in the expression for $E$, one decaying as $1/(\delta r)^{2}$ corresponding to the normal electrostatic fields, and the other as $1/\delta r$ and in proportion to acceleration $\beta^{2}$, corresponding to radiation. The expression for $B$ also contains the same two terms (crossed with the unit vector $n$) one decaying as the inverse square of the separation distance, proportional to velocity, and normal to the line connecting the source and receiver. This as pointed earlier, is seen as the usual magnetic field from the current generated by a moving charge. The second term in the $B$ expression is a magnetic field decaying as the inverse (not inverse square) of the separation distance and to acceleration. This is the magnetic component of the EM radiation vector. The radiation terms in $E, B$ disappear completely if the acceleration $\beta^{2}$ is made zero as it should. Note also that the direction of force vector is given by $-\beta$, which is
in the direction of the un-retarded/original source position as pointed before and is required by orbit stability- see this quote from [11]

“Note that the \((\mathbf{n} - \mathbf{\beta})\) part of the first term updates the direction of the field toward the instantaneous position of the charge, if it continues to move with constant velocity \(c\mathbf{\beta}\).”

3 The Retarded Potential for Newton’s Gravity

The static forces between two masses are given by Newton’s law of gravity; \(f=-K_g m_1 m_2/r^2\), where \(f\) is force, \(m\) mass, \(r\) separating distance, and \(K_g=G=6.67x10^{-11}\) Nm²/kg² is the coupling constant- known as the universal gravitation constant. The law of Newton is a direct consequence of momentum conservation as shown in Bertrand theorem and further explained in [1], when the velocities involved are small compared to \(c\). If the speeds are large, it is necessary to take into account the fact that the forces now take time to travel between two interacting point masses, and the inverse square potential must be modified accordingly as done in the case of the Coulomb forces.

To further continue and derive the gravity force fields in the case of motion using the retarded integral, we need to sort out few things first to ensure a complete correspondence between the gravity case and the EM case. We recall first that similar masses attract whereas similar charges repel which results in the sign change \(K, K_e\) in the inverse square formulae. One might think this to be a small difference till we realize that it belongs to the product \(m_1 m_2\) in the forces formula, which is a second order/square term. In the potential formula on the other hand, we have only one \(m\). Thus, we need to use an imaginary number that squares to \(-1\) and gives a negative sign. This can be solved easily however, if we put \(i m\) for \(m\) and consider masses to be imaginary charges as suggested before in [9] for the charges instead. Since mass is the generators of the field, we see that the fields themselves become imaginary. This of course is not an unwelcomed result- as masses and charges are fundamentally different and need to be in different worlds to stop their fields mixing up. The resulting forces from their interaction are real however and can be added.

The next thing to do is find the correspondence between the various fields of gravity and electromagnetism. Masses would clearly correspond to charges, and \(mv\) would correspond to \(\rho v\) [7]. Thus, an electric current would correspond to a mass current or momentum flux with units \(\text{kg/m}^2\text{s}\). The electric force on a charge is; \(f=qE\), and the field is defined as the force per unit charge; \(E=f/q\). In the mechanical case this would be the force per unit mass; \(E_m=f/m\) with units as \(\text{N/kg=m/s}^2\), which is acceleration- with the subscript \(m\) denoting mechanical fields. Thus, the \(E_m\) field is an acceleration field. The magnetic force on a charge can be taken from; \(f=q v \times B\). This makes the units for \(B_m\); \(\text{N.s/kg.m=1/s}\), which is angular frequency. The units for the magnetic vector potential \(A\) can be obtained from \(B = \nabla \times A\). This gives the units for \(A_m\) as; \(\text{N.s}\). This is an interesting result, as we get the same units for both the EM and gravity vector potential and the two are in pure mechanical units \(\text{Newton and second- also the units for momentum } mv; \text{kg(m/s)}(\text{kgm/s}^2)\text{s}=\text{N.s}\).
The vector potential is thus seen to correspond to the mechanical momentum. This agrees with results from the Poynting vector \( \mathbf{P} = \mathbf{E} \wedge \mathbf{H} / c^2 \) in electromagnetism, which is a pure momentum vector along the propagation of an EM wave and has the same units.

The remaining vectors \( \mathbf{D}_m, \mathbf{H}_m \) that correspond to displacement and field strength in EM can be related via similar constitutive relations in empty space; \( \mathbf{D}_m = \varepsilon_m \mathbf{E}_m \), \( \mathbf{H}_m = (1/\mu_m) \mathbf{B}_m \), and we now need to define the permittivity and permeability for the gravity system. Despite the fields being imaginary, these values can/must remain real. The units for \( \varepsilon \) in electromagnetism can be obtained from \( \nabla \cdot \mathbf{D} = \rho \), which gives; \( C.m^2 \) since the units for the divergence and curl are essentially \( m^{-1} \). Accordingly, the units for \( \mathbf{D}_m \) should be; \( kg.m^2 \). The units for \( \mathbf{H}_m \) can be obtained from \( \nabla \times \mathbf{H} = \partial \mathbf{D}/\partial t \), giving \( \mathbf{H}_m : kg.m^3/s \). In summary we have;

\[
\begin{align*}
\mathbf{E}_m : m/s^2, & \quad \mathbf{B}_m : 1/s; \\
\mathbf{D}_m : kg.m^2, & \quad \mathbf{H}_m : kg.m^3/s .
\end{align*}
\]

There is one more equation that was not used in the unit identification and can be used to check our unit construction. It is; \( \nabla \times \mathbf{E} = \partial \mathbf{B}/\partial t \), wherein the units of both sides come out to be; \( 1/s^2 \) as can readily be checked by direct substitution. From \( \mathbf{D}_m = \varepsilon_m \mathbf{E}_m \), the units for \( \varepsilon_m \) become: \( (kg.m^2)/(m^2/s^2) = kg.m.s^2 \). From the formula \( \mathbf{H}_m = (1/\mu_m) \mathbf{B}_m \) we get \( \mu_m = \mathbf{B}_m/\mathbf{H}_m \) and the units as; \( (1/s)(kg.m^2/s) = kg^{-1}.m^{-2} \).

The speed of light \( c^2 = 1/\varepsilon_m \mu_m \) can further be used to check this last unit construct. Here we get; \( 1/(kg.m.s^2)(1/kg.m^2) = (m/s)^2 \) as it should. To summarize, we have;

\[
\begin{align*}
\varepsilon_m : kg.m.s^2, & \quad \mu_m : kg^{-1}.m^{-2}, \\
\varepsilon_m = 1/4\pi G = 1.931x10^9 \ kg.m.s^2, & \quad \mu_m = 4\pi G/c^2 = 9.3261x10^{-1} \ kg^{-1}.m^{-3}.
\end{align*}
\]

The value for \( \mu_m \) is very low showing the feebleness of gravity compared to EM coupling when the masses involved are modest. They do become quite large however when we go to terrestrial scales.

### 4 Force Fields and Conservation Equations

Maxwell equations were originally built using a collection of empirical laws and formulae. The first was the law for force acting between electrostatic charges. It was formulated by Coulomb to agree with experiments and it resembled another empirical law- namely the Newton’s law of gravitation- and it worked just as good.

The Newton’s law as we know, was inspired by Kepler’s three observational laws on the motion of planets. The second base of Maxwell equations was Ampère’s force law between electric currents- or equivalently, between moving electric charges. The third is the Lorenz relation that finds the force on a charge from the
sum of the static contribution of Coulomb forces and the dynamic contribution of Ampere forces. The fourth pillar is the displacement current hypothesis introduced by Maxwell to complete his set of equations for the electromagnetic phenomenon. So, despite the impressive and super accuracy of Maxwell equations, they remained essentially empirical in origin at start.

When we come to Einstein GR, we find that it depended on few experimental observations; that gravitation is equivalent to acceleration, that the speed of light is constant for all frames, and that this is also the speed of propagation of the gravity forces. Then he added to the observation that the laws of physics are unchanged for different inertial frames, which was preserved by the covariant derivative. Thus, whether you are standing on earth, on a cruising train, a falling apple and a projectile take the same path in all the cases.

After years of working with Maxwell equations, people realized that these elegant equations are in fact derivable from two simple conservation equations that of the charges when static and when in motion and creating an electric current [7]. With the similarity between the EM and gravity phenomena through the inverse square force law, infinite range, and the single speed of propagation, one is compelled to ask if similar equations can also be derived for gravity starting from the conservation of mass. This is possible in principle as the starting set of equations given in [7] are purely mathematical and don’t refer specifically to electric charges. Masses can replace charges in this case which is taken up next.

5 The Field Equations from a Wave Equation

The method used in [7] is that if we assume the existence of a continuity equation which is the dynamic version of charge conservation, there mathematically follows a set of field equations similar to those of Maxwell. We shall follow a different path and show that the existence of a wave equation also leads to Maxwell type field equations. We follow in this a reverse construction that Maxwell himself used in deriving wave equations for each field component from the full set of field equations. To do this consider for example, the wave equation for a vibrating string or a vibrating metal bar. The first is an example of a transverse wave and the second of a longitudinal wave. The two follow the same formulation, which is the same as the vibration of a mass spring system- we shall neglect gravity. The only difference between the first two cases and the third is that a vibrating string or a bar can be imagined as a collection of many mass spring sections stacked next to each other resulting in the problem changing from an ordinary differential equation- varying in amplitude and time, to a PDE in two variables with amplitude changing with space and time.

Any vibration is caused by a process of a perpetual (if no energy is lost) exchange of energy between two stores. A spring for example, is a store of potential energy and the mass is the store of kinetic energy. The vibration process occurs with the emptying and filling of these two stores into each other. When the spring is at maximum compression the potential store is full, and when the mass at its maximum
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speed, the kinetic energy store is full. The tension in a string and the stress in a bar take the place of the spring as the stores of potential energy. The amount of energy exchanged is determined by what was initially put in the system above equilibrium. Any addition/subtraction of energy in the meantime constitutes a new initial/starting position.

For small amplitudes away from the steady position in a string or a bar, the equation for the displacement \( A \) is widely known and given by;

\[
\frac{\partial}{\partial x} \left( T \frac{\partial A}{\partial x} \right) = \frac{\partial}{\partial t} \left( R \frac{\partial A}{\partial t} \right), \text{ or } (T/R) \frac{\partial^2 A}{\partial x^2} = \frac{\partial^2 A}{\partial t^2} \text{ for constant } T/R=c^2 \tag{3a, b}
\]

Where \( \partial \) stands for partial differentiation with respect to the subscripts \( x \) for space and \( t \) for time. \( T \) is the initial tension of a string or stress in a bar. \( R \) is the longitudinal density and \( c \) is the wave speed in the medium concerned. If \( T, R \) are constants, the equation takes the usual second order vibration equation in one space variable and time. The left side of the equation represents changes in potential energy and the right-side changes in kinetic energy.

Next, we define new field variables and coefficients related to \( A \). We choose; \( E, B, H, D \), such that; \( B=\partial A/\partial x, H=TB, E=\partial A/\partial t, \text{ and } D=RE \). This is then used to transform the original equation to; \( \partial H/\partial x = \partial D/\partial t \). By differentiating the expression \( E=\partial A/\partial t \) w.r.t \( x \) and exchanging the order of differentiation on the right side using \( B=\partial A/\partial x \) we get; \( \partial E/\partial x = \partial B/\partial t \). Putting \( \epsilon \) for \( R \) and \( 1/\mu \) for \( T \) we get;

\[
\frac{\partial E}{\partial x} = \frac{\partial B}{\partial t}, \quad \frac{\partial H}{\partial x} = \frac{\partial D}{\partial t}, \quad D=\epsilon E, \quad B=\mu H \tag{4a-d}
\]

These equations look like Maxwell equations in 1D. The newly defined variables and coefficients are to be fixed next when needed. By inspection, we can extend this to 3D. The \( x \) derivative becomes the curl or \( \nabla \Lambda \), and the 1D wave equations (3a,b) takes the form;

\[
\nabla \Lambda (T \nabla \Lambda A) = \partial/dt (R \partial A/\partial t), \text{ and for constant } T/R=c^2, \quad c^2 \nabla \Lambda \nabla \Lambda A = \partial^2 A/\partial t^2 \tag{5a, b}
\]

Where; \( c^2= T/R = 1/\epsilon \mu \) is the wave speed squared. As in the 1D version we put;

\( B = \nabla \Lambda A, \quad H = TB, \quad E = \partial A/\partial t, \quad D = RE \), and get;

\[
\nabla \Lambda H = \partial D/\partial t, \quad \Lambda E = \partial B/\partial t, \quad .B = 0, \quad .D = \rho, \quad \text{with} \quad D=\epsilon E, B=\mu H. \tag{6a-e}
\]

The \( \nabla B =0 \) equation comes from the definition \( B=\nabla \Lambda A \) plus the fact that the divergence of any curl is identically zero. The formula for \( \nabla .D = \rho \) is taken as the definition of the electric charge \( \rho \) as normally done in the literature.
Accordingly, we can conclude that it is possible to obtain equations like those of Maxwell from a single vector wave equation. These results, however, give only similar but not exact copies of the field equations of Maxwell. This is because there should be a negative sign in the relation (6b) that is \( \nabla \Lambda E = -\partial B/\partial t \). As we shall see, this set of equations turned out to be suitable for the gravity fields we are trying to find. To show this, we reconstruct the wave equation back from the fields-like Maxwell did in his theory, and see where the negative sign comes from. So, take the curl of the first of (6) and substituting from the second and get;

\[
\nabla \Lambda \nabla H = \nabla \Lambda \partial D/\partial t, \nabla \Lambda \nabla \Lambda (1/\mu)B = \partial (\nabla \Lambda E)/\partial t, \text{ giving; } c^2 \nabla \Lambda \nabla \Lambda B = \partial^2 B/\partial t^2 \\
(7)
\]

which is an equation in \( B \) only- that is after we exchanged the time and space derivatives, and put \( c^2=1/\epsilon \mu \). We can similarly take the curl of the second equation and substitute from the first to get;

\[
\nabla \Lambda \nabla \Lambda E = \nabla \Lambda \partial B/\partial t, \nabla \Lambda \nabla \Lambda E = \partial (\nabla \Lambda E)/\partial t, \text{ giving; } c^2 \nabla \Lambda \nabla \Lambda E = \partial^2 E/\partial t^2 \\
(8)
\]

In empty space there are no electric charges leading to both \( \nabla E = \nabla B = 0 \). For this case the next vector equality holds; \( \Lambda \nabla \Lambda = \nabla (\nabla \Lambda) - \nabla^2 = -\nabla^2 \). This gives three vector none-wave equations (because of the negative signs);

\[
c^2 \nabla^2 B = -\partial^2 B/\partial t^2, \text{ and } c^2 \nabla^2 E = -\partial^2 E/\partial t^2, \text{ } c^2 \nabla^2 A = -\partial^2 A/\partial t^2 \quad (9a-d)
\]

These can be turned into wave equations only if we put for example; \( t=it \). That is we changed the time to an imaginary variable as done in GR wherein the coordinate signature becomes \((+++-)\), with \((it)^2 = -(t^2)\) and this cancels the first negative sing. The conclusion here is that field equations can be constructed for gravity forces, but the resulting fields don’t have the form of a wave equation like the EM fields. They in fact form elliptic equations instead. In other words, in matter we don’t have propagation but staying at the same starting point situation- the place where the disturbance was initiated. This in fact is a direct result of similar masses attract whereas similar electric charges repel. In other words, there exists a wave equation for gravity only if we used imaginary fields. It is worth noting here that in the original string vibration equation, we also don’t have true matter propagation, as matter is only made to oscillate at the same position. A famous example on this is the experiment of putting small floats on the surface of a propagating wave in water and seeing the floats oscillating up and down but never change position while the wave does travel from one end to the other and may reflect and come back. EM waves on the other hand always propagate unless we have a standing wave. This
makes a standing wave like a matter wave which is clearly correct, since a standing/trapped energy is equivalent to a mass.

6 Complex Potentials

We have seen that the major difference between gravity fields and EM fields lies in the sign of the square terms in their respective wave equations or the inverse square law, and this difference can be removed by taking one real while the other imaginary. This fact appeared in the inverse square laws as well as in wave equations derived from real vibration problems. The two type of fields can now combine to form a single complex potential. We shall take the electric charge to be real and the mass to be an imaginary charge for this purpose. This is because EM is an older idea and it always represented propagating waves. Matter fields (not particles) give way to elliptic equation and can be made waves only in an imaginary sense. This is quite convenient outcome and it means there is little chance of mixing/adding the two fields. Note also that this is not uncommon in the field of differential equations. The fluid flow in a nozzle for example, can change from elliptic to parabolic to hyperbolic in the describing equation in a single supersonic flow nozzle- according to the type of flow. In the hyperbolic(wave) case, disturbances propagate out and away, while they stay in the same region when the equations are elliptic.

Because the coupling constant values differ between EM and gravity, it is necessary to consider this before combining the two fields into a single complex field. Accordingly, we take the generalized complex charge to be; \( M = (m/\sqrt{K_m} + i q/\sqrt{K}) \). Then the unified inverse square potential becomes; \( A = Mr/r^2 \), and the combined Newton/Coulomb complex force between two complex charges becomes; \( F = M_1 M_2 \, r/r^2 \), where \( A, F \) are complex vectors. Because \( M_1, M_2 \) are complex, their product will be containing terms corresponding to the usual real Coulomb and Newton’s forces, in addition to an imaginary force term given by; \( i K_m K q \, m/r^2 \). This imaginary force is not so small. For two electrons for example, the ratio of the imaginary force to the electrostatic force at the electron radius is \( 4.9 \times 10^{-22} \), whereas the ratio of the gravitational force to the electrostatic force is \( 2.4 \times 10^{-43} \). This is a marked increase in effect and might prove important in cosmological set up if one can identify particles that are under such imaginary forces. A strong charge or currents in addition to a large masses in one place can lead to a big force. It should be stressed here that this imaginary force is the result of real masses and charges, but being imaginary, its effects can’t be added to the real forces. Again, the presence of imaginary quantities is not unheard off in physics. The current in a non-resistive electric circuit for example, can be imaginary compared to the one in a resistance and it is real enough and capable of causing loading problems on powerlines and instruments. This is the reason for the continuous efforts to correct the power factor and reduce the imaginary part to improve the performance of electrical circuits.
7 Negative Mass

In addition to an imaginary mass, we can in theory have a negative mass too. As far as we know there are no negative masses. But we can’t be very sure of that for two reasons. One is the observed symmetry of space in every other aspect requires negative masses to balance positive masses as in the case of electric charges and it is not forbidden by the laws of gravity that we already have. However, we expect negative masses to attract, while negative and positive masses to repel. This means that a negative mass could be around but pushed out of and away from (our) positive mass. It may be residing around normal mass in outer space for example. As negative masses attract each other, they could be existing in mass in the form of lines or sheets outside of normal mass- as a cosmic web for example. In a celestial setup, the effect of a negative mass would be to weaken/shield gravity, very much like dissimilar electric charges do. And indeed, some researchers [3] have used this negative mass idea to show that a simulation of the cosmological equations that included negative mass is able to reproduce the distribution of what is observed in the sky including the cosmic web shapes. It also explained the flatness in the rotation curves of galaxies. And it helps explaining the expansion of the universe, which is normally attributed to dark energy.

If we now admit a negative mass, we get a complete similarity between electric and gravity effect from all respects with the additional requirement of taking mass to be imaginary charges (or the reverse if needed). The retarded integral and the complex inverse square containing negative mass will result in a plethora of new forces-many of which haven’t been investigated before and are well worth doing.

8 Comparing the GI with the GM Results

The gravity-inertia (GI) equations (7-9) are a copy of Maxwell equations when imaginary fields and negative masses are admitted and when considering the equivalent gravitational permittivity and permeability of empty space as derived above. The gravito-magnetic (GM) equations are investigated thoroughly in [8], with the starting point being GR in the weak gravity approximation. Despite the use of different methods, the equations look similar to ours (eqs. 6- in this reference) apart from the appearance of a negative sign in the induced magnetic field. This results in wave equations- ie. hyperbolic like those of EM and unlike ours. One wonders if it is a case of an overlooked sign. As shown above, a correspondence between the absence of the negative sign in the induced field and the need for multiplying with an imaginary number to make Newton’s gravity law work in the same way as Coulomb’s law is necessary and also agrees with the work in [9] as quoted above. We believe this point is worth checking as it affects work on gravitational waves understanding.

9 Conclusions

In [1] it was concluded that all matter attributes like mass, charge, gravity etc. can
emerge as a result of EM radiation condensing into matter as topological EM solitons. This amounted to a fundamental connection between Newton’s gravity and Coulomb’s electric forces. The equality of the speeds of propagation, the infinite ranges of the two, and the inverse square behavior of the forces are clearly not accidental. Starting from this point, we showed that gravity can be represented by fields similar to those of EM. The fact that similar masses attract whereas similar charges repel showed up as a negative sing in the square terms and in the wave equation derived for matter fields. This required these fields to be imaginary-squaring to -1 at the forces level to keep them real. A wave-like equations derived from such fields in the same way Maxwell did for his fields, turned out to be elliptic only and not hyperbolic. They become a wave only on choosing imaginary fields. The name gravity-inertial equations (GI) is given to differentiate the GI from similar fields derived from GR in the weak field approximation and already given the name gravito-magnetic (GM) equations. These equations differ from ours only in that the wave equation being a proper hyperbolic equation and the magnetic induction term being negative exactly like in EM. We nonetheless note that the sign difference is original cannot be arbitrary- as it is tied with the difference in the physical description of the GI and EM forces - namely that similar masses attract whereas similar charges repel. It is hoped that the present results can contribute to better understanding of the nature of the gravity force and simplify working in this subject. It could encourage more people to attempt solutions for various new interesting situations in outer space applications where gravity is dominant but there could still be significant contribution from EM forces- see this quote from [6]; "Considering the strength of the electromagnetic force in comparison with that of gravity surely indicates a need for the two basic schools of thought concerned with problems of astronomy and astrophysics to come together for open-minded consideration of the problems facing those disciplines today". This possibility becomes visible when gravity and EM potentials are unified into a single complex potential and a single law is written combining the Newton’s and Coulomb’s effects. If this is done and negative mass is added too, a plethora of forces can emerge that are worth investigating.

References


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