Modulating the Dynamics of

a Mathematical Model of an Electrosensory

Neuron by the Membrane Capacitance

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Abstract

In a previous study, the electrical activity of electrosensory neurons from a weakly electric fish was described by the ghostbursting model, which was constructed on the basis of the Hodgkin-Huxley formalism. This mathematical model is described by a system of nonlinear ordinary differential equations. This study focuses on three parameters of this ghostbursting model, i.e., the injected current, somatic membrane capacitance, and dendritic membrane capacitance. Via a numerical simulation, this study shows how the dynamics of the model are regulated by changes to these parameter values. For changes to somatic and dendritic membrane capacitances, both qualitative and quantitative differences in the modulation of the ghostbursting model's dynamics are revealed.

1 Introduction

In response to changes in external stimulation levels, the mathematical model known as the ghostbursting model can show three different dynamical states, i.e.,

a quiescent state, repetitive spiking state, and bursting state [1]. The ghostbursting model is described by a system of six coupled nonlinear ordinary differential equations (ODEs), constructed on the basis of the Hodgkin–Huxley formalism, and divided into somatic and dendritic compartments. The time evolution of the membrane potential of each compartment is regulated by several parameters including the membrane capacitance and several ionic conductances. Previous studies have revealed the effects of varying the ionic conductances on the dynamical states of the ghostbursting model [1-3]; however, the effect of varying the membrane capacitance has yet to be reported. However, changes in membrane capacitance values are known to affect the dynamics of nonlinear dynamical systems such as pituitary cell models [4-6] and a cortical pyramidal neuron model [7]. To understand the characteristics of the ghostbursting model more completely, it is therefore necessary to elucidate how its dynamical states are modulated by variations in membrane capacitance. Because the membrane capacitance of both the somatic and dendritic compartments is relevant in the ghostbursting model, the present numerical study investigates how the dynamical states of the model are regulated by changes to these two membrane capacitances.

2 Materials and Methods

In the present investigation, the ghostbursting model [1] is numerically simulated. The dynamics of this model are described by a system of six coupled ODEs:

$$\frac{dV_s}{dt} = \frac{1}{C_s}[I_s - 0.18(V_s + 70)$$

$$-55\left(\frac{1}{1+e^{-(V_s+40)/3}}\right)^2 (1-n_s)(V_s - 40) - 20n_s^2(V_s + 88.5) - \frac{1}{0.4}(V_s - V_d)] \quad (1)$$

$$\frac{dn_s}{dt} = \frac{1}{0.39}\left(\frac{1}{1+e^{-(V_s+40)/3}} - n_s\right) \quad (2)$$

$$\frac{dV_d}{dt} = \frac{1}{C_d}[-0.18(V_d + 70)$$

$$-5\left(\frac{1}{1+e^{-(V_d+40)/5}}\right)^2 h_d(V_d - 40) - 15n_d^2 p_d(V_d + 88.5) - \frac{1}{0.6}(V_d - V_s)] \quad (3)$$

$$\frac{dh_d}{dt} = \frac{1}{1+e^{(V_d+52)/5}} - h_d \quad (4)$$

$$\frac{dn_d}{dt} = \frac{1}{0.9}\left(\frac{1}{1+e^{-(V_d+40)/5}} - n_d\right) \quad (5)$$

$$\frac{dp_d}{dt} = \frac{1}{5.0}\left(\frac{1}{1+e^{(V_d+65)/6}} - p_d\right) \quad (6).$$

In these ODEs, V_s (mV) is the membrane potential of the somatic compartment, n_s is the activating variable of the potassium conductance of the somatic compartment, V_d (mV) is the membrane potential of the dendritic compartment, h_d

is the inactivating variable of the sodium conductance of the dendritic compartment, n_d is the activating variable of the potassium conductance of the dendritic compartment, and p_d is the inactivating variable of the potassium conductance of the dendritic compartment. Three specific control parameters are the focus of this study: the injected current to the somatic compartment I_s (μA/cm^2), the somatic membrane capacitance C_s (μF/cm^2), and the dendritic membrane capacitance C_d (μF/cm^2). When varying these parameters, I_s is varied from 5.4 to 6.0 μA/cm^2 or from 7.6 to 9.6 μA/cm^2. The default values of C_s and C_d are 1.0 μF/cm^2; C_s and C_d are varied either from 95% to 105% of the default values (Figure 1) or from 0.6 to 1.4 μF/cm^2 (Figure 2). Equations (1)–(6) are explained in detail in Doiron et al. [1].

Under the initial condition, i.e., $V_s = -70$ mV, $n_s = 0.00005$, $V_d = -70$ mV, $h_d = 0.973$, $n_d = 0.002$, and $p_d = 0.697$, we solved Equations (1)–(6) numerically using the free and open source software Scilab (http://www.scilab.org/).

3 Numerical Results

I_s is the most basic parameter of the ghostbursting model, and the effects of varying I_s on the dynamics of the model have previously been reported [1]. When I_s is small, a quiescent state is the dynamical state of the model. When I_s is increased to a certain threshold (i.e., a repetitive spiking threshold), the dynamical state changes from a quiescent to a repetitive spiking state. When I_s is further increased to a certain threshold (i.e., a bursting threshold), the dynamical state again changes from a repetitive spiking to a bursting state. In this study, the repetitive spiking threshold was 5.8 μA/cm^2 irrespective of the C_s value (Figure 1A) or C_d value (Figure 1B). However, the bursting threshold changed such that $8.4 \rightarrow 8.6 \rightarrow 8.8$ μA/cm^2 when C_s was changed from 95% $\rightarrow$ 100% $\rightarrow$ 105% (Figure 1C); the bursting threshold changed such that $9.6 \rightarrow 8.6 \rightarrow 7.8$ μA/cm^2 when C_d was changed from 95% $\rightarrow$ 100% $\rightarrow$ 105% (Figure 1D). The dynamical states of the ghostbursting model in the (C_s, C_d) parameter space were also investigated under conditions in which I_s was fixed at 8.6 μA/cm^2 (Figure 2). When C_d was small (i.e., $C_d = 0.6$ or 0.8 μF/cm^2), the model showed a repetitive spiking state irrespective of the C_s value. However, when C_d was large (i.e., $C_d = 1.2$ or 1.4 μF/cm^2), the model showed a bursting state irrespective of the C_s value. When C_d was an intermediate value ($C_d = 1.0$ μF/cm^2), the model showed a repetitive spiking or bursting state depending on the C_s value: a bursting state was shown under conditions in which C_s was small (i.e., $C_s = 0.6$, 0.8, or 1.0 μF/cm^2), whereas a repetitive spiking state was shown under conditions in which C_s was large (i.e., $C_s = 1.2$ or 1.4 μF/cm^2).

4 Discussion

In previous studies, a comparison was made between the somatic and dendritic parameters of the ghostbursting model [2, 3]. Although these studies focused on

several ionic conductances and the leak reversal potential, they did not focus on the membrane capacitance. Thus, membrane capacitance was the focus of this study, in which a comparison was made specifically between the somatic and dendritic membrane capacitances. Importantly, the results of this study reveal that, when varying the somatic, and dendritic membrane capacitances in the model, both qualitative and quantitative differences exist in terms of the effects on the bursting threshold.

Previous studies investigated the effects of changing membrane capacitance on the dynamics of pituitary cell models [4-6]; however, in contrast to the ghostbursting model, which is described by a two-compartment model, the pituitary cell models are single-compartment models. In addition, Wan et al. [7] reported the effect of varying membrane capacitance on the dynamics of a mathematical model of cortical pyramidal neurons. Similar to the ghostbursting model, this model is a two-compartment model based on the Hodgkin–Huxley formalism. The authors revealed that the dynamical states of the cortical pyramidal neuron model are a repetitive spiking state and a bursting state when the dendritic membrane capacitance is small and large, respectively. This conclusion is similar to the results of this study; however, the study of Wan et al. [7] differs from this study when the dendritic membrane capacitance is an intermediate value. Specifically, the dynamical state of the cortical pyramidal neuron model is (1) a repetitive spiking state when the somatic membrane capacitance is small and (2) a bursting state when the somatic membrane capacitance is large, whereas the dynamical state of the ghostbursting model is (1) a bursting state when the somatic membrane capacitance is small and (2) a repetitive spiking state when the somatic membrane capacitance is large. This disparity might be due to differences between the models in terms of the ionic currents contained in the dendritic compartments: the cortical pyramidal neuron model contains a persistent sodium current and slow potassium current, whereas the ghostbursting model contains a fast sodium current and delayed rectifying potassium current. In addition, in this study, changes were made not only to the membrane capacitance but also to the current injected into the somatic compartment. These changes revealed the sensitivity of the repetitive spiking and bursting thresholds to variations in membrane capacitance. However, Wan et al. [7] fixed the injected current at a specific value; thus, they did not report the sensitivity of the thresholds to variation in membrane capacitance.

5 Conclusions

The present numerical analysis of the ghostbursting model focused on changes to three control parameters: I_s, C_s, and C_d. The results revealed (1) how the repetitive spiking and bursting thresholds are modulated by changes in C_s and C_d, and (2) how the dynamical states of the model are dependent on C_s and C_d under conditions in which I_s is a specific set value.

(A)

I_s (μA/cm²)	5.4	5.6	5.8	6.0
95% C_s	×	×	○	○
100% C_s	×	×	○	○
105% C_s	×	×	○	○

(B)

I_s (μA/cm²)	5.4	5.6	5.8	6.0
95% C_d	×	×	○	○
100% C_d	×	×	○	○
105% C_d	×	×	○	○

(C)

I_s (μA/cm²)	7.6	7.8	8.0	8.2	8.4	8.6	8.8	9.0	9.2	9.4	9.6
95% C_s	○	○	○	○	●	●	●	●	●	●	●
100% C_s	○	○	○	○	○	●	●	●	●	●	●
105% C_s	○	○	○	○	○	○	●	●	●	●	●

(D)

I_s (μA/cm²)	7.6	7.8	8.0	8.2	8.4	8.6	8.8	9.0	9.2	9.4	9.6
95% C_d	○	○	○	○	○	○	○	○	○	○	●
100% C_d	○	○	○	○	○	●	●	●	●	●	●
105% C_d	○	●	●	●	●	●	●	●	●	●	●

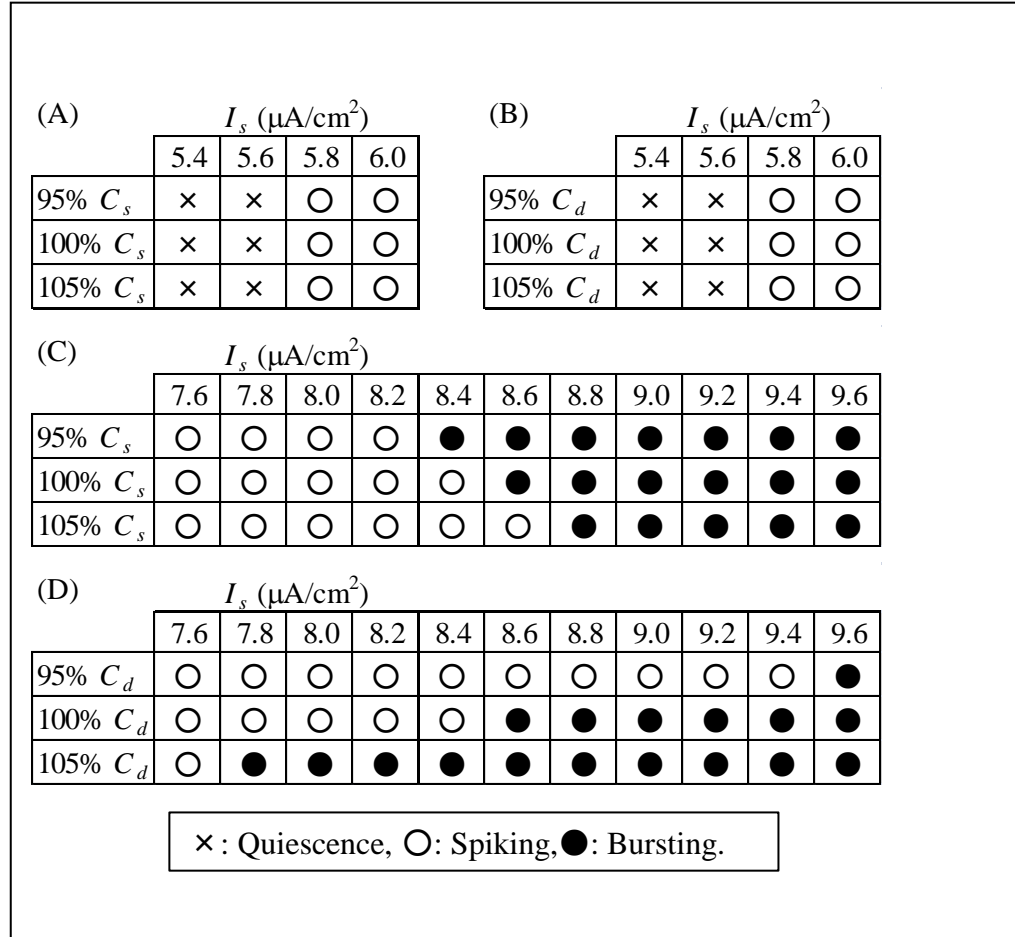×: Quiescence, ○: Spiking, ●: Bursting.

Figure 1. The dependence of the dynamical states of the ghostbursting model on the injected current to the somatic compartment (I_s), somatic membrane capacitance (C_s), and dendritic membrane capacitance (C_d). The sensitivity of the repetitive spiking threshold to the variation in (A) C_s and (B) C_d. The sensitivity of the bursting threshold to variation in (C) C_s and (D) C_d. In (A–D), × indicates a quiescent state, ○ indicates a repetitive spiking state, and ● indicates a bursting state.
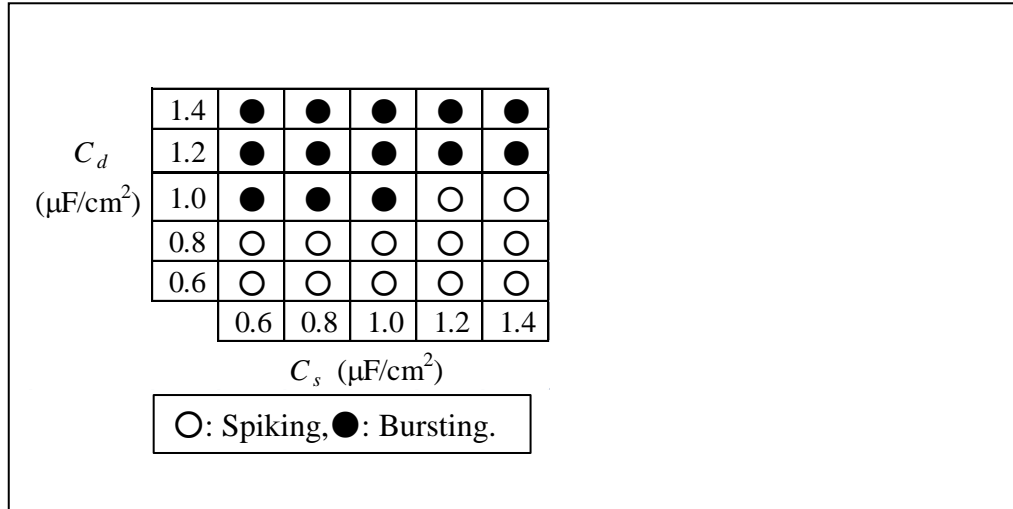
Figure 2. The dependence of the dynamical states of the ghostbursting model on somatic membrane capacitance (C_s) and dendritic membrane capacitance (C_d) under conditions in which the injected current to the somatic compartment (I_s) is fixed at 8.6 μA/cm^2. $\circ$ indicates a repetitive spiking state; $\bullet$ indicates a bursting state.

References

[1]   Doiron, B., Laing, C., Longtin, A. and Maler, L., Ghostbursting: A novel neuronal burst mechanism, *Journal of Computational Neuroscience*, **12** (2002), 5-25. https://doi.org/10.1023/a:1014921628797

[2]   Shirahata, T., The relationship of sodium and potassium conductances with dynamic states of a mathematical model of electrosensory pyramidal neurons, *Applied Mathematics*, **7** (2016), 819-823. https://doi.org/10.4236/am.2016.79072

[3]   Shirahata, T., A numerical study of the relationship between the ghostbursting model and the leak current, *Advanced Studies in Theoretical Physics*, **14** (2020), 65-72. https://doi.org/10.12988/astp.2020.91248

[4]   Vo, T., Bertram, R., Tabak, J. and Wechselberger, M., Mixed mode oscillations as a mechanism for pseudo-plateau bursting, *Journal of Computational Neuroscience*, **28** (2010), 443-458.

[5] Teka, W., Tabak, J., Vo, T., Wechselberger, M. and Bertram, R., The dynamics underlying pseudo-plateau bursting in a pituitary cell model, *Journal of Mathematical Neuroscience*, **1** (2011), 12.
https://doi.org/10.1186/2190-8567-1-12

[6] Vo, T., Bertram, R. and Wechselberger, M., Bifurcations of canard-induced mixed mode oscillations in a pituitary lactotroph model, *Discrete and Continuous Dynamical Systems*, **32** (2012), 2879-2912.
https://doi.org/10.3934/dcds.2012.32.2879

[7] Wang, L., Liu, S., Zhang, J. and Zeng, Y., Burst firing transitions in two-compartment pyramidal neuron induced by the perturbation of membrane capacitance, *Neurological Sciences*, **33** (2012), 595-604.
https://doi.org/10.1007/s10072-011-0819-6