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A Purely Probabilistic Approach to Quantum Measurement and Collapse

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Abstract

The probability theory presents various interpretations - frequentist, subjective, axiomatic, Bayesian, logical etc. - which exhibit profound diversities, while the definitive comprehensive theoretical frame does not appear at the horizon. The quantum universe is intrinsically probabilistic, and several times Karl Popper underscored the importance of the probability fundamentals as requisite to quantum mechanics (QM).

This research has followed Popper's lesson. The first step went through the probability foundations, here the second step illustrates how those theoretical results apply to QM. In particular, this paper is arranged as follows, Section 2 recalls some definitions and theorems about probability that we have published. Section 3 derives the definitions of particles and waves from the concept of probabilistic outcome. Section 4 puts forward a new scheme about the wave collapse and the measurement problem. Section 5 discusses physical experiments supporting the achievements inferred from the probability theory.

It is important to emphasize that the indeterministic concepts lead the reader to revisit vexed questions from an innovative perspective which the cited experiments substantiate.

Keywords: Probability theory, quantum dualism, wave collapse, quantum measurement, two-slits experiments, experiments with macromolecules

1 Introduction

There is an apparent temporal priority in science: as first, mathematicians set up the necessary formal instruments, as second physicists employ those instruments. If mathematicians misfire, physicists have scarce possibilities of success, as they lack the language and the logical tools to analyze and calculate the phenomena. Physicists can handle single applications with slapdash formal instruments, but do not succeed in formulating a satisfactory and complete theory until they get the appropriate mathematical instruments. The history of science has several cases that illustrate this natural order.

In 1654 Blaise Pascal inaugurated the modern probability calculus that however is still in a troubled state. The various theories - e. g. frequentist, subjective, axiomatic, logical etc. - provide diverging conclusions while the definitive theoretical frame lies beyond the horizon. Karl Popper [10] many times claimed this failure impairs the progress of quantum mechanics (QM). He was convinced that the complete theory of probability is the necessary prerequisite to QM and wrote: "I had always been convinced that the problem of interpretation of the quantum theory was closely linked with the problem of the interpretation of probability theory in general." Throughout the 1950s and 1960s he greatly intensified his research and devised a physical interpretation called *propensity* [4].

Quantum computing, nuclear medicine, atomic engineering, astronomy and other fields employ quantum physics with remarkable success. Scientists advance at the application level, whereas theorists are stuck to the starting point. They meet inextricable knots in the very first steps [1] deeply involved with the probability theory. The following issues are waiting a satisfactory answer:

(a) The quantum dualism,
(b) The wave collapse,
(c) The measurement problem.

The lesson of Popper encouraged us to go deep into the probability domain in advance of addressing questions (a), (b) and (c) and the present research exhibits a style far different from modern inquiries.

We went through the probability foundations in the first stage [12] and now we employ *exclusively* those theoretical results in tackling the mentioned issues. In consequence of this unusual method, this work does not assess physical parameters such as speeds, momenta, spins, energies etc. This paper is not an essay on quantum mechanics in strict sense, rather it could be watched as an application of the probability theory to QM.

2 A Summary

Let us recall briefly the mathematical results that have been published and will be used in the subsequent sections.

In accordance with the axiomatic frame, we assume probability P is a *set-function* defined over the sample space and satisfying the *non-negativity, normalization* and *finite additivity* axioms. As we mean to address physical questions, we specify three points [3]. As first, the noun *event* stands for the phenomenon, the trial or the system E which brings forth the *outcome e*. The event and the outcome have close relations but are distinct physical entities since the former produces the latter. As second, we suppose the generic event E starts at the instant $t_o = 0$ and finishes at t_e that is the delivery time of e, hence we obtain the time intervals T1 with $0 < t < t_e$, and T2 with $t \geq t_e$. As third, the calculated value P is valid before t_e, while the relative frequency F is the empirical probability measured after t_e

$$0 \leq P(E) \leq 1;$$
$$0 \leq F(E) \leq 1. \tag{1}$$

Testing is a cornerstone in science, and P, defined in abstract, must be supported by physical experiments. Various authors share the idea that the probability of certain and impossible events can be easily checked, whereas fluctuations oppose difficulties to testing aleatory facts. The *theorem of large numbers* (TLN) proves that the relative frequency approximates the probability of the long-term event $P(E_\infty)$ when the number of trials tends toward infinity (n=∞) [13]. This means that the frequentist $P(E_\infty)$ is an authentic physical quantity and has objective significance. When n=1, the *theorem of a single number* (TSN) demonstrates the probability of the individual random event $P(E_1)$ is out of control. The frequency does not match with $P(E_1)$ therefore $P(E_1)$ has no physical significance and should be rejected from the scientific domain. Subjectivists and Bayesians recycle $P(E_1)$ and use it to qualify the personal degree of credence about E_1 [14].

TLN and TSN show how $P(E_\infty)$ and $P(E_1)$ broadly differ but apply to disjoint material situations. The frequentist and subjective probabilities do not conflict as usually credited and the present frame includes them both.

The second pair of theorems published in [15] calculate the properties of the result e that has the *indeterminate status* $e^{(I)}$ when one of the followings are true

$$0 < P(e) < 1; \quad 0 < F(e) < 1. \tag{2}$$

The result e has the *determinate status* $e^{(D)}$ when an extreme qualifies it

$$P(e) = 0; F(e) = 0,$$
$$P(e) = 1; F(e) = 1. \tag{3}$$

The disjoint expressions (2) and (3) lead to

$$e = e^{(I)} \text{ OR } e^{(D)} \tag{4}$$

Obviously, the probability of event $P(E)$ and the probability of result $P(e)$ are identical in numerical terms. The assessment of (2) and (3) turns out to be easily manageable, in particular, *if the event is random then the outcome e is indeterminate* *in* T1

$$0 < P(e) < 1. \qquad 0 \leq t < t_e. \tag{5}$$

This statement will provide meaningful conclusions later.
In consequence of TLN and TSN we distinguish the single result e_1 from the long-term result e_∞ that is a collection of individual results

$$e_\infty = \{e_{1h}, e_{1j}, e_{1k}, \ldots\} \tag{6}$$

The *theorem of continuity* (TC) proves that the outcome e_∞ of the long-term aleatory event keeps the indeterminate status in T1 and T2

$$e_\infty = e_\infty^{(I)}, \qquad 0 \leq t. \tag{7}$$

The *theorem of discontinuity* (TD) holds that the outcome of the single event switches from the indeterministic status to the deterministic status at the end of T1

$$e_1^{(I)} \rightarrow e_1^{(D)}, \qquad t \geq t_e. \tag{8}$$

As an example, $P(H_1) = 0.5$ proves that the result heads H_1 is indeterminate when the coin flies in the air (= T1). When the coin lands, the relative frequency $F(H_1)$ is either zero or one, namely heads collapses from the indeterminate to the determinate status in T2.
TD fits and completes TSN in point of logic. TD spells out how e_1 is no longer random at the end of the experiment and this conclusion matches with TSN demonstrating that the single random outcome cannot be tested. The two theorems illustrate one of the most bizarre phenomena in probability and statistics from two different stances and reach consistent conclusions. Mathematicians have overlooked the universal property (8) so far and inevitably quantum theorists meet unsurmountable obstacles from the logical-mathematical viewpoint.

3 Quantum Dualism

Every quantic entity behaves like a wave in certain circumstances, while in others as a particle, and these properties never appear simultaneously. Several scientists share the wave-particle dualism as the concept has worked well in physics, however its meaning has not been satisfactorily explained. Interests and bouts of controversy regarding the dualism appear to breakout intermittently since its early original formulation. The Copenhagen interpretation gave formal explanation of the wave-particle duality on the basis of Born's probabilistic view and the uncertainty principle. In substance the Copenhagen circle presented an agnostic stance and held that our knowledge, based on macroscopic notions, cannot underpin the phenomena pertaining to the microworld [8]. This inquiry adopts exclusively the notions from

(1) to (8) and puts forward an original viewpoint.

3.1 Let us begin with the ensuing remarks.

□ - The classical probability P and the quantum probability Pr do not coincide, yet they have two fundamental properties in common. Both P and Pr qualify the likelihood of the event while the integer and decimal values of P and Pr have identical meanings. Therefore, definitions (2) and (3) can be imported in the quantum context keeping their significance.

□ - The *quantum* ξ *is a discrete portion of energy (and in case of matter)* in the sense that the magnitude of the energy takes on only discrete values. The outcome e is a subset of the event space hence for the axiomatic theory the ξ is *a subset.*

From (2) and (3) we have that ξ has the *indeterminate status* $\xi^{(I)}$ when

$$0 < P(\xi) < 1; \quad 0 < F(\xi) < 1. \tag{9}$$

The quantum has the *determinate status* $\xi^{(D)}$ when

$$\begin{gathered} P(\xi) = 0;\ F(\xi) = 0, \\ P(\xi) = 1;\ F(\xi) = 1. \end{gathered} \tag{10}$$

3.2 Two probability distributions – This purely probabilistic confines attention to time and space and overlooks parameters such as energies, momenta, spins etc. that are typical of physics.
Quanta are discrete quantities of energy and nothing prevents such discrete quantities from occupying diverse volumes. We observe that a particle occupies only a spot in the space whereas the wave has the property of being diffused all around. This input leads us to establish that the quantum ξ has the volume V_P nearly null or otherwise has the volume V_W which in principle has no boundary

$$\begin{gathered} V_P \approx 0, \\ V_W \approx \infty. \end{gathered} \tag{11}$$

Let us employ the spatial property (11) to detail $P(\xi)$ in (9) and (10). We call $P_\xi(x,y,z)$ the probability of finding energy (and matter) in (x,y,z) at a given time, and from (11) we get the following alternative distributions:

❖ The *probability distribution function* $P_{P\xi}$ has the ensuing integer values that qualifies *the determinate status of* ξ taking place in (x_*,y_*,z_*) at a given instant

$$\left.\begin{aligned} P_{P\xi}(x,y,z) &= 1 \quad \text{if } (x,y,z) = (x_*,y_*,z_*). \\ P_{P\xi}(x,y,z) &= 0 \quad \text{if } (x,y,z) \neq (x_*,y_*,z_*). \end{aligned}\right\} \quad \xi = \xi^{(D)} \tag{12}$$

- The *probability distribution function* $P_{W\xi}$ has only decimal values when *x, y* and *z* vary between minus infinite and plus infinite; they qualify *the indeterminate status of* ξ placed everywhere

$$\left. P_{W\xi}(x,y,z) \neq 1;\ P_{W\xi}(x,y,z) \neq 0 \quad \text{when } x, y, z \in (-\infty, +\infty). \right\} \quad \xi = \xi^{(I)} \tag{13}$$

Obviously $P_{P\xi}(x,y,z)$ and $P_{W\xi}(x,y,z)$ conform with the normalization rule.
The step function $P_{P\xi}$ depicts the particle concentrated in a point. The continuous distribution $P_{W\xi}$ characterizes the wave diffused all over the space. The indeterminate status $\xi^{(I)}$ is similar to a cloud more or less dense at the given time *t*.
Expression (4) entails that the statuses just defined are mutually exclusive and fit with the *complementary principle*

$$\xi = \text{Particle OR Wave.} \tag{14}$$

3.3 Definitions (2) and (3) employ the probability *P* as a yes-not quantity and one cannot calculate the distributions (12) and (13), only *Pr* can be calculated with precision. The squared wavefunction $|\Psi|^2$ provides the exact probability amplitude of *Pr*. The *functions* $P_{P\xi}$ *and* $P_{W\xi}$ *are essentially conceptual references* which present limits of use in calculations. This restriction although do not impede to proceed. These mathematical distributions call to mind the Boltzmann entropy which explains fundamental aspects of thermodynamics and does not turn out to be a manageable instrument of calculus.

3.4 Completing the inventory – The theorems of large numbers and a single number prove that probability has different properties depending on the number of trials, hence each status of ξ must be related to the number of occurrences that ξ has. Assuming n=1, (12) and (13) describe an individual portion of energy/matter and we get

$$\xi_{\mathbf{1}}^{(D)} = \text{A particle.} \tag{15}$$

$$\xi_{\mathbf{1}}^{(I)} = \text{A wave or wavelet.} \tag{16}$$

Assuming $n\rightarrow\infty$, from (6), (12) and (13) we get

$$\xi_{\infty}^{(D)} = \{\xi_{\mathbf{1h}}^{(D)}, \xi_{\mathbf{1j}}^{(D)}, \xi_{\mathbf{1k}}^{(D)}, \ldots\} = \text{Flow of particles.} \tag{17}$$

$$\xi_{\infty}^{(I)} = \{\xi_{\mathbf{1g}}^{(I)}, \xi_{\mathbf{1f}}^{(I)}, \xi_{\mathbf{1s}}^{(I)}, \ldots\} = \text{Stream of waves.} \tag{18}$$

3.5 Physical meanings – At this point perhaps the reader is wondering: When a quantum is a wave, what is waving?
A wave is defined as a portion of energy (or even matter) spreading all around, hence energy and matter have irregular distribution and fluctuate with time. In principle, this property fits with the De Broglie hypothesis.

Where was the particle in advance of the wave collapse?
Eqn. (14) holds that quanta can be either particles or waves. One status excludes the other one and vice versa. In detail, the wave $\xi_{\mathbf{1}}^{(I)}$ does not cohabit with the particle $\xi_{\mathbf{1}}^{(D)}$, and the function $P_{W\xi}$ does not describe a particle whose position is uncertain. The distribution $P_{W\xi}$ does not say anything about the possible positions

of $\xi_1^{(D)}$ since $\xi_1^{(D)}$ *does not exist* and the query is simply nonsensical here.

How are quanta made?
Particles are determinate quanta and factually occupy specific locations, no matter they are confined in a box or move in the space. Technology offers an assortment of sensors that allow physicists to check $\xi_1^{(D)}$ and $\xi_\infty^{(D)}$ with precision. Also $\xi_\infty^{(I)}$ has objective significance and actually the intense beam of waves complies with the laws of classical optics (read Section 5). In summary, the flow of particles $\xi_\infty^{(D)}$, the individual particle $\xi_1^{(D)}$ and the stream of waves $\xi_\infty^{(I)}$ have precise physical meanings and can be tested.
It remains the wavelet. The theorems of a single number and discontinuity demonstrate that $\xi_1^{(I)}$ cannot be measured, and the universal experience corroborates this pair of statements. The wavelet described as energy diffused in the space is clear in principle, the distribution $P_{W\xi}$ is not ambiguous although $\xi_1^{(I)}$ cannot be directly tested. Therefore, the observer can but seek indirect evidence of the wavelet. This experimental tactic is not new in science, for instance, it is very common in astronomy where several phenomena cannot be accessed. It is important to underscore how the principles of probability reduce the importance and the weight of the measurement issue that appears to be dramatic in modern QM. In fact, the present probabilistic interpretation proves that the wavelet is not an erratic particle moving randomly but a diffused state of energy.

3.6 Consistent sets of notions – A group of scientists including Einstein, Born, Ballentine and others, is convinced of the connection of $|\Psi|^2$ with the physical reality; the wavefunction has physical meaning. Another group including Bohr, Heisenberg, Dirac and others, maintain the wavefunction has no real significance. For them there are neither waves nor particles, nor indeed anything specifiable at the physical level [6].

Several quantum scientists argue about the irreconcilable features of particles and waves and reach conflicting conclusions. Also the current theoretical frame points out the irreconcilable features of determinate and indeterminate quanta but they do not conflict. On one hand, there are integer values of probability, the determinate status $\xi^{(D)}$ which is local and is embodied by a particle that is a condensed portion of energy/matter. On the other hand, there are decimal values of probability, the indeterminate status of $\xi^{(I)}$ which is not local and actualizes the quantum in a wave that is a diffused portion of energy/matter (Figure 1). The webs of consistent tenets are very distant from the physical viewpoint, yet the present theory connects them. The concept probability joins the two conceptual nets and demonstrates how they pertain to the same domain of knowledge.

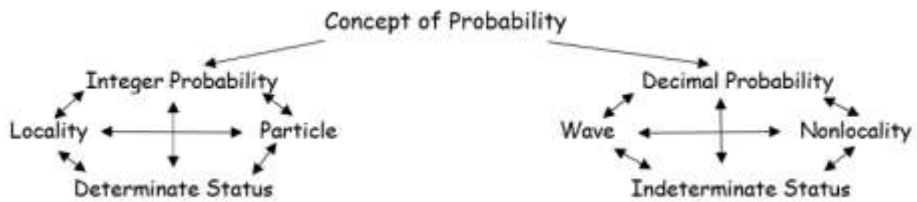

Figure 1 – Conceptual map of quantum dualism.

Quantum theorists take different hypotheses which are not perfectly formalized due to the insufficient support of the probability theory and arrive to conflicting conclusions. The unclear foundations of the probability calculus prevent scientists from analyzing the hallmarks of QM. Parts of the assumptions remain hidden whereas the present theoretical frame explicates all the probabilistic features of quanta.

4 Free Space

The present study, probability driven, addresses the base issues of QM and draws attention to the most straightforward phenomenon that is the *free motion* consisting in one or more quanta ξ which fly from the emitter toward a detector. As classical mechanics begins with the uniform linear motion, so the free motion can be considered the initial event of quantum mechanics. As the calculus of derivatives qualifies the speed changes of bodies, so the calculus of probability qualifies the status changes of quanta.

The free motion occurs in the space free of any force, perturbation, energy exchange etc. As in classical mechanics the uniform linear motion ceases when a force affects it, so the free flight ceases as soon as an external determinant acts on it. Quanta can go through slits, prisms, holes, etc. that do not result in the energy state modification. We also assume that entanglement, spinning and other quantum effects do not regard the free space.

We share the idea that the free flight is an aleatory event starting with the gun **A** which casts the quantic entity ξ toward the sensor **S**; ξ moves during the interval T1 and stops due to the measurement process at t_ξ, notably the interval T2 lasts just one instant

$$\mathrm{T1} = (0, t_\xi]; \quad \mathrm{T2} = t_\xi.$$

4.1 Continuity and discontinuity – The free flight is intrinsically random and from (5) we get that ξ_∞ and $\xi_\mathbf{1}$ take the indeterminate status in the interval T1

$$\xi_\infty = \xi_\infty^{(\mathrm{I})}, \qquad 0 \leq t < t_\xi. \tag{19a}$$
$$\xi_\mathbf{1} = \xi_\mathbf{1}^{(\mathrm{I})}, \qquad 0 \leq t < t_\xi. \tag{19b}$$

If (19a) is true, the theorem of continuity specifies that the outcome of the long-term event remains aleatory in T2

$$\xi_\infty = \xi_\infty^{(\mathrm{I})}, \qquad 0 \leq t. \tag{20}$$

Under the hypothesis (19b), the theorem of discontinuity proves that the indeterminate outcome of the single event changes status in T2, notably when the motion finishes, $\xi_\mathbf{1}$ becomes certain

$$\xi_\mathbf{1}^{(\mathrm{I})} \rightarrow \xi_\mathbf{1}^{(\mathrm{D})}, \qquad t = t_\mathrm{x}. \tag{21}$$

In short, when a field of force affects the beam of electrons, protons etc., $\xi_\infty^{(\mathrm{I})}$ keeps the indeterminate status, conversely the wavelet $\xi_1^{(I)}$ collapses and becomes a particle.

4.3 The first postulate of quantum mechanics associates a wavefunction with a particle moving in a conservative field of force. The wavefunction provides the probabilities for the possible results of measurements.
The current probabilistic study presents a far different perspective. The wavelet $\xi_1^{(I)}$ describes a diffused portion of energy that (14) states as alternative to the particle; therefore, there is no particle whose position is uncertain during T1. There is no erratic corpuscle wandering in the space in advance of the transition (21). *The collapse consists in the condensation of the diffused portion of energy $\xi_1^{(I)}$ into the localized portion $\xi_1^{(D)}$.*

4.4 Why does the wave collapse?
The theorem of discontinuity proves that $\xi_1^{(I)}$ switches into the determinate status when the free movement comes to the end. The free motion ceases because a physical factor, a perturbation, an interference or measurement acts on $\xi_1^{(I)}$. The free motion is no longer free and finishes in consequence of the intrusion.

When exactly does the free flight finish?
Any exchange of energy triggers the end of the free flight and so the collapse. Whatever energy change results in the close of the free movement and at the same time, in the probabilistic status switching.
The measurement process turns out to be the most common (but not unique) perturbation causing the halt of $\xi_1^{(I)}$. A measure is a practical motive of collapse, it is *the operational cause and not the general cause of the individual wave change*. Measuring is the most common factual antecedent of (21) and does not constitute the universal determinant. This conclusion dismantles one of the most puzzling problems of quantum mechanics.

5 Experiments to Validate the Theory

The present section goes through some experiments already published in the literature that support the theoretical conclusions reported in the previous sections.

5.1 Random emission – Expressions (19a) and (19b) state that quanta emitted by a random source are waves and this conclusion is not so common in the literature. Quite a number of scientists hold that quantum emitters send out particles and it is necessary to supply accurate evidence about this fundamental detail.
We specify that the gun **A** complies with the present theory if it is governed by the irregular dynamics of atoms. In practice **A** may be a thermionic tungsten filament, a furnace, a radioactive source, a fluorescent tube, a laser system etc. The emitted quanta are uncertain and thereby are waves because of the internal chaotic state of the source **A**. Besides this general specification, detailed inquiries demonstrate that the emitted quanta are waves.

As first, we cite the equations of Maxwell that describe the wave behavior of the electromagnetic field and thereafter, justifies the wave nature of photons that are discrete portions of electromagnetic energy. As second, we mention the equation of Richardson [11] who calculated the intensity J of the current emitted from the heated filament **A**

$$J = A_0\left(1-r\right)\lambda_R T^2 e^{\frac{-W}{kT}}. \tag{22}$$

Where A_0 is a constant, T the temperature of the electrode, W is the extraction work, k the Boltzmann constant, and λ_R a parameter typical of the metal. The *reflection factor r* is due to a 'mirror effect'. The electrons exiting from the thermionic source behave as waves thrown back from the surface of the metal in conformity with (19a) and (19b).

Clinton Davisson and Lester Germer [20] measured the intensity of electrons emitted by **A** and reflected by a nickel surface. As the angle θ between the detector and the surface changes, so the intensity of the reflected beam complies with the *Bragg law*

$$n\,\lambda = 2\,d\,\sin\theta. \tag{23}$$

Where n is an integer and d the distance between the atomic layers. The phase shift causes constructive and destructive interference and beyond reasonable doubts the Bragg equation proves that flying electrons behave as waves in accordance with (19a) and (19b).

As regard the classification of quanta, experimentalists use an *intense* (*or strong*) *beam* of quanta that gives body to the stream (18). The *weak beam* consisting of a wavelet a time realizes (16). Frequently the wavelet is produced by the same device that emits the intense beam. For example, a physicists can reduce the time frequency of quanta, and passes from type (18) to (16) using a *beam attenuator* or another tool. The physical continuity of the emission process yields that if the intense beam is made of waves (apparent macroscopic observation can bring evidence of this property), then also the individual quantum is a wave.

5.2 Detection of quanta – Any measurement process results in loss of energy and the flight is no longer free. The wave $\xi^{(I)}$ interacting with a sensor is deprived of energy. Technical manuals give the details of the detection process, that we summarize in the case of photodetectors [19].

The λ-thresholds typical of each device serve to classify photodetectors since the wave length is bound to the energy of the incident photons

$$E = h\,c\,/\,\lambda. \tag{24}$$

Let us recall the classification of photodetectors descending from (24).

Thermal detectors measure the power of incident radiation via the heating of a material. A thermal detector does not react to the single photon, but rather to the integrated effect of a number of photons. Thermal detectors operate between around 700 nm and 1 mm (infrared).

Photoelectric detectors make the largest group. Electrons of the metal surface become free by energy absorption obtained by streams of incident photons. Because of the spectrum of technical solutions, photodiodes operate with a broad range of wave lengths from about 200 nm to about 1,700 nm.

Compton detectors. The ingoing photons bounce off target electrons and the wavelength of outgoing photons increases since they transfer energy to the recoiling electrons. This type of sensors works with high energy impinging radiation, approximately with wavelengths ranging from about 1 nm to about 1/1000 nm.

All the photosensors absorb energy and for this reason the free motion of photon wave necessarily finishes with the measurement process.

5.3 The Double Slit Experiment – The double-slit experiment (Figure 2) is often deemed as the real, pedagogically clean and fundamental quantum experiment. The next sub-sections go into two versions of the experiment which cast an intense beam and a weak beam of photons, electrons etc. The third sub-section discusses the double slit experiment which uses macromolecules.

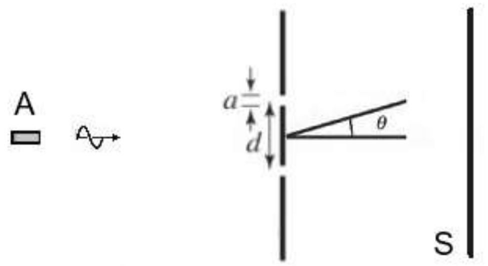

Figure 2 – The double-slit experiment setup.

5.3.1 Interpretation of the intense beam – When **A** emits an intense beam, the detector-screen **S** (e.g. a photographic plate, a fluorescent screen etc.) exhibits a continuous diffraction pattern [16] (Figure 3).

Figure 3– Continuous diffraction pattern
created by an intense laser light through two slits.

The Fraunhofer equation [18] describes the intensity of the spectrum formed by the wave crossing two slits

$$I(\theta) = I_{IN} \cos^2 \alpha \left(\frac{\sin \beta}{\beta} \right)^2 . \tag{25}$$

Where I_{IN} is the on-axis intensity of the incident wave; α and β are two parameters depending on the distance between the slits *d* and the width of each slit *a* (Figure 2). Eqn. (25) qualifies two overposing effects. The first trigonometric function [= ($\cos^2\alpha$)] describes the *interference* resulting from two waves exiting from the slits. The second function [= $(\sin \beta/\beta)^2$] assesses the *diffraction* caused by the narrow width of the slits. The continuous spectrum (Figure 3) is consistent with (25) and brings evidence that the quanta moving in the range (**A**, **S**], which corresponds to the time interval T1, are waves

$$\xi_\infty = \xi_\infty^{(I)}, \qquad 0 \leq t < t_\xi. \tag{26}$$

The spectrum also indicates that the wave stream is still in S at the instant t_ξ – the interval T2 lasts solely an instant – and we write

$$\xi_\infty = \xi_\infty^{(I)}, \qquad t = t_\xi. \tag{27}$$

Eqn. (26) together with (27) prove that the intense beam consists of waves during the intervals T1 and T2, notably quanta remain indeterminate from the emission onward and confirm the theorem of continuity.

5.3.2 Interpretation of the weak beam – When **A** emits a quantum at a time, the screen **S** exhibits a spot [2]. The more is the number of quanta sent one by one, the clearer appears the discrete diffraction pattern on the screen (Figure 4)

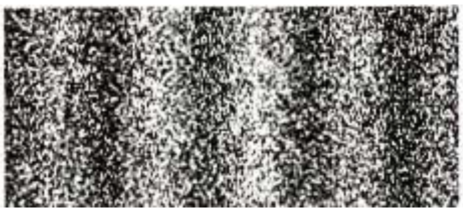

Figure 4 – Discrete diffraction pattern
created by a weak beam of electrons through two slits [19].

Various explanations of these empirical results have been proposed, the present probabilistic inquiry yields a completely innovative interpretation as follow.
The gun **A** casts a quantum and **S** exhibits a dot which brings evidence that a particle is over the screen **S** in T2

$$\boldsymbol{\xi_1} = \boldsymbol{\xi_1}^{(D)}, \qquad t = t_\xi. \tag{28}$$

When the operator repeats the experiment several times and the gun **A** sends K quanta one by one, the screen exhibits a diffraction pattern composed of K dots. As K becomes larger, so the discrete spectrum progressively comes to sight. This phenomenon consists of two overlapping effects.

I] Every wavelet emitted by **A** (Section 5.1) interferes with itself that is to say each incoming wavelet flies in the time interval T1

$$\xi_{\mathbf{1}} = \xi_{\mathbf{1}}^{(\mathrm{I})}, \qquad 0 \leq t < t_\xi. \tag{29}$$

Joining (28) together with (29) we get the collapse of the single wave which confirms the theorem of discontinuity

$$\xi_{\mathbf{1}}^{(\mathrm{I})} \to \xi_{\mathbf{1}}^{(\mathrm{D})}, \qquad t = t_\xi. \tag{30}$$

II] The ensemble of K ($K >> 1$) individual quanta emitted one by one makes the long-term outcome $\xi_\infty^{(\mathrm{I})}$ as in (18)

$$\xi_\infty^{(\mathrm{I})} = \{\xi_{\mathbf{1g}}^{(\mathrm{I})}, \xi_{\mathbf{1f}}^{(\mathrm{I})}, \xi_{\mathbf{1s}}^{(\mathrm{I})}, \ldots\} \tag{18}$$

Counting the number $m(x,y)$ of dots per unit surface area $s(x,y)$ of the screen **S** we obtain the *density function*

$$F_S(x,y) = \frac{m(x,y)}{s(x,y)}. \tag{31}$$

The density is the empirical parameter which offers two services:

a) $F_S(x,y)$ varies with the coordinates (x,y) in **S**, and conforms to (9)

$$0 < F_S(x,y) < 1. \tag{32}$$

Hence (18) has the indeterminate status, notably it is the wave stream $\xi_\infty^{(\mathrm{I})}$.

b) With increasing exposure time, the density of dots shows the fringes which are consistent with eqn. (25) [17]. The spatial frequency $F_S(x,y)$ gives the details of the physics of waves.

Points a) and b) spell out how the wave $\xi_\infty^{(\mathrm{I})}$ keeps the indeterminate status during T1 and T2, and corroborate the theorem of continuity, while every wavelet collapses.

5.3.3 Use of large molecules – After the eighties of the past century physicists began to employ quanta of increasing size to cast against the two slits. They used neutrons, helium atoms, C_{60} fullerenes, and Bose-Einstein condensates. Eibenberger and colleagues [5] have created fringes of interference launching molecules of about 10^4 atomic mass units, rough speaking about 5000 protons and 5000 neutrons. Hackermüller and colleagues [7] inquired the wave nature of biological molecules.

How does the present theory explain these phenomena?

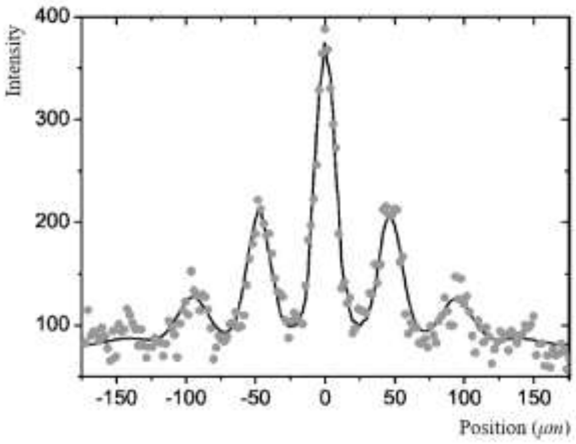

Figure 5 – Diffraction pattern created by a weak beam of C_{60} with two slits [9].

The mentioned ensemble of tests perfectly matches with the present theory that formalizes the particle/wave dualism using the determinate and indeterminate statuses of energy and matter. A quantic entity takes on a discrete value consisting of an integer multiple of one quantum and this frame does not posit limits to the multipliers. The wavelet can even make up a large molecule. It is natural to wonder: There is a threshold to create wavelets?
Can the quantum boundary be determined with precision?

The present theory is consistent with de Broglie who hypothesized the waves of matter have the length λ_B as the inverse of the momentum

$$\lambda_B = \frac{h}{p} = \frac{h}{mv} \tag{33}$$

If the mass m is very 'large', then the wavelength is extremely short, thereby λ_B leads to the boundaries of the quantum dualism. In practice, experimentalists will launch larger and larger molecules, doing so they will obtain extremely short wavelength to the extent that one can no longer control λ_B. They will find out the empirical threshold of the dualism particle/wave and a boundary between the classical and quantum contexts.
The probability theory together with the scheme of de Broglie provides a precise answer to the vexed issues about classical/quantum limits and the correspondence principle in particular.

6 Conclusion

The probability theory currently presents distant and seemingly irreconcilable interpretations. The sad state of this mathematical field can but affect in negative terms quantum mechanics that is intrinsically probabilistic. The theorists need precise cognitive orientation and mathematical instruments in order to tackle basic issues such as the quantum dualism, the wave collapse and the measurement process. It is evident how unresolved *base probability problems* raise or aggravate *base quantum problems*. In consequence of these methodological observations, as first we went through the probability theory and next, we have attacked quantum questions.

This paper describes the particle as a discrete part of energy/matter condensed in a point of the space, while a wave is a part diffused all around. The first concept is synonym with locality, the second with non-locality. The determinate and indeterminate statuses refer to quantic entities that can be as tiny as photons or even as large as macromolecules.

In consequence of the theorems of the large numbers and a single number there are sequels of waves and particles on one side, and one wavelet and one particle on the other side. They present different properties that do not lead to conflicting conclusions since the number *n* gives the criterion to treat each typology of quanta.

The theorem of discontinuity demonstrates that the single wave switches from the indeterminate to the determinate status at the end of its flight. The wavelet condenses in a point and becomes a particle. For the present theory, collapsing is a natural event which occurs whenever the movement of $\xi_1^{(I)}$ is non longer free. The measurement process is an operational and material cause, and not the general cause of the wave collapse.
The wavelet cannot be tested yet various experimental methods allow to circumvent this limit by means of indirect methods.
It is necessary to pinpoint that experiments employing quanta which are entangled, spinning, relativistic etc. lie beyond the present theory, in particular they turn out to be incompatible with the concept of free motion.
The theoretical results about the probability foundations apply to classical and quantum phenomena as well, they suggest a comprehensive and unifying view of physics which scientists are searching since decades. They provide a completely new vision including intuitive notions which respond to the expectations of eminent authors. We have other theoretical results that will be published as soon as possible.

References

[1] G. Parisi, G. Auletta, *Foundations and Interpretation of Quantum Mechanics. In the light of a critical-historical analysis of the problems and of the synthesis of the results*, World Scientific, 2001. https://doi.org/10.1142/4194

[2] R. Bach, D. Pope, S. Liou, H. Batelaan, Controlled double-slit electron diffraction, *New Journal of Physics*, **15** (3), (2013) 1-7. https://doi.org/10.1088/1367-2630/15/3/033018

[3] M. Burgin, P. Rocchi, Basic concepts of the theory of ample probability, *UPI Journal of Mathematics and Biostatistics*, **2** (1) (2019) 1-28.

[4] F. Del Santo, Karl Popper's Forgotten Role in the Quantum Debate at the Edge between Philosophy and Physics in 1950s and 1960s, *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, **67** (2019), 78-88. https://doi.org/10.1016/j.shpsb.2019.05.002

[5] S. Eibenberger, S. Gerlich, M. Arndt, M. Mayor, J. Tüxen, Matter-wave interference with particles selected from a molecular library with masses exceeding 10000 amu, *Phys. Chem. Chem. Phys*., **15** (2013), 14696-14700. https://doi.org/10.1039/c3cp51500a

[6] S. Gao, *The Meaning of the Wave Function. In Search of the Ontology of Quantum Mechanics*, Cambridge University Press, 2017. https://doi.org/10.1017/9781316407479

[7] L. Hackermüller, S. Uttenthaler, K. Hornberger, E. Reiger, B. Brezger, A. Zeilinger, M. Arnd, Wave nature of biomolecules and fluorofullerenes, *Phys. Rev. Lett*., **91** (2003), 90408. https://doi.org/10.1103/physrevlett.91.090408

[8] A. Messiah, *Quantum Mechanics*, Dover Publications Inc., 1999.

[9] O. Nairz, M. Arndt, A. Zeilinger, Quantum interference experiments with large molecules, *American Journal of Physics*, **71** (4) (2003), 319-325. https://doi.org/10.1119/1.1531580

[10] K. Popper, Propensity interpretation of probability, *British Journal for the Philosophy of Science*, **10** (37) (1959), 25-42. https://doi.org/10.1093/bjps/x.37.25

[11] O.W. Richardson, *Thermionic Emission from Hot Bodies*, Wexford College Press, 2003.

[12] P. Rocchi, *The Structural Theory of Probability: New Ideas from Computer Science on the Ancient Problem of Probability Interpretation*, Kluwer/Plenum, 2003. https://doi.org/10.1007/978-1-4615-0109-1

[13] P. Rocchi, L. Gianfagna, An essay on the double nature of the probability, *Advances in Pure Mathematics*, **2** (5) (2012), 305-309. https://doi.org/10.4236/apm.2012.25041

[14] P. Rocchi, *Janus-Faced Probability*, Springer, 2014. https://doi.org/10.1007/978-3-319-04861-1

[15] P. Rocchi, O. Panella, Some probability effects in the classical context, *Open Physics*, **18** (1) (2020), 512-516. https://doi.org/10.1515/phys-2020-0178

[16] J. San Roman, C. Ruiz, J.A. Perez, D. Delgado, C. Mendez, L. Plaja, L. Roso, Non-linear Young's double-slit experiment, *Optics Express*, **14** (7) (2006), 2817-2824. https://doi.org/10.1364/oe.14.002817

[17] E.A. Santos, F. Castro, R. Torres, Huygens-Fresnel principle: Analyzing consistency at the photon level, *Physical Review A*, **97** (4) (2018), 043853. https://doi.org/10.1103/physreva.97.043853

[18] A. Siciliano, *Optics: Problems and Solutions*, World Scientific, 2006. https://doi.org/10.1142/6097

[19] A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, H. Ezawa, Demonstration of single-electron buildup of an interference pattern, *American. J. Physics*, **57** (1989), 117-120. https://doi.org/10.1119/1.16104

[20] F. Weinert, Davisson-Germer Experiment, In: Greenberger D., Hentschel K., Weinert F. (eds), *Compendium of Quantum Physics*, Springer, 2009. https://doi.org/10.1007/978-3-540-70626-7_45

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