Advanced Studies in Theoretical Physics
Vol. 14, 2020, no. 1, 1 - 8
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/astp.2020.9416

Deriving Quantised Inertia Using Horizon-Widths in the Uncertainty Principle

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Abstract

In this paper we show that a model for inertial mass (quantised inertia) can be derived exactly and simply by assuming that the property of inertia is caused by gradients in the energy in the Unruh radiation field seen by a object, and that this energy is determined by the uncertainty principle, with the uncertainty in position being given by the width of the Rindler horizons seen by the object. This is then an alternative derivation for quantised inertia which predicts galaxy rotation without dark matter. It is also shown that quantised inertia predicts an expression close to the simple empirical interpolation function used in MoND.

Keywords: Quantised inertia, uncertainty principle, Dark matter, Dark energy

1 Introduction

Physics is split between relativity, which uses smooth spacetime and demands locality, and quantum mechanics which is modeled using wave-particles and seems to demand non-locality [1, 2, 3, 4, 5]. The first example of combining both theories at astronomical scales was [6]. He suggested that the black hole event horizons would separate virtual particles, one falling into the black hole and one escaping, producing Hawking radiation. Fluid analogues of this process have been shown to occur [7].

Due to the equivalence of gravity and acceleration, [8], [9] and [10] showed that when an object accelerates it sees an information horizon (like an event horizon) appears in its frame of reference on the side opposite to the acceleration. This happens because information is limited to the speed of light by relativity and cannot get to the object from behind that horizon. This horizon similarly separates paired virtual particles producing acceleration-dependent Unruh radiation. Unruh radiation is now widely accepted, but see [11]. Unruh radiation may already have been observed [12].

The phenomena of inertial mass has never been well understood, just assumed as Newton's first law. An early inspiring attempt to explain inertial mass with quantum vacuum was made by [13]. This scheme is unconvincing however because it needs an arbitrary cutoff. Also, [14] asked whether Unruh radiation might be a way to derive inertial-MoND (Modified Newtonian Dynamics), but concluded that Unruh radiation could not be the cause of inertia, being isotropic.

In [15, 16] is proposed a new hypothesis to explain inertial mass called quantised inertia. In this model the inertia of an object is explained by the Unruh radiation it sees when it accelerates, but also the relativistic horizon that forms in the opposite direction to the acceleration vector disallows some wavelengths of the Unruh radiation on that side of the object giving rise to an anisotropic radiation pressure that predicts the inertial mass quite well, see [16, 17]. So inertia in quantised inertia comes from a combination of relativity (the horizons) and quantum mechanics (the Unruh waves). A new feature is that when accelerations are tiny the Unruh waves lengthen and are also disallowed by the cosmic horizon, this time equally in all directions (Hubble-scale Casimir effect) [15]. This causes a new loss of inertia when accelerations are tiny. Quantised inertia modifies the usual inertial mass m to m_i as follows:

$$m_i = m\left(1 - \frac{2c^2}{|a|\Theta}\right). \tag{1}$$

Here, c is the speed of light, the Θ is the co-moving diameter of the observable cosmos and $|a|$ is the acceleration. Eq. 1 predicts that when accelerations are large, or at least terrestrial in size (eg: $9.8m/s^2$) the second term in the brackets becomes tiny and standard inertia is recovered, but when the acceleration is very low, for example at the edges of galaxies (when a is tiny) the second term in the bracket becomes larger and the predicted inertial mass decreases in a new way. Quantised inertia explains galaxy rotation without dark matter

[18, 23] and cosmic acceleration without dark energy [15, 19].

In a similar way, applying quantum mechanics on a large scale [21] derived the form of Newtonian gravity from the uncertainty principle and [22] derived a formula close to Eq. 1 (quantised inertia) by using relativistic horizons in the uncertainty principle but the expression derived was 26% too large. The Hawking and Unruh temperatures were derived exactly with a similar method by [24] with the new proposal that what is important is not the horizon distance but the half-circumference (width) of the horizon, see also [25]. The advance in this paper is to build on [22] using [24], and derive quantised inertia exactly (and its successes such a predicting galaxy rotation without dark matter [23] and show that the result is close to the simple interpolation function of MoND but without needing an arbitrary constant.

2 Quantised Inertia from Uncertainty

First we are going to compute the energy uncertainty of the Unruh radiation. Using the momentum-position uncertainty principle we get

$$\Delta p \Delta x \sim \hbar/2. \tag{2}$$

For the photons of Unruh radiation E=pc so

$$\Delta E \Delta x \sim \hbar c/2. \tag{3}$$

The energy uncertainty of these photons is then $\Delta E \sim \hbar c/(2\Delta x)$. The assumption (as in [22]) is that when a particle accelerates and a relativistic Rindler horizon forms then this destroys all the particle's potential knowledge of space beyond the horizon and decreases the uncertainty in position Δx. From Eq. 3 we would then expect the uncertainty in energy to go up. We are going to see that this energy uncertainty predicts the modified inertial mass of one particle. As we have said the inertial mass of a particle is due to the Unruh radiation bath it sees when it accelerates. The relativistic (so called Rindler) horizon that appears in the opposite direction to the acceleration vector damps the Unruh radiation on that side leading to an anisotropic radiation pressure that looks like inertial mass. On the other side we have the Hubble horizon or cosmic horizon that also damps the Unruh radiation from this opposed side. Hence we have that the resulting energy uncertainty of these photons is

$$\Delta E = \Delta E_2 - \Delta E_1 = \frac{\hbar c}{2\Delta x_2} - \frac{\hbar c}{2\Delta x_1} \tag{4}$$

where Δx_1 and Δx_2 are the positional uncertainties in the Hubble horizon and in the Rindler horizon respectively. Hence we have

$$\Delta E = \frac{\hbar c}{2}\left(\frac{1}{\Delta x_2} - \frac{1}{\Delta x_1}\right). \tag{5}$$

Now we consider these relativistic horizons. For an object at a minimal acceleration (a zero acceleration is forbidden by quantised inertia) the maximum uncertainty in position has to be due to the cosmic horizon. The uncertainty in position is a spatial magnitude of dimension one. The vacuum fluctuations produce the appearance of a particle-antiparticle. If this appearance occurs near the cosmic horizon one of the pair can cross the event horizon while the other escapes becoming real and producing what is called Unruh radiation. For this radiation the Unruh temperature is associated with the cosmic horizon. The uncertainty of the crossing point of the cosmic horizon determines the uncertainty in the position. We assume here, as in [24] that this is the half-circumference of the Hubble-sphere $\Delta x_2 = \pi R_U$ where R_U is the radius of the cosmos. Hence $\Delta x_1 = \pi\Theta/2$ where $\Theta = 2R_U$ (R_U=cosmic radius), so that

$$\Delta E = \frac{\hbar c}{2}\left(\frac{1}{\Delta x_2} - \frac{2}{\pi\Theta}\right). \tag{6}$$

If an object is then given an acceleration, a, then a Rindler horizon forms at a distance away of $d = c^2/a$. As before, the Unruh temperature associated with the Rindler horizon is a consequence of the vacuum fluctuations that appear close to the Rindler horizon. The uncertainty of the crossing point of the Rindler horizon determines the uncertainty in the position. So the new uncertainty of the crossing point is given by $\Delta x_1 = \pi c^2/a$ so that

$$\Delta E = \frac{\hbar c}{2}\left(\frac{a}{\pi c^2} - \frac{2}{\pi\Theta}\right). \tag{7}$$

Rearranging we get

$$\Delta E = \frac{\hbar\, a}{2\pi c}\left(1 - \frac{2c^2}{a\Theta}\right). \tag{8}$$

The acceleration 'a' is associated with the Unruh radiation from the Rindler horizon whose temperature is given by

$$T = \frac{\hbar\, a}{2\pi c k_B}. \tag{9}$$

where k_B is the Boltzmann constant. Now we take into account that the typical energy of a photon from the Unruh radiation is $E_p = k_B T$. Using this and Eq. 8 we can replace the 'a' in the factor, to get

$$\Delta E_p = E_p\left(1 - \frac{2c^2}{a\Theta}\right). \tag{10}$$

Consequently the energy uncertainty of a photon from the Unruh radiation is given by (10). Let $E_m = mc^2$, the energy of the accelerated particle. Let $N = E_m/E_p = E_m/(k_B T)$, the number of photons equivalent to the energy of the particle with mass m. Then we have

$$\Delta E_m = N\Delta E_p = NE_p\left(1 - \frac{2c^2}{a\Theta}\right) = E_m\left(1 - \frac{2c^2}{a\Theta}\right). \tag{11}$$

Now using $E_m = mc^2$ for the particle we get:

$$\Delta m = m\left(1 - \frac{2c^2}{a\Theta}\right). \tag{12}$$

This is the same as Eq. 1. The important point is that quantised inertia (Eq. 1) can be derived taking into account the position uncertainty of both the Rindler and cosmic horizons. Another way to deduce Eq. 1 is given in [28].

3 Discussion

The simple MoND function [26, 27], which has been derived empirically from galaxy rotation data, modifies Newton's second law (or in another variant, gravity) such that

$$F = m\left(\frac{1}{1 + \frac{a_0}{a}}\right)a. \tag{13}$$

In which the implied modified inertia mass is

$$m_i = m\left(\frac{1}{1 + \frac{a_0}{a}}\right). \tag{14}$$

The Maclaurin series expansion is

$$m\left(\frac{1}{1 + \frac{a_0}{a}}\right) = m\left(1 - \frac{a_0}{a} - \frac{a_0^2}{a^2} - ...\right). \tag{15}$$

Therefore the modified inertial mass predicted by quantised inertia (Eq. 11) is equal to the first two terms of the expansion of the simple MoND function, except that the value of MoND's adjustable parameter a_0 is predicted by quantised inertia itself as being $2c^2/\Theta$. Both MoND and quantised inertia solve the galaxy rotation problem without dark matter [18, 23], but quantised inertia does it without arbitrary adjustment, and it also explains the observed cosmic acceleration without needing dark energy [15, 19].

4 Conclusion

A recent model for inertia called quantised inertia, which predicts galaxy rotation without needing dark matter, and some other anomalies, can be derived exactly by assuming that the property of inertia is caused by gradients in the energy in the Unruh radiation field seen by the object, and that this energy is determined by the uncertainty principle, with the uncertainty in position being given by the width of the relativistic horizons seen by the object.

It is also shown that quantised inertia predicts an expression close to the simple empirical interpolation function used in MoND.

Acknowledgements. The second author is partially supported by a MINECO/ FEDER grant number MTM2017-84383-P and an AGAUR (Generalitat de Catalunya) grant number 2017SGR-1276.

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Received: May 12, 2019; Published: July 14, 2019