Cosmologic Log Periodic Solutions of Interactive Spinorial and Scalar Fields in an Anisotropic Space-Time of Petrov D

R. Alvarado

CINESPA, Escuela de Física
Universidad de Costa Rica, Costa Rica

This article is distributed under the Creative Commons by-nc-nd Attribution License.
Copyright © 2019 Hikari Ltd.

Abstract

This study obtained two exact cosmologic solutions to the Einstein equations considering the interactive, non-linear and self-consistent spinorial and scalar fields in an anisotropic homogeneous symmetry of Petrov D. For this proposed interaction, it is obtained that the metric tensor, the fields, the volumetric density of the energy and the pressure are log periodic functions or related to them. It is determined that in proximities to \( t \to 0 \), a solution is singular with characteristics similar to its analogous of Kasner; for said proximity, the other one is not singular and it is even immediately the metric of the flat world for some values of \( t \). The parameter of Hubble \('H'\) and the deceleration parameter \('q'\) were analyzed and it was obtained that the parameter of Hubble is indefinite for both solutions when \( t \to 0 \), and it depends on \( t \), to some extent, as a log periodic function with values of \( H \in ]0, \sqrt{C_1/3}[, \) where \( C_1 \) is a constant that indicates the maximum value that the density of the energy of the interactive scalar and spinorials fields have; on the other hand, the deceleration parameter is a log periodic function that can take positive or negative values, maximum or minimum. It is determined that the function of the scalar field, for the proposed interaction, is part of the components of the spinor, and it is observed that the influence of the scalar field in the phase of the components of the spinor varies depending on time, being mostly non-linear as the configuration of the space-time is different to the flat world, for example, when is close
to its equivalent dark energy, and mostly lineal for times when tends to be close to the flat world.

**Keywords:** cosmology, Einstein, exact, fields, log periodic, solution

## 1 Introduction

Based on the data from the background radiation obtained by the COBE, WMAP, and PLANCK satellites, and the discovery of the universe acceleration [1], [2], cosmology has become more dynamic in these and other aspects that have been already discussed on [3].

On the other hand, the study of the physic fields in cosmology, such as the scalar, the spinorial and the electromagnetic fields, in combinations of ideal fluids, isolated, or in their interactions, have relevance because of the possibility of investigating the different states of the matter for which the Universe could pass through, its influence over the possible singularities, and the changes for which the anisotropic space-time configuration could pass through. The importance of said studies has been mentioned and investigated, for example, in [4], [5], [6], and [7]. Moreover, the interest for the cyclic solutions or with cyclic characteristics has increased; some examples have been studied and discussed in different articles (for example [8], [9]). On [8], it is analyzed the possibility that, in an anisotropic space-time of Petrov D, matter, due to a temporal perturbation, changes its state so that after some time, tends to come back to its initial state, without completely reaching it, but without implying that the effective entropy does not increase, leaving place to an almost oscillatory state, for which the changes of the matter are gradual.

Some processes of scientific interest are ruled by log periodic functions, in other words, they change according to $sin(ln(f(t, x, y, z)))$; said processes happen in different areas of nature and life like earthquakes and icequakes [10], [11], besides the modulation in the concentration of ions in groundwater in earthquakes [12]; in finances [13], in small ferromagnetic interactions on hierarchical systems [14]; in biology [15] and in processes of correction of vibration and discrete fractal structure waves on [16]. Because of this, the interest of studying the possibility that the Universe is ruled that way emerges. The possibility that cosmologic models are guided through log periodic functions is what this study analyzes.
2 The Symmetry of the Anisotropic Space-Time of the Petrov D Type and the Einstein Tensor

2.1 The Symmetry

The symmetry that will be used in this work is the anisotropic and homogeneous of the Petrov D type, which has the pattern [3]

\[ ds^2 = F dt^2 - t^{2/3}K (dx^2 + dy^2) - \frac{t^{2/3}}{K^2} dz^2, \] (1)

where \( F \) and \( K \) are the functions of \( t \).

The components of the Einstein tensor [3] \( G^\beta_\alpha = R^\beta_\alpha - 1/2 \delta^\beta_\alpha R \) different than zero of (1), are

\[ G^0_0 = \frac{4K^2 - 9t^2K^2}{12t^2K^2F}, \] (2)

\[ G^1_1 = -\frac{3Kt\dot{K} \left( 2F - \dot{F}t \right) + 3Ft^2 \left( 2K\dot{K} - 5K^2 \right) + 4K^2 \left( \dot{F}t + F \right)}{12t^2K^2F^2}, \] (3)

\[ G^2_2 = G^1_1, \] (4)

\[ G^3_3 = -\frac{6Kt\dot{K} \left( 2F - \dot{F}t \right) - 3Ft^2 \left( 4K\dot{K} - \dot{K}^2 \right) + 4K^2 \left( \dot{F}t + F \right)}{12t^2K^2F^2}, \] (5)

where the points over the functions represent derivatives of the time.

2.2 Spinorial Fields and Interactive Scalars

The interactive spinorial and scalar fields of this study are determined by the following lagrangian:

\[ \mathcal{L} = \frac{1}{2} (\bar{\psi} \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi) - m\bar{\psi}\psi + \frac{1}{2} \varphi \varphi^\alpha \Phi(S), \] (6)

where \( \psi \) is the spinor, \( S = \bar{\psi}\psi \) is the invariant scalar of the spinorial field, the function \( \Phi(S) \) is a non-linear function of the spinorial field, and \( \varphi \) is the function of the scalar field; moreover, \( \bar{\psi} \) is the hermitian conjugated spinorial function of Dirac, which is defined as \( \bar{\psi} = \psi^* \gamma^0 \) (that will be defined later on.
$\bar{\gamma}^\beta$) and where the matrices $g_{\mu\nu}\gamma^\nu = \gamma_\mu$ can be defined, regarding the metric of the pattern,

$$\gamma_\mu = e_\mu^\eta \bar{\gamma}^\eta, \quad g_{\mu\nu} = e_\mu^\eta e_\nu^\delta h_{\eta\delta}, \quad \gamma^\mu$$

(7)

where $h_{\eta\delta}$ is the tensor of Minkowski of the flat world with signature $(+,-,-,-)$, and the matrices $\bar{\gamma}_\eta$ are the matrices of Dirac defined for the flat space-time, $e_\mu^\eta, e_\mu^\delta$ is the set of tetrads of vector 4. The matrices $\bar{\gamma}^\eta$ are taken from the pattern:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad \bar{\gamma}^1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

(8)

$$\begin{bmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, \quad \bar{\gamma}^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

(9)

The covariant spinorial derivative $\nabla_\mu$ is defined as

$$\nabla_\mu \psi = \frac{\partial}{\partial x^\mu} \psi - \Gamma_\mu \psi,$$  

(10)

where $\Gamma_\mu$ are the spinorial matrices of the coefficient of analogous relation, which is vinculated to the symbol of Christoffel, in the following way

$$\Gamma_\mu = \frac{1}{4} g_{\rho\delta}( (e_\sigma^\eta )_{,\rho} e_\eta^\mu - \Gamma^\rho_{\mu\sigma}) \gamma^\delta \gamma^\sigma.$$  

(11)

From the function of Lagrange (6), and the metric (1), the equations of the spinorial field, the ones of the scalar field, and the components of the stress-energy tensor are obtained. These equations and components can be expressed as

$$\frac{i}{2} \gamma^\mu \nabla_\mu \psi + \frac{i}{2} \nabla_\mu \gamma^\mu \psi - m \psi + \frac{1}{2} \varphi,_{\alpha} \varphi,^{\alpha} \Psi'(S) \psi = 0, \Psi'(S) = \frac{d\Psi}{dS},$$  

(12)

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} [\sqrt{-g} g^{\mu\nu} \{ \varphi,_{\mu} \Phi(S) \}] = 0,$$  

(13)
\[ T^\rho_\mu = \frac{i}{4} \theta^{\rho \nu \mu} [\bar{\psi}_\gamma \nabla_\nu \psi - \nabla_\mu \bar{\psi}_\gamma \nu \psi + \bar{\psi}_\gamma \mu \nabla_\nu \psi - \nabla_\nu \bar{\psi}_\gamma \mu \psi] + \\
+ \varphi,_{\mu} \varphi^\rho \Psi(S) - \delta^\rho_\mu L. \quad (14) \]

From (7), the set of tetrads, can be taken the following way:

\[ e^0_0 = \frac{1}{e^0_0} = \sqrt{F}, \quad e^1_1 = e^2_2 = \frac{1}{t^{1/3} \sqrt{K}}, \quad e^3_3 = \frac{1}{t^{1/3} K}. \quad (15) \]

3 Solution

It will be assumed that the spinor \( \psi \) and the scalar \( \varphi \) depend only on \( t \), therefore, from the equation of the scalar field (13), it is obtained

\[ \varphi = \int \frac{C_{sc} \sqrt{F}}{t \Phi} dt + \varphi_0. \quad (16) \]

The equation of the spinor field takes the pattern

\[ \frac{\dot{\psi}_r}{\sqrt{F}} + \frac{\psi_r}{2t \sqrt{F}} + i \left( m - \frac{C_{sc} 2 \Phi S}{2t^2 \Phi^2} \right) \psi_r = 0, \]

\[ \frac{\dot{\psi}_{r+2}}{\sqrt{F}} + \frac{\psi_{r+2}}{2t \sqrt{F}} - i \left( m - \frac{C_{sc} 2 \Phi S}{2t^2 \Phi^2} \right) \psi_{r+2} = 0, \quad r = 1, 2, \quad (17) \]

where the point represents the partial derivative of \( t \). From (17), it is obtained that

\[ \bar{\psi} \psi = S = \frac{C_{sp}}{t}, \quad (18) \]

where \( C_{sp} \) is a constant of integration.

From (18), and considering that

\[ \Phi = C_{sc} 2 S^2 C_{sp}^{-2} \left( C_1 - C_1 \sin \left( \ln \left( \frac{S}{C_2} \right) \right) - 2 Sm \right)^{-1}, \quad (19) \]

the non-linear interaction in the lagrangian (6), takes the pattern

\[ L_{no-lin} = \frac{1}{2} \varphi,_{\alpha} \varphi^\alpha \Phi(S) = \frac{F}{2} C_1 + \frac{F}{2} C_1 \sin \left( \ln \left( \frac{tC_2}{C_{sp}} \right) \right) - \frac{C_{sp} m F}{t}, \quad (20) \]

where \( C_2 \) and \( C_1 \), are constants, whose physical sense is determined by energetic and temporal concepts that will be analyzed later on. The components of the spinor, of (17), can be expressed as

\[ \psi_r = \alpha_r e^{-phas} \sqrt{t}, \quad \psi_{r+2} = \alpha_{r+2} e^{phas} \sqrt{t}, \quad r = 1, 2, \quad (21) \]
where \( \alpha_1, \alpha_2, \alpha_3 \) and \( \alpha_4 \), and the constant relates to \( C_{sp} \) in the way \( C_{sp} = \alpha_1^2 + \alpha_2^2 - \alpha_3^2 - \alpha_4^2 \) and

\[
\text{phas} = \int \left( i\sqrt{F}m + \frac{i\sqrt{FC_{sc}}\overline{\Phi}}{2\Phi^2C_{sp}} \right) dt + \text{phas}_0, \tag{22}
\]

where \( \text{phas}_0 \) is a constant.

The components of the tensor of stress-energy (14), take the pattern

\[
T^0_0 = \frac{\partial^2 \Phi}{2F} + \frac{mC_{sp}}{t}, \quad T^k_k = -\frac{C_{sc}^2\overline{\Phi}}{2t\Phi^2} - \frac{\partial^2 \Phi}{2F}, \quad k = 1, 2, 3, \tag{23}
\]

for this reason, the components of the Einstein tensor (3) and (5) should be the same, from where it is obtained that

\[
K = K_0C_k \int \frac{F^{1/2}}{t} dt, \tag{24}
\]

where \( K_0 \) and \( C_k \) are constants, which without losing their generalities, can be taken as (24) \( K_0 = 1 \) and the constant \( C_k = \pm 2/3 \), which provides two possible kinds of expansion (higher over an axis or higher over the perpendicular plane of the axis), as it will be shown.

Considering (24), the independent equations of Einstein, of (2), (3) and (23), take the pattern

\[
-\frac{1}{3Ft^2}F - \frac{C_1}{C_{sp}} + \sin \left( \ln \left( \frac{tC_2}{C_{sp}} \right) \right) C_1 = 0, \tag{25}
\]

\[
-\frac{\dot{F}t + F^2 - F}{3F^2t^2} - \frac{C_1}{2} \left( \cos \left( \ln \left( \frac{tC_2}{C_{sp}} \right) \right) + 1 + \sin \left( \ln \left( \frac{tC_2}{C_{sp}} \right) \right) \right) = 0, \tag{26}
\]

the solution that meets the equations (25) and (26), is

\[
F = \frac{1}{\frac{3C_1}{2} t^2 \left( 1 + \sin \left( \ln \left( \frac{tC_2}{C_{sp}} \right) \right) \right) + 1}, \tag{27}
\]

from which it is defined (24), (22) and (16). The no-null components of the stress-energy tensor (23), which represent the volumetric density and pressure take the pattern

\[
T^0_0 = \mu = \frac{C_1}{2} \left( 1 + \sin \left( \ln \left( \frac{C_2 t}{C_{sp}} \right) \right) \right),
\]

\[
T^1_1 = T^2_2 = T^3_3 = -P = \frac{C_1}{2} \cos \left( \ln \left( \frac{C_2 t}{C_{sp}} \right) \right) + T^0_0, \tag{28}
\]

and meet the equality \( T^\mu_\nu = 0 \).

From (28) and (18), the constant \( C_2 = S_0 = C_{sp}/t_0 \) is obtained, so that the density \( \mu = \mu_{max}/2 \) and \( \mu_{max} = C_1 \), where \( \mu_{max} \) is the maximum value of the energy-volumetric density.
4 Analysis of the Solution

4.1 Kretschmann Invariant

The possible singularity, when \( t \to 0 \), in the obtained solutions, can be investigated using the Kretschman invariant, which can be expressed as \( Krets = R^{\alpha\beta\gamma\tau} R_{\alpha\beta\gamma\tau} \). For the used symmetry, considering (24) two possible invariants are taken, depending on the positive or negative value of \( C_k = \pm 2/3 \), which follow the pattern

\[
Krets_{\pm} = \frac{-24 F^3 + 24 F \dot{F} \dot{t} (1 - F) + 36 F^4 + 20 F^2 \pm 32 F^{7/2} + 9 \dot{F}^2 t^2}{27 F^4 t^4}. \tag{29}
\]

The invariant \( Krets_{\pm} \), considering (27) is

\[
Krets_{\pm} = \frac{32}{27} t^{-4} + \frac{2C_1}{9t^2} \sin \left( \frac{1}{2} \ln \left( \frac{tC_2 e^{\pi/2}}{C_{sp}} \right) \right)^2 \left( 12 t^2 C_1 + 8 + 9 C_1 \cos \left( \ln \left( \frac{tC_2}{C_{sp}} \right) \right) t^2 \right) + \frac{1}{12} C_1^2 \cos \left( \ln \left( \frac{tC_2}{C_{sp}} \right) \right)^2 \pm \frac{32}{27t^4} \sqrt{3 t^2 C_1 \sin \left( \frac{1}{2} \ln \left( \frac{tC_2 e^{\pi/2}}{C_{sp}} \right) \right)^2 + 1} \tag{30}
\]

when \( t \to 0 \), it is obtained that

\[
Krets_{+} \approx \frac{64}{27t^4} \tag{31}
\]

and for \( Krets_{-} \), it is obtained

\[
Krets_{-} \approx -\frac{1}{4} C_1^2 \left( \cos \left( \ln \left( \frac{tC_2}{C_{sp}} \right) \right) \right)^2 + 2 C_1^2 \sin \left( \frac{1}{2} \ln \left( \frac{tC_2 e^{\pi/2}}{C_{sp}} \right) \right)^2 \left( \cos \left( \ln \left( \frac{tC_2}{C_{sp}} \right) \right) + 2 \right) \tag{32}
\]

The solution with positive sign in (24) is singular when \( t \to 0 \) and it behaves in an analogous manner to the solution of Kasner \( E_D \) in said proximities [3], however, with the negative sign it does not; moreover, the value of the invariant \( Krets \) can tend to zero in low or not values of \( t \to t_n = C_{sp} e^{(3\pi/2 + 2n\pi)} / C_2 \), \( n \in \mathbb{Z} \), which represents a behavior close to the models of ideal fluids of the Phantom type [3] in the proximities to \( t \to 0 \). The solution with negative sign in (24) is immediately the flat world when \( t = t_n \), for said values, is the tensor of Riemann \( R^{\alpha\beta\gamma\tau} \left( t_n \right) = 0 \).
4.2 Tendencies of the Fields and the Metric of Temporal Points

Because the solutions are log periodic, there exists an infinite amount of values of \( t \), for the ones the space-time and the fields tend to behave in a similar manner that they did in the past, which does not mean, for example, that the space expands and shrinks as it usually does in cyclic models [9]; this allows an equivalent or close start to the previous cycle. Said cyclic process is presented in different spatial scales and the space always increase its size to allow the possibility of repeating processes, where the fields (the matter) gets involved, but in higher scales.

There exists, fundamentally, two sets of tendencies in the solutions (24), with their respective time intervals, and they differ in a great manner; one correspond to the solutions when \( t \to 0 \). This set can be found in the proximities to values of (33), for which \( F \to 1 \). The other set is present when

\[
F \to \frac{1}{\frac{3C_1}{2} t^2 \left(1 + \sin \left(\frac{tC_2}{C_{sp}}\right)\right)}.
\]  

(34)

In its general form, the solution of the function of the scalar field (16) is

\[
\varphi = \int \frac{\left(-2mC_{sp} + C_1 t \left(1 + \sin \left(\frac{tC_2}{C_{sp}}\right)\right)\right) \sqrt{2}}{C_{sc} \sqrt{3 t^2 C_1 \left(1 + \sin \left(\frac{tC_2}{C_{sp}}\right)\right)} + 2} dt + \varphi_0,
\]  

(35)

and the components of the spinor (17) or (21) are

\[
\psi_r(t) = \frac{\alpha_r e^{-\text{phas}}}{\sqrt{t}}, \quad \psi_{r+2}(t) = \frac{\alpha_{r+2} e^{\text{phas}}}{\sqrt{t}}, \quad r = 1, 2, \quad \text{and}
\]

\[
\text{phas} = -\frac{i C_{sc} \varphi}{C_{sp}} + i \int \frac{C_1 \cos \left(\ln \left(\frac{tC_2}{C_{sp}}\right)\right) t \sqrt{2}}{2 C_{sp} \sqrt{\left(3 t^2 C_1 \left(1 + \sin \left(\frac{tC_2}{C_{sp}}\right)\right)\right) + 2}} dt + \text{phas}_0
\]  

(36)

4.2.1 Tendency \( t \to 0 \) or \( t \to t_n \)

In the close vicinity of \( t \) with \( t_n \), the functions (24), (27), (35) and (36) can be written as:

\[
K \approx K_n = K_{0n} t^{C_k}, \quad F \approx 1, \quad \varphi \approx \varphi_n = -\frac{2mC_{sp}}{C_{sc}} t + \varphi_{0n}, \quad \text{and}
\]

\[
\text{phas} \approx \text{phas}_n = -\frac{i C_{sc}}{C_{sp}} \varphi_n + \text{phas}_{0n},
\]  

(37)
where \( K_{0n}, \varphi_{0n}, \) and \( \text{phas}_{0n} \) are integration constants.
In these cases (when \( t \to t_n,0 \) or \( t_{-n} \)), the function of the scalar field tends to
\[
\varphi \to \varphi_{In} = -2 \frac{m C_{sp} t_n}{C_{sc}} + \varphi_{0n}.
\]
and the phase (36) of the spinor tends to
\[
\text{phas}_{n} = -i \frac{C_{sc} \varphi_{In}}{C_{sp}} + \text{phas}_{0n}.
\]

The metric (1), of each of these solutions, tends to the Kasner solution [3] \( E_{D_1} \), when it is taken with the positive sign in (24), or like the flat world \( E_{D_0} \), in the case that the negative sign is taken. The stress-energy tensor tends to be null in both solutions.

### II Tendency

The second tendency is present if it is met (34), that is more frequent in high values of \( t \). The function of the scalar field \( \varphi \) tends to \( \varphi_{II} \), which is defined as
\[
\varphi_{II} = \frac{4 m C_{sp}}{\sqrt{3C_1 C_{sc}}} \ln \left( \frac{2 C_1 + C_1 |\cos(\xi)|}{|\sin(\xi)|} \right) \text{sgn} \left( \sin \left( 2 \xi \right) \right) +
\]
\[
-4 \sqrt{C_1 t} \cos \left( \xi + \arctan(2) \right) \text{sgn} \left( \sin \left( \xi \right) \right) \sqrt{15 C_{sc}} + \varphi_0, \approx
\]
\[
\approx -4 \sqrt{C_1 t} \cos \left( \xi + \arctan(2) \right) \text{sgn} \left( \sin \left( \xi \right) \right) \sqrt{15 C_{sc}} + \varphi_0
\]

where \( \xi = \frac{1}{2} \ln \left( \frac{C_{sp} t}{C_{sc}} \right) + \frac{\pi}{4} \).

The phase (36) of the spinor tends to
\[
\text{phas} \to \text{phas}_{II} = -i \frac{C_{sc} \varphi_{II}}{C_{sp}} \left( 1 + \frac{1}{2} \tan \left( \xi + \arctan(2) \right) \right) + \text{phas}_0.
\]

The functions (24), tend to \( K_{\pm} \to e^{\pm \Gamma} \), where \( (\Gamma \in \mathbb{R}) \), it is defined as
\[
\Gamma = (-1)^{k+1} \left( \frac{-\frac{2}{9} - \frac{2i}{9}}{\sqrt{6}} \sqrt{C_1 t} \left( i \sin \left( \frac{1}{2} \rho \right) + \cos \left( \frac{1}{2} \rho \right) \right) L(\rho) \right) -
\]
\[
\sqrt{6} \text{Im} \left( \frac{-\frac{2}{9} - \frac{2i}{9}}{C_{sp} \sqrt{C_1 t} e^{2(k+1)\pi}} \Phi \left( i, 1, \frac{1}{2} + i \right) \right) + \Gamma_0
\]
\[
L(\rho) = \Phi \left( -\sin(\rho) + i \cos(\rho), 1, \frac{1}{2} + i \right), \Phi \left( z, s, \alpha \right) \text{ is the Lerch transcendent function and } \rho = \ln \left( \frac{C_{sp} t}{C_{sc}} \right). \text{ The values of } k \text{ in } (42), \text{ are defined so that if}
\]
The function $\Gamma$ in (42), tends to zero when $t \to \infty$, $(t \not\to C_{2s}e^{(3/2+2n)\pi})$. Because of this, the metric (1) tends to be isotropic with time increase, in the case of tendency II. The function $F$, in (34), tends to behave, for non-prolonged periods of time, close to $F \approx 1/(\alpha t^2)$, where $\alpha$ is constant for the chosen interval; therefore, the function of $\sin \left( \ln \left( \frac{C_2 t}{C_{sp}} \right) \right)$ slightly changes if the values of time are high and the intervals of time $t \in [-t_m + T, T + t_m]$ of this analysis are relatively short. The metric (1), for the mentioned case, with an approximate value of time $t = T \neq t_n$, tends to

$$ds^2 \approx d\eta^2 - \left( e^{W(-(e^{\eta_0}/e^{\eta_m})+\eta_0)} \right)^{2/3} (dx^2 + dy^2 + dz^2) \approx d\eta^2 - e^{2\eta_0/3} (dx^2 + dy^2 + dz^2),$$

(43)

where $W(z)$ is the function $W$ of Lambert, $\tau = \frac{3}{4} C_1 \cos \left( \ln \left( \frac{C_2 T}{C_{sp}} \right) \right) / \alpha^2$, $\alpha = \sqrt{3} \sqrt{C_1} \left| \sin \left( \frac{1}{2} \ln \left( \frac{C_2 t}{C_{sp}} \right) + \frac{\pi}{4} \right) \right|$, $\epsilon = 1 + 2|\eta_0 - \eta_m|$, $\eta_0 = \eta(T)$, $\eta_m = \eta(t_m)$ and $\eta = \frac{\ln(t)}{\alpha} + \frac{\pi}{4}$.

For this reason, for non-prolonged time intervals with high values of $t$, the metric presents a behavior similar to its analogous of dark energy [3].

The function $K$, from (24), in general, tends to a maximum constant value of $(K < K_{max})$, when $t \to \infty$; therefore, the metric tends to be isotropic.

5 Hubble Parameters and Deceleration

The Hubble parameters $H$ and of deceleration $q$ are defined, for an anisotropic symmetry of Petrov D with the way obtained in [17]. For the solution of this work, the Hubble parameter follows the pattern

$$H = \frac{1}{3t} \sqrt{\frac{3}{2} t^2 C_1 \left( 1 + \sin \left( \ln \left( \frac{C_2 t}{C_{sp}} \right) \right) \right) + 1},$$

(44)

and the deceleration parameter $q$, is

$$q = -1 - \frac{9 t^2 C_1 \cos \left( \ln \left( \frac{C_2 t}{C_{sp}} \right) \right) - 12}{12 t^2 C_1 \sin \left( \frac{1}{2} \ln \left( \frac{C_2 t}{C_{sp}} \right) + \frac{\pi}{4} \right)^2 + 4}.$$

(45)

The parameters of Hubble (44) and deceleration (45) tend to $H \to \frac{1}{3t}$ and $q \to 2$, respectively, when $t \to 0$, therefore, the Hubble parameter, is indefinite.
in $t = 0$; for this reason, it is similar to what happens to some models of the ideal fluid when $P = \lambda \mu$ [17]. For time proximities $t_n$ (tendency I), it is given that the parameter (44) tends to be a function of $t$, similar to the one presented when $t \to 0$. For the tendency II, the situation changes and the parameters tend to be log periodic functions of the pattern

$$H \to H_{II} = \sqrt{\frac{C_1}{3}} \sin \left( \frac{1}{2} \ln \left( \frac{C_2 t}{C_{sp}} \right) + \frac{\pi}{4} \right),$$

and

$$q \to q_{II} = -1 - \frac{3 \cos \left( \ln \left( \frac{C_2 t}{C_{sp}} \right) \right)}{4 \sin \left( \frac{1}{2} \ln \left( \frac{C_2 t}{C_{sp}} \right) + \frac{\pi}{4} \right)^2}.$$  

The Hubble parameter tends to a maximum value of $H \to H_m = \sqrt{C_1/3}$, which is achieved when $t = t_n = \frac{C_{sp} e^{\frac{\pi}{4}} + 2n\pi}{C_2}$, that are the times in which the maximum values of energetic density are obtained $T_0 = C_1$. The maximum $(q(t_{max}) > 0)$ and minimum $(q(t_{min}) < 0)$ values of the deceleration parameter (45) are obtained relatively close to the value of $t_n$; they satisfy equality

$$3 t^2 C_1 \sin \left( \frac{1}{2} \ln \left( \frac{C_2 t}{C_{sp}} \right) + \frac{\pi}{4} \right)^2 - 5 \cos \left( \ln \left( \frac{C_2 t}{C_{sp}} \right) - \arctan \left( \frac{3}{4} \right) \right) = 4$$

and belong to the order $\frac{9}{8} t_n C_1 \sigma$ (if $t_n$ is big), where $\sigma = |-t_n + t_{extr}|$ and $t_{extr}$ is $t_{max} \approx t_n - \frac{2}{\sqrt{3} \sigma}$ or $t_{min} \approx t_n + \frac{2}{\sqrt{3} \sigma}$ from (48). The parameter of deceleration $q$ tends to zero for values of $t \to t_n + \frac{8}{9C_1 t_n}$.

6 Conclusions

Cosmologic solutions of the log periodic type with a symmetry of Petrov D were obtained. Said solutions represent a Universe which undergoes cycles where the volumetric density of the energy $\mu$ and the pressure $P$ of the interaction of non-lineal, and self-consistent scalar and spinorial fields are log periodic functions of time. The function of the scalar field $\varphi$ determines in a great way the phase of spinor; said function is log periodic most part of the time (tendency II), although there are intervals of short periods (tendency I), in which their behavior is lineal in terms of time. The non-lineal element of interaction of the fields in the lagrangian is characterized for being log periodic in regards to the spinorial invariant $S = \overline{\psi} \psi$; and it generates an increase of the effective mass of the spinorial field, is doubling it for $t \to t_n$. 
The obtained solution represents a Universe which expands without facing a process of spatial contraction; it is cyclic in the sense that the energetic density $\mu$ and the pressure $P$ of the studied fields are log periodic; for this reason, in different scales they rely on past values that will repeat, but with a time pause longer that the last time due to the logarithmic cyclic scale, moreover, the space, with the increase of $t$ tends to be isotropic too.

For some values of $t$, it is given that the tensor of stress-energy is null, which leaves room for two possible options, depending on the sign of the constant $C_k$ in (24): a) when the sign is positive, it is given an space-time, immediately of the Kasner type $E_{D_1}$ [3], and b) when the sign is negative, it is obtained, immediately, the flat world $E_{D_0}$; for the last case, as it is expected $R^\mu_{\nu\alpha\beta} = 0$ in these instances. Both solutions are analyzed in the proximities to $t \rightarrow 0$.

In the case when $C_k = +2/3$, it is given that the Kretschmann invariant is singular the same way that the Kasner solution $E_{D_1}$ [3], when $t \rightarrow 0$; and the solution when $C_k = -2/3$, the Kretschmann invariant is not singular (happens to $R^\mu_{\nu\alpha\beta}$) because of an infinite succession of time closely related, in which both the Kretschmann invariant and the tensor of Riemann are denied, and for which the space-time is immediately flat.

There were obtained the parameters of Hubble $H$ and deceleration $q$, which do not change in relation to the value of $C_k$, and it is concluded that the parameter of Hubble is inversely proportional to time, for some time intervals (I tendency), and singular when $t \rightarrow 0$, but most of the time (II tendency) it behaves as a log periodic function whose maximum value is $H_m = \sqrt{\mu_m/3}$, where $\mu_m$ is the maximum value of $\mu$. The deceleration parameter $q$ is equal to $q_I = 2$, for values of $t \rightarrow 0$ or the I tendency, and it is a log periodic function for the values of the II tendency; the values of $q$ are positive, null or negative, depending on the value of $t$. Its minimum values or (relative) maximums are approximately close to one another, $q$ has a null value, located almost between the maximum and (relative) minimum values in each log periodic cycle. All this happens in time intervals close to time values for which $\mu$ and $P$ tend to zero (I tendency). In relation to this, the solutions present periods when the Universe decelerates and periods when it accelerates, so that the values of maximum deceleration and acceleration are close and followed by the other.

References


Received: August 14, 2019; Published: September 1, 2019