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A Geometric Version for the Description of the Dynamics of Different Models

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Abstract

In many of our works, and over the years, the dynamics of different models were analyzed from a geometric point of view. In this work the advantages of the applications of this formalisms are studied comparatively for models that have diverse origins, from gravity and supergravity models, through models of condensed matter, to models of productive systems. The geometrical approach it is very useful tool for the study of the dynamics, and it is possible to apply it with the correct interpretation of the variables in game. In this paper we try to study the scope of this application.

Keywords: Geometric Action; Canonical Exterior Formalism; Gravity; Productive System

1 Introduction

1.1 Gravity and Strings Models

For years there has been an extensive bibliography that treats the geometric approaches to gravity and supergravity models in different dimensions, and from various points of view. In these works, the first applications of the canonical exterior formalism (FCE) to the mentioned models proved to be very effective. Therefore, and given its similarity to the models that we will try to discuss, we will do a joint analysis (gravity or supergravity and strings, obtained from the previous ones by dimensional reduction).

In all these models, the geometrical structure is analyzed in the framework of the canonical exterior formalism. In this context, the motion equations of the dynamical field and the constraints are found and analyzed. It can be seen how the use of the canonical exterior formalism is more adequate and simple because of its manifest covariance in all the steps. The relationship between the form brackets defined in the canonical exterior formalism and the Poisson-brackets is defined. Later on, the Dirac-brackets are written by using the second class constraints provided by the canonical exterior formalism. As it can be seen the canonical exterior formalism allows to show how the later canonical quantization of the model is facilitated.

A brief description allows to clarify the ideas. Initially, it is important to mention that the analysis of the role of the different fields is fundamental in all models.

Two-dimensional gravity and supergravity models were constructed from different point of view, and as already said there is a vast literature on the subject matter, for example, linear gravity theories based on the Riemann scalar curvature R. The first model of two-dimensional gravity was constructed by Jackiw and Teitelboim by means of dimensional reduction of the usual Einstein-Hilbert action in $(2+1)$ dimensions, [1, 2, 3].

The geometrical structures of the different models are generally the de Sitter or anti-de Sitter groups. All these models have the remarkable property of possessing a topological and gauge invariant formulation. Instead, the "string-inspired" models are based on the extended Poincare group. Given the possibility of obtaining solutions of "black-hole" this models and your study, becomes interesting from the quantum point of view.

The two-dimensional reduction of the invariant action is done by means of the Kaluza-Klein ansatz, decomposing the three-dimensional metric into a two-dimensional metric, with a gauge field $A = A_\mu dx^\mu$ and a scalar field ξ. The dimensional reduction procedure yields a two-dimensional topological theory. In order to give the discussion to a global level, the action is written by using target space coordinates. As it can be seen, the use of such coordinates brings some advantages from classical as well as quantum point of view, [4, 5, 6, 7, 8].

Besides, as it is well known the 2D conformal supergravity is the proper framework for the description of superstring theories, [9, 10, 11]. This intuitive idea is originated by observing that two is the dimension of the world-sheet (WS) spanned by a one-dimensional object while propagating in an external space-time, named target manifold M_{target}. The two-dimensional manifolds play an important role as they are responsible for the fundamental geometric structure in superstring theory. Likewise, in order to make local the graded algebra, the two-dimensional "vielbein" (V) and the two-dimensional "gravitino" (ξ) are needed.

The gravitational field must be interpreted as a Lagrangian multiplier for the corresponding constraints giving the vanishing condition of the matter fields stress-energy tensor. Consequently, the whole gravitational formalism reduces to a theory of boundary conditions in two-dimension and so, only its topology is the matter of interest. In this context, the Lagrangian formalism in components and Noether theorem acquire a singular importance.

1.2 t-J Model

On the other hand, different geometrical formalisms were applied to the study of the t-J model (the Faddeev-Jackiw and Lagrangian formalisms).

In this sense, a Lagrangian formalism in which the field variables are directly the Hubbard operators it is a natural representation to treat the electronic correlation effects, [12, 13, 14]. In this approach the Hubbard operators representing the real physical excitations are treated as indivisible objects. The t-J model is one of the better candidates to explain the phenomenology of High-Tc superconductivity, and it contains the main physics of doped holes on an antiferromagnetic background. Our starting point was the construction of a particular family of first-order constrained Lagrangians by using the Faddeev-Jackiw (FJ) symplectic method, [15], in the supersymmetric version, [16, 17]. The t-J model is usually studied in the framework of the slave-particle representations, [18]. As is known the slave-particle models exhibit a local gauge invariance which is destroyed in the mean field approximation. This local gauge invariance has associated a first-class constraint which is difficult to handle in the path-integral formalism. In the t-J model, in which spin and charge degrees of freedom are present, the Hubbard operators represent the real physical excitations, and verifies the graded algebra $spl(2,1)$ given by:

$$[\hat{X}_i^{\alpha\beta}\,,\,\hat{X}_j^{\gamma\delta}]_\pm = \delta_{ij}(\delta^{\beta\gamma}\hat{X}_i^{\alpha\delta} \pm \delta^{\alpha\delta}\hat{X}_i^{\gamma\beta})\ . \tag{1}$$

the indices α, β, γ, δ run in the values $+$, $-$, and 0; the $+$ sign must be used when both operators are fermion-like, otherwise it corresponds the $-$ sign, and i, j denote the site indices.

The purpose is to find the family of first-order Lagrangians which can be mapped in the slave-fermion representation, written in terms of fermion-like and boson-like Hubbard operators. The family of Lagrangians and the constraint structure of the model will be determined by using the Faddeev-Jackiw (FJ) symplectic method, [15]. The set of constraints is second-class one, [13, 14]. Then, by means of the path-integral technique, the correlation generating functional is written in terms of the effective Lagrangian, which results non-polynomial. Therefore, we study the general perturbative formalism. By defining proper propagators and vertices, the standard Feynman diagrammatics, graphic representation of the dynamics of the model, is given. Besides, the ghost fields needed to render the model renormalizable are introduced.

1.3 Productive System

In the case of productive systems, the study and analysis of the advantages of the application of a geometric approach for the study of dynamics is in full development. In previous works, and following the line of argument of several authors in recent years, we explore the application of technique from the Field Theory.

To model the dynamics of a productive system, different theoretical frameworks have been defined and used from Physics, (M. Estola 2013 and M. Estola and A.A. Dannenberg 2016), [19, 20, 21]. In these works, the dynamics of a production are modeled after Newton's second law and the application of the Newtonian and Lagrangian formalisms. In these cases, the dynamics of the neo-classical theory (state of static equilibrium) is studied, and it is shown that this theory corresponds to the particular situation of the mentioned formalisms (zero force).

Again, it is important to mention that in order to follow the reasoning and the way of constructing exact arguments used by Physics, [22], the meanings of the co-related magnitudes between Physics and Economics have a fundamental weight for the development of consistent ideas.

Probably this set of concepts, definitions and procedures do not have the same weight as in Physics, but they can be applied to a certain number of models, or have a specific universe of application.

I. Fisher (2006, original work in 1892) planted in his doctoral thesis a vector formulation for economics, first published work where the correspondences between Physics and Economics, [23], are explicitly defined.

Then, and with a more general vision coming from the Field Theory, it would be possible to model the dynamics of one productive system that can be described from a trajectory, in coordinates $(q, \dot{q})$, of a given configuration space. By correctly defining the Lagrangian of the model, applying the techniques from the Field Theory, it is possible to obtain the equations of motion.

It is essential to emphasize the importance of correctly defining the relationship between magnitudes, in this case the concepts of kinetic and potential energies of the economic productive system, in consistency with the conceptualizations coming from Physics, as generators of the dynamics of the system.

M. Estola et all (2013 and 2016) agree with I. Fisher (2006), that the economic kinematics can be described according to the movement of a point representative of the "position" of an economic quantity in a coordinate system (coordinates q and $\dot{q}$ of a configuration space), from:

$$Q_i(t) = Q_i(t_0) \; + \; \int q_i(s)\; ds \tag{2}$$

The above equation represents the temporal evolution, which in the case of Physics is obtained by means of the integral of the Lagrangian in time, called the functional action S. The action in Physics is an abstract concept, and it contains all the dynamic information and the system interactions. The temporal evolution takes place through a trajectory, in which the action has an extreme (a minimum).

M. Estola, add in their works the idea of the existence of factors that resist the changes of said economic quantities, modeling this inertia according to Physics. Thus, it defines the "forces" and "inertial masses" of the economic magnitudes.

2 Theoretical Frameworks

We can start mentioning that in the geometrical picture, closed string as a one-dimensional loop moving in a smooth target manifold M_{target} was systematically studied. Furthermore, every consistent 2D conformal field theory corresponds to a possible string vacuum and it is a suitable starting point for the string perturbation theory. So, it is possible to regard as a string vacuum only those consistent conformal theories which are generated by embedding scalar functions $X^{\mu}(\xi^{\alpha})$ from the world-sheet to that target space, where $X^{\mu} \,\epsilon\, M_{target}$ and $\xi^{\alpha} \,\epsilon\, World - Sheet$. In the two-dimensional framework, the embedding scalar functions $X^{\mu}(\xi)$ must be viewed as scalar fields coupled to the 2D gravitational field with metric $g_{\alpha\beta}(\xi)$. The coupling is realized in such a way that the classical action must be invariant under diffeomorphisms and Weyl transformations, relating two different 2D conformal metrics. In the case of superstrings, the two-dimensional action contains a convenient set of left-handed and right-handed 2D-fermions.

We can say then that a consistent conformal theory implies that the classical conformal theory mantains the classical Virasoro algebra also at the quantum level. This is possible by choosing the field content adequately, such that, after quantization all the central charges a_i and the coboundaries b_i corresponding to the different fields in the theory, sum up to zero. In summary, these quantum conformal theories, given by well defined choices of the target space, are suitable string vacuum.

Let's remember that the geometric structure underlying heterotic superstring is that of $N = 1$, $D = 2$ conformal supergravity, i.e, the superspace named $(1, 0)$. The geometry of this superspace of two bosonic coordinates z and $\bar{z}$ and a single Majorana-Weyl fermionic coordinate θ, is described by a supervielbein (V^+, V^-, ξ) and an $S0(1, 1)$ connection ω. The one-forms (V^+, V^-, ξ) provide a basis for the cotangent space. The one-forms V^+ and V^- are the inner directions and the one-form ξ is the outer direction in the cotangent space. Once the basis (V^+, V^-, ξ) was given, it is possible to write the torsion and the curvature of $(1, 0)$ superspace as follows:

$$T^+ = dV^+ + \omega \ \wedge V^+ = \frac{i}{2}\, \xi \ \wedge \ \xi \tag{3}$$

$$T^- = dV^- - \omega \ \wedge \ V^- = 0 \tag{4}$$

$$T^o = d\xi + \frac{1}{2}\, \omega \ \wedge \ \xi = \tau \, V^+ \ \wedge \ V^- \tag{5}$$

$$R = d\omega = \mathcal{R} \, V^+ \ \wedge \ V^- - i\tau \, \xi \ \wedge \ V^- \ , \tag{6}$$

where in the right hand side of the above equations are written the correspondent parametrization of torsion and curvature consistent with the corresponding Bianchi identities. In Eqs. (5,6) the superfield $\tau(z, \bar{z}, \xi)$ is the field strength of the two-dimensional gravitino that provides a complete description of the heterotic geometry, and $\mathcal{R}$ in Eq. (6), is the curvature that equals twice the spinor derivative of τ.

Now it is possible to describe a classical 2D superconformal theory by the Wess-Zumino-Witten action, which can be formally written as follows:

$$S = \int_{\Sigma_g} d^2\zeta \ det \, V(\zeta) \, \mathcal{L} \, (V^{\pm} \, , \, \xi(\zeta) \, , \, \varphi^i(\zeta)) \ , \tag{7}$$

where the integral is defined over the Riemann surface Σ_g which is a 2D real manifold. The one-forms $V^{\pm}$ and ξ are respectively the vielbein and the gravitino fields which are the supergravity background fields, and $\varphi^i(\zeta)$ is a convenient set of matter fields.

The geometric action of the $(1,0)$ σ model was proposed from several years ago. In order to construct of such superconformal theory in the exterior canonical picture, in Eq. (7) we take as matter fields $\varphi^i(\zeta)$ the components of a superfield $g(z, \bar{z}, \theta)$ which describes the injection:

$$g(z, \bar{z}, \theta) : SWS {\rightarrow} G \ , \tag{8}$$

where G is a simple group manifold. All the geometrical quantities of $M_{target} = G$ are constructed in terms of the left-invariant or right-invariant one-forms $(\Omega = g^{-1}\, dg)$. This theory is called the Wess-Zumino-Witten model.

So, the Lie algebra-valued one-forms Ω and $\bar{\Omega}$ are decomposed along a basis t_A of the Lie algebra associated to the group manifold G. It is obvious that Ω^A and $\bar{\Omega}^A$ satisfy the Maurer-Cartan equations.

And since the one-forms Ω^A and $\bar{\Omega}^A$ depend on the superspace coordinates $(z \, , \, \bar{z} \, , \, \theta)$, they can be written along a complete superspace basis of one-forms:

$$\Omega^A = \Omega^A_+ \, V^+ + \Omega^A_- \, V^- + \lambda^A \, \xi \ , \tag{9}$$

and similarly for $\bar{\Omega}^A$.

So, the starting point is to consider the following geometric action:

$$\begin{aligned} S_(\alpha) &= \frac{k}{8\pi}\{\int_{M_2}[(\Omega^A-\lambda^A\xi)\wedge(\Omega_{A+}\,V^+-\Omega_{A-}\,V^-)+2i\lambda^A\,\nabla_\alpha\,\lambda_A\,\wedge\,V^+ \\ &+ \lambda^A\,\Omega_A\,\wedge\,\xi-\frac{4i}{3}(\frac{1}{2}+\alpha)\,f^{ABC}\,\lambda_A\,\lambda_B\,\lambda_C\,\xi\,\wedge\,V^+ \\ &+ \Omega^A_+\,\Omega_{A-}\,V^+\wedge V^-]+\frac{1}{6}(1+2\alpha)\int_{M_3}f^{ABC}\,\Omega_A\,\wedge\,\Omega_B\,\wedge\,\Omega_C\}\,. \end{aligned} \tag{10}$$

The covariant differential of the two-dimensional spinor λ^A is given by:

$$\nabla_{(\alpha)}\,\lambda^A=\mathcal{D}\,\lambda^A+\omega^{AB}_{(\alpha)}\,\lambda_B\,, \tag{11}$$

where is well defined:

$$\mathcal{D}\,\lambda^A\equiv d\lambda^A+\frac{1}{2}\,\omega\,\lambda^A\,. \tag{12}$$

Another similar model studied from a geometric point of view was the σ model for superstrings of type II. In this case, the geometric canonical exterior formalism on group manifold, for the heterotic supersymmetric (1,1) σ model is constructed. This is done by starting from a classical 2D superconformal theory described by the Wess-Zumino-Witten model, where the world-sheet geometry is the (1,1) superspace. In this framework, the motion equations of the dynamical field and the constraints are found and analyzed from the geometric point of view. Again, it can be seen how the use of the FCE is more adequate and simple because of its manifest covariance in all the steps. The relationship between the form brackets defined in the FCE and the Poisson-brackets is given. Later on, the Dirac-brackets are written by using the second class constraints provided by the own formalism. Also, it can be seen the FCE allows facilitates the canonical quantization of the model.

In this model, the geometric structure underlying in the superspace $(1,1)$ with superconformal algebra of $2D$, $N=2$. This superalgebra contains: translations V^a, conformal boosts K^a, Q-supersymetry ψ, S-supersymetry ϕ, Lorentz rotations ω^{ab} and dilatations W.

For the dynamics of the t-J model, we find the family of first-order Lagrangians, which map in the slave-fermion representation, written in terms the Hubbard operators, which represent the real physical excitations, and verifies the graded algebra $spl(2,1)$. The family of Lagrangians and the constraint structure of the model will be determined by using the Faddeev-Jackiw symplectic method.

By last, and following M. Estola's line of argument, in the sense that the dynamic of a production is modeled from Newton's second law and the application of Lagrangian and Newtonian formalisms, we presents and analyzes the possibility of applying more general techniques, typical of field theory such as the FCE, to the study of the dynamic mentioned.

The FCE plays an important role, because of it is more simple and compact structure can be used as an interesting geometrical formalism to derive and analyze the equations of motions, and thus obtain the Hamiltonian density, generator of the temporary evolutions of the generic functional.

3 Formalism for Gravity, Supergravity and Strings Models

The FCE was constructed and applied to different models of gravity and supergravity in diverse dimensions, as well as their coupling to matter supermultiplets and to the Yang-Mills field [24, 25, 26, 27, 28, 29]. In general this formalism permits to find and study constraints, equation of motion and all the dynamical properties of such systems in a more simple way that following the usual Lagrangian method. The FCE is covariant in all its steps because it is constructed by using only operation of the exterior algebra. In all case the gravity or supergravity fields are dynamics ones. The idea was to work by first time with the FCE applied to the description of the heterotic supersymmetric sigma model in which the supergravity field is a non-dynamical one.

If we will consider the $\alpha = -\frac{1}{2}$ case, which corresponds to choose a metric connection for which the torsion is zero, in such case, the Lagrangian density is written as follows:

$$\begin{aligned} \mathcal{L} &= \frac{k}{8\pi}\,[(\Omega^A - \lambda^A\zeta)\wedge(\Omega_{A+}\,V^+ - \Omega_{A-}\,V^-) + 2i\lambda^A\,\nabla\,\lambda_A\,\wedge\,V^+ \\ &+ \lambda^A\,\Omega_A\,\wedge\,\zeta + \Omega^A_+\,\Omega_{A-}\,V^+\wedge V^-] \end{aligned} \tag{13}$$

In the Lagrangian density (13) the auxiliary two 0-forms fields Ω^A_+, Ω^A_- are non-geometrical objects and are introduced with the purpose of obtaining rheonomic equations of motion, i.e., equations compatible with the Bianchi identities as it is required by the group manifold approach [24, 25].

In order to obtain the equation of motion, instead of the WZW field g contained in the one-form field Ω^A, we can use as dynamical variable the tangent variation.

Therefore, Eq. (13) is our starting point in order to construct the first-order FCE. We define the canonical conjugate momenta to each one of the dynamical field variables $\mu^\Sigma = (y^A,\ \lambda^A,\ V^+,\ V^-,\ \zeta,\ \Omega^A_+,\ \Omega^A_-)$ for the compound index Σ. By means of the functional variation of the Lagrangian with respect to the "velocities" $d\mu^\Sigma$, the canonical conjugate moments are defined by:

$$\Pi_{\Sigma} = \frac{\delta \mathcal{L}}{\delta(d\mu^{\Sigma})} \ . \tag{14}$$

In the FCE, it is necessary to define a suitable operation involving forms, capable of replacing the role of the classical Poisson brackets. Therefore, the graded form-brackets operation between pairs of canonical variables is defined and it is given by:

$$\left(\mu^{\Sigma} \ , \ \Pi_{\Lambda}\right) = (-1)^{a+1+|A|} \, \delta^{\Sigma}_{\Lambda} \ , \tag{15}$$

where a and $\mid A \mid$ are respectively the degree and the Fermi grading of the form μ^{Σ}.

In the FCE, the conserved first-class dynamical quantity describing the dynamics of the system is the extended Hamiltonian H_T, and it is the bosonic two-form defined by:

$$H_T = H_{can} + \Lambda^{\Sigma} \ \wedge \ \Phi_{\Sigma} \ , \tag{16}$$

where the Lagrange multipliers Λ^{Σ} can be unambiguously determined. When the fundamental equation of motion in the FCE is taken into account, it is possible to write the Hamiltonian equations for pairs of canonical variables:

$$d\mu^{\Sigma} = \left(\mu^{\Sigma} \ , \ H_T\right) \tag{17}$$

$$d\Pi^{\Sigma} = \left(\Pi^{\Sigma} \ , \ H_T\right) \tag{18}$$

From Eq. (16) and by using Eq. (15) the following general result is obtained:

$$\Lambda^{\Sigma} = d\mu^{\Sigma} \ . \tag{19}$$

In Eq. (15) the canonical Hamiltonian is given by: $H_{can} = d\mu^{\Sigma} \ \wedge \ \pi_{\Sigma} - \mathcal{L}$, and explicitly by:

$$\begin{aligned} H_{can} \ &= \ dy^A \ \wedge \ P_A + d\lambda^A \ \wedge \ Q_A + dV^+ \ \wedge \ \Pi_+ + dV^- \ \wedge \ \Pi_- \\ &+ \ d\zeta \ \wedge \ \Pi_\zeta + d\Omega^{+A} \ \wedge \ \mathcal{P}_{+A} + d\Omega^{-A} \ \wedge \ \mathcal{P}_{-A} - \mathcal{L} \end{aligned} \tag{20}$$

3.1 Equations of motion

The field equations of motion in the FCE are given by the consistency conditions on the primary constraints, i.e:

$$d\Phi^{\Sigma} = (\Phi^{\Sigma} \ , \ H_T) \approx 0. \tag{21}$$

As it was commented above the vielbein and the gravitino are not dynamical fields in 2D, therefore the motion equation for the supergravity background fields V^+,V^- and ζ will be not considered. The supergravity background fields play the role of Lagrange multipliers associated to the primary constraints of the theory, that is the superstress-energy tensor and the supercurrent. In fact, the superstress-energy tensor and the supercurrent one-forms are respectively defined by making the variation of the action (10) with respect to the supervielbein (V^+, V^-, ζ). As it is known in the classical theory these quantities are weakly zero ones. The variables Ω^A_+ and Ω^A_- are introduced to enforce the rheonomic. From the quantum point of view they are used to construct the BRST-charge parametrization.

Therefore, the main equations are those for the fields y^A and λ^A which respectively read:

$$
\begin{aligned}
d\Phi^M &= (\Phi^M \; H_T) \\
&= (P^M \;,\; H_{can}) + \Lambda^B \;\wedge\; (\Phi^M \;,\; \Phi_B) + \Sigma^B \;\wedge\; (\Phi^M \;,\; \Psi_B) \\
&+ \; \Lambda_+ \;\wedge\; (\Phi^M \;,\; \varphi_+) + \Lambda_- \;\wedge\; (\Phi^M \;,\; \varphi_-) + \Sigma_\zeta \;\wedge\; (\Phi^M \;,\; \varphi_\zeta) \\
&+ \; \Delta^A_+ \;\wedge\; (\Phi^M \;,\; \Theta^A_+) + \Delta^A_- \;\wedge\; (\Phi^M \;,\; \Theta^A_-) \\
&+ \; \text{weakly zero terms} = 0 \;,
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
d\Psi^M &= (\Psi^M \; H_T) \\
&= (Q^M \;,\; H_{can}) + dy^B \;\wedge\; (\Psi^M \;,\; \Phi_B) - d\lambda^B \;\wedge\; (\Psi^M \;,\; \Psi_B) \\
&+ \; dV^+ \;\wedge\; (\Psi^M \;,\; \varphi_+) + dV^- \;\wedge\; (\Psi^M \;,\; \varphi_-) + d\zeta \;\wedge\; (\Psi^M \;,\; \varphi_\zeta) \\
&+ \; d\Omega^A_+ \;\wedge\; (\Psi^M \;,\; \Theta^A_+) + d\Omega^A_- \;\wedge\; (\Psi^M \;,\; \Theta^A_-) \\
&+ \; \text{weakly zero terms} = 0 \;.
\end{aligned}
\tag{23}
$$

After considering the explicit expressions of the form-brackets between constraints, and by replacing, we have:

$$
\begin{aligned}
d\Phi^M &= -\Big(d\Pi^A - f^{ABC}\,\Omega^B \;\wedge \Pi^C + \mathcal{D}\lambda^A \;\wedge \zeta + \lambda^A\, T^o \\
&+ \; f^{ABC}\,\Omega^B\,\lambda^C \;\wedge\; \zeta - 2if^{ABC}\,\lambda^B\,\mathcal{D}\,\lambda^C \;\wedge\; V^+ + \frac{1}{2} f^{ABC}\,\lambda^B\,\lambda^C\,\zeta \;\wedge\; \zeta \\
&- \; if^{ABC}\,f^{CDE}\,\Omega^B\,\lambda^D\,\lambda^E \;\wedge\; V^+\Big) \\
&+ \; \text{weakly zero terms} = 0 \;,
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
d\Psi^M &= -\Big(-4i\nabla\,\lambda^A \;\wedge V^+ + \zeta \;\wedge\; \Pi^A + \zeta \;\wedge\; \Omega^A + \lambda^A\,\zeta \;\wedge\; \zeta\Big) \\
&+ \; \text{weakly zero terms} = 0 \;.
\end{aligned}
\tag{25}
$$

Having the same structure they can be decomposed into four independent sectors corresponding to the inner-inner direction $V^+ \wedge\ V^-$ the inter-outer directions $V^+ \wedge\ \zeta$ and $V^- \wedge\ \zeta$ and the outer-outer direction $\zeta \wedge\ \zeta$.

The first step is to consider the Maurer-Cartan two-form equation and the one-forms decomposed along the supergravity background one-form fields (V^+,V^-,ζ).

By straightforward calculation it can be shown: i) that the coefficients of the components $V^+\ \wedge\ \zeta\,, V^-\ \wedge\ \zeta$ and $\zeta\ \wedge\ \zeta$ cancel automatically when the rheonomic parametrization is introduced. On the other hand, the cancelation of the component $V^+\ \wedge\ V^-$ gives rise to the following condition: $\mathcal{D}_-\ \Omega^A_+ + \mathcal{D}_+\ \Omega^A_- - \tau\ \lambda^A - 2if^{ABC}\ \lambda^B\ \mathcal{D}_-\ \lambda^C - i\ f^{ABC}\ f^{CDE}\ \Omega^B_-\ \lambda^D\ \lambda^E = 0.$ ii) that the coefficients of the components $V^+\ \wedge\ \zeta\,, V^-\ \wedge\ \zeta$ and $\zeta\ \wedge\ \zeta$ cancel automatically, while the cancelation of the component $V^+\ \wedge\ V^-$ gives rise to the following condition: $\mathcal{D}_-\ \lambda^A - \frac{1}{2}f^{ABC}\ \lambda^B\ \Omega^C_- = 0.$ iii) considering the different projections for the Maurer-Cartan equation, the following conditions are found; the coefficient cancelation of the components $V^+\ \wedge\ \zeta$ and $V^-\ \wedge\ \zeta$, the cancelation of the coefficient of $V^+\ \wedge\ V^-$ gives rise to the Bianchi identity, the coefficient of $\zeta\ \wedge\ \zeta$ cancel automatically.

Therefore, the conclusion is that the motion field equations (23) and (24) for the fields y^A and λ^A, are reduced to the two differential equations, in the points i) and ii), and the remaining conditions are all geometrical ones.

4 Formalism for the t-J Model

For this model, we will start summarizing the main definitions as the starting point for our perturbative formalism.

It is important mention that the Faddeev-Jackiw symplectic quantization method (FJ) is formulated on actions only containing first order time derivatives, whereby we consider the following first order Lagrangian written in terms of the Hubbard $\hat{X}$-variables, X-variables:

$$L = \sum_i a_{i\alpha\beta}(X)\dot{X}^{\alpha\beta}_i - \mathbf{V^{(0)}}(X)\ . \tag{26}$$

where the coefficients $a_{i\alpha\beta}$ are unknown and they are determined in such a way that a graded algebra (1) for the Hubbard $\hat{X}$-operators must be verified. Is important to remark that at this level the X-variables must be treated as classical fields.

In the Faddeev-Jackiw language the symplectic potential $\mathbf{V^{(0)}}(X)$ is defined by:

$$\mathbf{V^{(0)}}(X) = H(X) + \lambda^a\Omega_a\ , \tag{27}$$

where λ^a are appropriate Lagrange multipliers for the constraints Ω_a.

Therefore, the constraints are given by:

$$\Omega_a = \frac{\partial \mathbf{V^{(0)}}(\mathbf{X})}{\partial \lambda^a} \,, \tag{28}$$

In equation (26) $H(X)$ is the usual t-J Hamiltonian:

$$H(X) = \sum_{i,j,\sigma} t_{ij}\, X_i^{\sigma 0} X_j^{0\sigma} + \frac{1}{2}\sum_{ij} J_{ij}(X_i^{+-}X_j^{-+} - X_i^{++}X_j^{--}) - \mu \sum_{i,\sigma}\, X_i^{\sigma\sigma} \tag{29}$$

where a term depending on the chemical potential μ was added.

In equation (28) t_{ij} and J_{ij} are respectively the hopping and the effective exchange parameters between sites i and j. The indices $\alpha\,,\beta$ take the values 0 (empty state) or spin index $\sigma = \pm$ (up and down states, respectively). The five Hubbard X-variables $X^{\sigma\sigma'}$ and X^{00} are boson-like and the four Hubbard X-variables $X^{\sigma 0}$ and $X^{0\sigma}$ are fermion-like. Once the FJ symplectic algorithm, [15], is implemented on the first-order Lagrangian (25), a particular solution of the differential equations bring the following values for the coefficients $a_{\alpha\beta}$ and the constraints Ω_a.

Taking into account that there are only four bosonic fields, only two bosonic constraints are possible (discussion in [13]), and the fermionic constraints turn out to be four. Only two of the fermionic constraints must be considered as independent. Therefore, in the t-J model under consideration, there are two bosonic constraints and two fermionic constraints.

The completeness condition is obtained as one of the bosonic constraints in consequence ρ must be identified with the hole density. We remember that such a condition has an important physical meaning, and it must be imposed to avoid at quantum level the configuration with double occupancy at each site. So, we must emphasize that by means of our approach the completeness condition appears as necessary by consistency.

Finally, and without loosing generality, we choose $\alpha = -1$. So, we can write the Lagrangian (25) as:

$$\begin{aligned} L(X,\dot{X}) \;\; &= \;\; i\sum_i \frac{(1+\rho_i)u_i - 1}{(2-v_i)^2 - 4\rho_i - u_i^2}\left(X_i^{-+}\dot{X}_i^{+-} - X_i^{+-}\dot{X}_i^{-+}\right) \\ &+ \;\; \frac{i}{2}\sum_{i,\sigma}\left(X_i^{\sigma 0}\dot{X}_i^{0\sigma} + X_i^{0\sigma}\dot{X}_i^{\sigma 0}\right) - H_{t-J}(X)\,, \end{aligned} \tag{30}$$

Which, and after some algebraic calculations for the coefficients, leads to the Euclidean Lagrangian L^E_{eff}, argument of the path integral that will give us the diagrammatics and Feynman rules of the model (dynamics of the model).

In this case a particular solution for the coefficients of the Lagrangian (25) leads to the following Euclidean Lagrangian:

$$L^E = -\frac{i}{2s}\sum_i \frac{S_{i1}\dot{S_{i2}} - S_{i2}\dot{S_{i1}}}{s+S_{i3}} + \sum_{i,\sigma}\Psi_{i\sigma}\dot{\Psi}^*_{i\sigma} + H_{t-J}\ , \tag{31}$$

and the set of second-class constraints:

$$\Omega = S_1^2 + S_2^2 + S_3^2 - s^2 = 0\ , \tag{32}$$

$$\Xi_1 = \Psi^*_-(S_1 + iS_2) - \Psi^*_+(s+S_3) = 0\ , \tag{33}$$

$$\Xi_2 = \Psi_-(S_1 - iS_2) - \Psi_+(s+S_3) = 0\ . \tag{34}$$

The correlation generating functional is obtained by integrating the fermionic constraints (32) and (33) and by using the integral representation for the delta function on the non-linear bosonic constraints (31). Therefore, the partition function writes:

$$\begin{aligned} Z &= \int \mathcal{D}S_{i1}\ \mathcal{D}S_{i2}\ \mathcal{D}S_{i3}\ \mathcal{D}\Psi_{i-}\ \mathcal{D}\Psi^*_{i-}\ \mathcal{D}\lambda_i\ (sdet\mathcal{M}_{AB})_i^{\frac{1}{2}} \\ exp &\left(-\int_0^\beta d\tau\ L^E_{eff}(S,\Psi)\right)\ , \end{aligned} \tag{35}$$

where $sdet\mathcal{M}_{AB}$ is the superdeterminant of the symplectic supermatrix $\mathcal{M}_{\mathcal{AB}}$, and $L^E_{eff}(S,\Psi)$ is defined by

$$\begin{aligned} L^E_{eff}(S,\Psi) &= -\frac{i}{2s}(1-\rho)\sum_i \frac{S_{i2}\dot{S_{i1}} - S_{i1}\dot{S_{i2}}}{s+S_{i3}} - \sum_i \lambda_i(S_{i1}^2 + S_{i2}^2 + S_{i3}^2 - s^2) \\ &- s\sum_i \frac{1}{s+S_{i3}}(\dot{\Psi}^*_{i-}\Psi_{i-} + \dot{\Psi}_{i-}\Psi^*_{i-}) + H(S,\Psi)\ . \end{aligned} \tag{36}$$

The first term in (35) shows the non-polynomial structure of the kinetic part of the Lagrangian. In (35) the total Hamiltonian H is defined by:

$$H = H_{t-J} - 2s\mu\sum_{i,\sigma}\frac{1}{s+S_{i3}}\Psi^*_{i-}\Psi_{i-}\ , \tag{37}$$

where the Hamiltonian H_{t-J} for the $t-J$ model is given by:

$$\begin{aligned} H_{t-J} &= \sum_{i,j} t_{ij}\Psi_{i-}\Psi^*_{j-}\left[1 + (\frac{S_{i1} - iS_{i2}}{s+S_{i3}})(\frac{S_{j1} + iS_{j2}}{s+S_{j3}})\right] \\ &- \frac{1}{8s^2}\sum_{i,j} J_{ij}(1-\rho_i)(1-\rho_j)[S_{i1}S_{j1} + S_{i2}S_{j2} + S_{i3}S_{j3} - s^2]\ , \end{aligned} \tag{38}$$

being $J_{ij} > 0$ for a ferromagnetic state and $J_{ij} < 0$ for an antiferromagnetic one.

Finally and summarizing, to describe the dynamics of the t-J model a first-order Lagrangian in terms of the Hubbard operators is define. In this way, the Hubbard operators assume the role of dynamic variables. This Lagrangian correspond to a dynamical second-class constrained system. Then, and consequently, the quantization is carried out by using the path-integral formalism. In this context the introduction of proper ghost fields is needed to render the model renormalizable. Later, the perturbative Lagrangian formalism is developed and it is shown how propagators and vertices, descriptors of the dynamic of the model, can be renormalized to each order.

5 Formalism for a Productive System Model

In the first order of this geometrical formalism the dynamics is described by the 1-form fields $q^A = (V^a, \omega^{ab})$, where the index $A = (a,\ ab)$. The fields V^a and ω^{ab} play the role of the coordinates of a configuration space. So, V^a represents a given quantity, and ω^{ab} is a field of geometric origin (in gravity, the dreibein and the Lorentz spin connection, respectively). The 2-forms $\dot{q}^A \equiv dq^A$ play the role of velocities. The curvature 2-forms corresponding to the above fields are called $R^A = (R^a, R^{ab})$, and are defined by:

$$R^A = dq^A - \frac{1}{2} C^A{}_{BC}\, q^C \wedge q^B \ , \tag{39}$$

where the graded structure constant C^A_{BC} and the constant symmetric Killing metric γ_{AD} are related by the equation:

$$C_{ABC} = \gamma_{AD} C^D_{BC} \ . \tag{40}$$

The explicit expressions for the curvatures are written:

$$R^a = dV^a + \omega^{ab} \wedge V_b \ , \tag{41}$$

$$R^{ab} = d\omega^{ab} + \omega^{ac} \wedge \omega_c{}^b \ . \tag{42}$$

5.1 Lagrangian density $\mathcal{L}$

The action in a three dimensional space (two spacial-one temporal) is defined by means of a Lagrangian density (3-form) given by:

$$\mathcal{L} \ = \ R^{ab} \wedge V^c \varepsilon_{abc} + d\omega^{ab} \wedge \omega_{ab} - \frac{2}{3} \omega^{ab} \wedge \omega_b{}^c \wedge \omega_{ca} \ , \tag{43}$$

where the usual Einstein-Hilbert term play the role of the "mass" term. The other terms are viewed as the "kinetic" terms, in higher derivative (second time derivative). An equivalent Lagrangian density is:

$$\begin{aligned} \mathcal{L} &= dV^a \wedge \omega^{bc} \varepsilon_{abc} + d\omega^{ab} \wedge \omega_{ab} \\ &\quad - \frac{2}{3} \omega^{ab} \wedge \omega_b{}^c \wedge \omega_{ca} + \omega^{ad} \wedge \omega_d{}^b \wedge V^c \varepsilon_{abc} , \end{aligned} \tag{44}$$

that differs of Eq. (34) in a total derivative. The canonical momenta (1-forms) π_A conjugate to the 1-forms field variables q^A obtained by the functional variation of the Lagrangian density (35) with respect to the 2-forms velocities $dq^A \equiv \dot{q}^A$ are given by:

$$\pi_A = \frac{\partial \mathcal{L}}{\partial (dq^A)} . \tag{45}$$

Therefore:

$$\pi_a = \omega^{bc} \varepsilon_{abc} , \tag{46}$$

$$\pi_{ab} = \omega_{ab} . \tag{47}$$

The set of primary constraints can be obtained from the Lagrangian density and they are the relationship between the field and momentum variables not depending on the velocities:

$$\Phi_a = \pi_a - \omega^{bc} \varepsilon_{abc} \approx 0 , \tag{48}$$

$$\Phi_{ab} = \pi_{ab} - \omega_{ab} \approx 0 , \tag{49}$$

where the symbol $\approx$ implies weakly zero.

It is necessary to define a suitable operation involving forms with the help of which the Hamiltonian equation of motion may be written. As it was shown in [24, 25] the Poisson brackets yields more information than the form brackets. They can be related by means of an integral relationship.

Starting from the Lagrangian (35) the canonical Hamiltonian can be defined:

$$\begin{aligned} \mathcal{H}_{can} &= dq^A \wedge \pi_A - \mathcal{L} = \\ &\quad -\omega^{ad} \wedge \omega_d{}^b \wedge V^c \varepsilon_{abc} + \frac{2}{3} \omega^{ab} \wedge \omega_b{}^c \wedge \omega_{ca} . \end{aligned} \tag{50}$$

Therefore, the total Hamiltonian can be defined as follows [25]:

$$\begin{aligned}\mathcal{H}_T &= \mathcal{H}_{can} + \Lambda^A \wedge \Phi_A = \\ & -\omega^{ad} \wedge \omega_d{}^b \wedge V^c \varepsilon_{abc} + \frac{2}{3}\omega^{ab} \wedge \omega_b{}^c \wedge \omega_{ca} \\ & +\Lambda^a \wedge \left(\pi_a - \omega^{bc}\varepsilon_{abc}\right) + \Lambda^{ab} \wedge (\pi_{ab} - \omega_{ab}) \ , \end{aligned} \tag{51}$$

where $\Lambda^A = \left(\Lambda^a, \Lambda^{ab}\right)$ are the Lagrange multipliers.

Now, it is necessary to introduce the fundamental equation of motion in the formalims, in analogy to classical mechanics, as was mentioned in the introduction, the following equation involving the form-bracket is introduced:

$$dA = (A \ , \ \mathcal{H}_T) + \partial A \ , \tag{52}$$

where $A = (q \ , \ \pi)$ is a generic polynomial in the canonical variables q^A and π^A The operator ∂ acts nontrivially on external fields only. Therefore, for the canonical variables:

$$\partial q^A = \partial \pi_A = 0 \ , \tag{53}$$

and also for constraints:

$$\partial \Phi_A = 0 \ . \tag{54}$$

Considering the equation (43) we can write the following Hamiltonian equations:

$$dq^A = \left(q^A \ , \ \mathcal{H}_T\right) \ , \tag{55}$$

and taking into account the expression (42) for $\mathcal{H}_T$, by straightforward calculation we find the following general results:

$$\Lambda_A = dq_A \ . \tag{56}$$

It is also necessary to prove whether there are secondary constraints in the theory. For this purpose, we must impose the consistency condition on the primary constraints. We must use (43) for Φ_A and impose the condition:

$$d\Phi_A = (\Phi_A \ , \ \mathcal{H}_T) \ \approx 0 \ , \tag{57}$$

where (45) was used.

Computing explicitly the form-bracket appearing in (48), we arrive to the general equation:

$$d\Phi_A = - \left[equation \ of \ motion\right] \ + \ \left(\Phi_A \ , \ \Lambda^B\right) \wedge \Lambda_B \ . \tag{58}$$

As $\left(\Phi_A\ ,\ \Lambda^B\right)\ \wedge\ \Phi_B$ is a weakly zero trem, (49) implies the lack of secondary constraints in the FCE. Moreover, the equation (49) guarantees that the Hamiltonian defined in (42) is a first class dynamical quantity. On the other hand, by using (48) and after lengthy algebraic manipulations, we find:

$$d\Phi_a = -R^{bc}\,\varepsilon_{abc} + \left(\Phi_a\ ,\ \Lambda^A\right)\ \wedge\ \Lambda_A\ , \tag{59}$$

$$d\Phi_{ab} = -\,2\,R_{ab} - R^c\,\varepsilon_{abc} + \left(\Phi_{ab}\ ,\ \Lambda^A\right)\ \wedge\ \Lambda_A\ . \tag{60}$$

These results and properties can be obtained from the FCE in a general form [24].

6 Conclusions

We conclude that due to it is intrinsic geometrical language, the FCE can be used as an interesting formal resource to understand the structure and the dynamics of different models, being able to analyze field theories such as: supergravity in diverse dimensions, the heterotic supersymmetric sigma model, which describes type II superstring, t-J model and a productive system. The first remark is that the FCE is not a proper canonical formalism because it does not propagate data defined on an initial surface as it is required by a standard mechanical system. However, as it can be seen from it is construction, that the FCE is a powerful method at classical level, due to the covariance in all its steps this formalism allows to find the equations of motion and the constraints in a very simple way without introducing complicate algebraic manipulations. Since all the primary constraints coming from the FCE are second-class ones, the Dirac brackets are easily defined by projecting these constraints on the surface Σ. The Hamiltonian of the system is treated as usual according to the Dirac prescriptions. The total Hamiltonian coming from the FCE (Eqs. 15, 37 and 50), it is evaluated as the generator of time evolution. The primary constraint obtained in the FCE also plays an important role in the construction of the proper Hamiltonian. Precisely, it is given in terms of the first-class constraints which close the constraint algebra. Therefore, all the Hamiltonian gauge symmetries remain determined and the apparent gauge degrees of freedom can be unambiguously removed leaving only the physical ones. When the model is considered from the quantum point of view this last step is necessary.

In the case of t-J model, the study of the dynamics was done by applying the Faddeev-Jackiw formalism, which allows a general treatment for systems containing the Hubbard operators as dynamical variables. A family of classical first-order Lagrangians describing these dynamical systems is found. Also, and in the framework of the path-integral formalism, it is possible to quantify the model, analyze the generating functional describing the dynamics of the t-J model and build the standard Feynman rules, a representation of the dynamics.

In the case of a productive economic system, have applied techniques of field theory following the approach taken by other authors in this area. The FCE we present for the analysis of the dynamics of the model is, from the point of view of the analytical mathematics, wider and more general. It includes the law of Euler Lagrange, the Lagrangian and Newtonian formalisms studied by others. With the right choice of meaning and correlation between the physical and economic variables, this formalism, coming from the field theory would give the possibility to study the model dynamics more general and complex.

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