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The Equivalence Principle, Cosmological Term, Quantum Theory and Measurability

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Abstract

The first part of this paper is devoted to analysis of the applicability limit of Einstein's Equivalence Principle (EP). It is noted that a natural applicability limit of this Principle, associated with the development of quantum-gravitational effects at Planck's scales, is absolute, its more accurate estimation being dependent on the processes under study and on the sizes of the corresponding particles. It is shown that, neglecting the applicability limit of EP, one can obtain senseless results on estimation of the relevant quantities within the scope of the well-known Quantum Field Theory (QFT). Besides, such neglect may be responsible for ultraviolet divergences in this Theory. In the second part of the work the author presents the general principles and mathematical apparatus for framing QFT in terms of the **measurability** notion introduced by the author earlier. In such QFT in the general case it is expedient to indicate the energy regions, where EP is valid and where it loses its force, in an effort to find a natural solution of the ultraviolet divergences problem in this theory that at low energies is very close to the initial well-known QFT.

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1 Introduction

This paper is a continuation of previous works by the author [1]–[6]. The first part is devoted to analysis of the applicability limit of Einstein's Equivalence Principle (EP). It is noted that a natural applicability limit of this Principle, associated with the development of quantum-gravitational effects at Planck's scales, is absolute, its more accurate estimation being dependent on the processes under study and on the sizes of the corresponding particles. It is shown that, neglecting the applicability limit of EP, one can obtain senseless results on estimation of the relevant quantities within the scope of the well-known Quantum Field Theory (QFT), in particular, of the cosmological term λ in General Relativity (GR). Besides, neglect of the applicability limit of EP may be responsible for ultraviolet divergences in OFT.
The idea that all the processes studied in QFT should be considered separately in two different energy ranges

$$\begin{aligned} E &\ll E_p \\ &\mathit{and} \\ E &\approx E_p \end{aligned} \tag{1}$$

is substantiated. Then the results earlier obtained by the author [1]–[6] are used. However now the author lifts some initial restrictions (limiting conditions) imposed in the above-mentioned papers. Specifically, it is not supposed initially that a theory involves some minimal length l_{min}; we start from the maximal momentum $p = p_{max}$,formula (10) in Section 4 (a certain maximal bound for the measured momenta), and then from this formula we can derive the length ℓ and the corresponding time $\tau = \ell/c$. ℓ is called the *primary* length, whereas τ is called the *primary* time. The whole formalism developed in [1]–[6] on condition that ℓ is a minimal length is fully valid for the case when ℓ is the *primary* length. It is important that there is a possibility to lift the formal requirement for involvement of l_{min} in the theory just from the start. The need for replacement of the minimal length l_{min} by the *primary* length ℓ according to the proposed approach is substantiated in the Section 4 (see the paragraph titled **Explanation**).
The principal idea of the above-mentioned works is as follows. Proceeding from the **measurability** notion, initially defined in [2] and also in Section 4 of this paper, we can reformulate quantum theory and gravity, removing from them the abstract infinitesimal variations $dt, dx_i, dp_i, dE, i = 1, ..., 3$ and replacing them by the quantities depending on the existent energies expressed in terms of the quantity ℓ. Within the scope of these terms, at low energies a theory becomes discrete, it is very close to the initial theory formulated in the continuous space-time. Actually, discreteness is revealed at high energies only. At the present time these theories are defined in the continuous space-time

paradigm but are associated with serious problems, in particular with the (ultraviolet and infrared) divergences
In Section 4 the primary elements of the mathematical apparatus based on this notion are recollected and elucidated.
Finally, in Sections 5,6 the general principles of framing QFT in terms of the **measurability** notion are given. It is noted that, from the viewpoint of the mathematical apparatus applicability, the condition (1) is quite natural for **"measurable"** QFT. It is shown that passage to higher or lower energies in such QFT is also naturally formulated in terms of the basic (numerical) parameters of the **measurability** notion. The main task is to convert the mathematical apparatus of the well-known QFT to continuous space-time in terms of **measurable** quantities. Provided this task be solved adequately, we should have the possibility to solve successfully the ultraviolet divergence in **"measurable"** QFT and hence in QFT itself.

2 Equivalence Principle Applicability Boundary in Canonical Theory

Einstein's Equivalence Principle (EP) underlies not only General Relativity (GR) [7]–[9] but also the fundamental physics as a whole. In the standard formulation it is as follows: ([9],p.68):
"at every space-time point in an arbitrary gravitational field it is possible to choose **a locally inertional coordinate system** *such that,within a sufficiently small region of the point in question, the laws of nature take the same form as in accelerated Cartesian coordinate systems in the absence of gravitation".*
Then in ([9],p.68) "...There is also a question, how small is **"sufficiently small"**. Roughly speaking, we mean that the region must be small enough so that gravitational field in sensible constant throughout it...".
However, the statement **"sufficiently small"** is associated with another problem. Indeed, let $\overline{x}$ be a certain point of the space-time manifold $\mathcal{M}$ (i.e. $\overline{x} \in \mathcal{M}$) with the geometry given by the metric $g_{\mu\nu}(\overline{x})$. Next, in accordance with EP, there is some **sufficiently small** region $\mathcal{V}_r$ of the point $\overline{x}$ with characteristic linear size r so that, within $\mathcal{V}_r$, we have

$$g_{\mu\nu}(\overline{x}) \equiv \eta_{\mu\nu}(\overline{x}), \tag{2}$$

where $\eta_{\mu\nu}(\overline{x})$ is Minkowskian metric.
In essence, **sufficiently small** $\mathcal{V}_r$ means that the region $\mathcal{V}^{'}$, for which $\overline{x} \in \mathcal{V}^{'}_{r'} \subset \mathcal{V}_r$ with $r' < r$ (here r, r' are characteristic spatial sizes of $\mathcal{V}_r$ and $\mathcal{V}^{\prime}_r$ correspondingly), satisfies (2) as well. In this way we can construct the

sequence

$$...\subset \mathcal{V}''_{r''} \subset \mathcal{V}'_{r'} \subset \mathcal{V}_{r},$$
$$... < r'' < r' < r \quad (3)$$

The problem arises, is there any lower limit for the sequence in formula (3)? The answer is positive. Currently, there is no doubt that at very high energies (on the order of Plancks energies $E \approx E_p$), i.e. on Plancks scales, $l \approx l_p$ quantum fluctuations of any metric $g_{\mu\nu}(\overline{x})$ are so high that in this case the geometry determined by $g_{\mu\nu}(\overline{x})$ is replaced by the "geometry" following from **space-time foam** that is defined by great quantum fluctuations of $g_{\mu\nu}(\overline{x})$,i.e. by the characteristic spatial sizes of the quantum-gravitational region (for example, [10]–[15]). The above-mentioned geometry is drastically differing from the locally smooth geometry of continuous space-time and EP in it is no longer valid [16]–[23].
From this it follows that the region $\mathcal{V}_{\overline{r},\overline{t}}$ with the characteristic spatial size $\overline{r} \approx l_p$ (and hence with the temporal size $\overline{t} \approx t_p$) is the lower (approximate) limit for the sequence in (3).
It is difficult to find the exact lower limit for the sequence in formula (3)–it seems to be dependent on the processes under study. Specifically, when the involved particles are considered to be point, their dimensions may be neglected in a definition of the EP applicability limit. When the characteristic spatial dimension of a particle is $\mathbf{r}$, the lower limit of the sequence from formula (3) seems to be given by the region $\mathcal{V}_{\mathbf{r}'}$ containing the above-mentioned particle with the characteristic dimensions $\mathbf{r}' > \mathbf{r}$, i.e. the space EP applicability limit should always be greater than dimensions of the particles considered in this region. By the present time, it is known that spatial dimensions of gauge bosons, quarks, and leptons within the limiting accuracy of the conducted measurements $< 10^{-18}m$. Because of this, the condition $\mathbf{r}' \geq 10^{-18}m$ must be fulfilled.In addition, the radius of interaction of particles $\mathbf{r}_{int}$ must be taken into account in quantum theory. And this fact also imposes a restriction on considering concrete processes in quantum theory. However, the interactions radii of all known processes lie in the energy scales $E \ll E_p$.
Therefore, it is assumed that the Equivalence Principle is valid for the locally smooth space-time and this suggests that all the energies E of the particles in the most general form meet the condition

$$E \ll E_p. \quad (4)$$

Then, if not stipulated otherwise, we can assume that the condition (4) is valid.

3 Quantum Field Theory,Ultraviolet Divergences and Cosmological Term Estimation in QFT

The canonical quantum field theory (QFT) is a local theory considered in continuous space-time with a plane geometry, i.e with the Minkowskian metric $\eta_{\mu\nu}(\overline{x})$ [24]– [26].
Actually, any interaction introduces some disturbances, introducing an additional local (little) curvature into the initially flat Minkowskian space $\mathcal{M}$. Then the metric $\eta_{\mu\nu}(\overline{x})$ is replaced by the metric $\eta_{\mu\nu}(\overline{x}) + o_{\mu\nu}(\overline{x})$, where the increment $o_{\mu\nu}(\overline{x})$ is small. But, when it is assumed that EP is valid, the increment $o_{\mu\nu}(\overline{x})$ in the local theory has no important role and, in a fairly small neighborhood of the point $\overline{x}$, formula (2) is valid.
Within the scope of the canonical QFT, the process of passage to more higher energies without a change in the local curvature has no limits [24]–[26], just this fact is the reason for ultraviolet divergences in QFT. But as follows from the previous section, this is not the case. Actually, on passage to the Planck energies $E \approx E_p$ (Planck scales $l \approx l_p$), the space in the Planck neighborhood $\mathcal{V}_{\overline{r},\overline{t}}$ of the point $\overline{x}$ one cannot consider flat even locally and in this case (as noted above) EP is not valid.
Then we introduce the following assumption:

Assumption 3.1
In the canonical QFT in calculations of the quantities it is wrong to sum (or same consider within a single sum) the contributions corresponding to space-time manifolds with locally nonzero or zero curvatures since these contributions are associated with different processes: (1) with the existence of a gravitational field that, in principle, can hardly be excluded; (2) in the absence of a gravitational field.

From the start, we can isolate the case when EP is valid (at sufficiently low energies, specifically satisfying the condition (4)) from the cases when EP becomes invalid (for example, Planck energies $E \approx E_p$).
Let us consider a widely known example when **Assumption 3.1** is not fulfilled leading to the senseless results.
In his popular lectures [27] at the Cornell University Steven Weinberg considered an example of calculating, within the scope of QFT, the expected value for the vacuum energy density $< \rho >$ that is proportional to the cosmological term λ. To this end, zero-point energies of all normal modes of some field with the mass m are summed up to the wave number cutoff $\Lambda \gg m$ for the selected normalization $\hbar = c = 1$ (formula (3.5) in [27]):

$$< \rho > \sim \int_0^\Lambda \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2}\sqrt{k^2+m^2} \simeq \frac{\Lambda^4}{16\pi^2}. \quad (5)$$

Assuming, similar to [27], that GR is valid at all the energy scales up to the Planck's, we have the cutoff $\Lambda \simeq (8\pi G)^{-1/2}$ and hence (formula (3.6) in [27]) leads to the following result:

$$<\rho>\propto 2\cdot 10^{71} GeV^4, \qquad (6)$$

that by 10^{118} orders of magnitude differs from the well-known experimental value for the vacuum energy density

$$<\rho_{exp}>\preceq 10^{-29}\text{g}/cm^3 \propto 10^{-47} GeV^4. \qquad (7)$$

Here G is a gravitational constant.
It is clear that in this case **Assumption 3.1** fails as Planck's scales and those close to them at lower energies are included into consideration. By the author's opinion, this is impermissible because for Planck's scales the quantum rather than classical gravity is true and the space even in a small neighborhood of the point is hardly flat. But in formula (5) for the cutoff $\Lambda \simeq (8\pi G)^{-1/2}$ this fact is not included because all calculations in the canonical QFT [26] are valid for the locally flat space and hence (5) in this case leads to senseless results.
Of particular interest is the **inverse problem**: if the experimental value of the vacuum energy density $<\rho_{exp}>$ is known from (7), substituting it into formula (5), we can estimate Λ_{exp} at the upper limit of integration by the above formula

$$<\rho_{exp}>\sim \int_0^{\Lambda_{exp}} \frac{4\pi k^2 dk}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m^2} \simeq 10^{-47} GeV^4. \qquad (8)$$

Note that Λ_{exp} may be found in other way. Denoting by Λ_{UV} the quantity $\simeq (8\pi G)^{-1/2}$ from formula (5), corresponding to the cutoff at Planck's scale $\approx 1,6\cdot 10^{-33} cm$ that is taken as the ultraviolet cutoff, denoting the required quantity $<\rho>$ by $<\rho_{UV}>$, by Λ_{IF} denoting the quantity from the same formula, that corresponds to the cutoff at the scale of a visible part of the Universe $\approx 10^{28} cm$, and the corresponding quantity $<\rho>$ denoting as $<\rho_{IF}>$ (infrared limit), in accordance with [28],[29], we obtain

$$<\rho_{exp}>=\sqrt{<\rho_{UV}><\rho_{IF}>}. \qquad (9)$$

Obviously, Λ_{exp} derived from formulae (8), (9) satisfies the condition (4) and in this case **Assumption 3.1** is fulfilled.

Remark 3.2
In this work we, in fact, consider two extremes:
a)low energies $E \ll E_p$ *and*
b)very high (essentially maximal) energies $E \approx E_p$.

Then it should be noted that, as all the experimentally involved energies E are low, they satisfy condition a). Specifically, for LHC maximal energies are $\approx 10TeV = 10^4 GeV$, that is by 15 orders of magnitude lower than the Planck energy $\approx 10^{19} GeV$.
Moreover, the characteristic energy scales of all fundamental interactions also satisfy condition a). Indeed, in the case of strong interactions this scale is $\Lambda_{QCD} \sim 200MeV$; for electroweak interactions this scale is determined by the vacuum average of a Higgs boson and equals $\upsilon \approx 246GeV$; finally, the scale of the (Grand Unification Theory (GUT)) M_{GUT} lies in the range of $\sim 10^{14}GeV - -10^{16}GeV$. It is obvious that all the above figures satisfy condition a).
Thus, only the expected characteristic energy scale of quantum gravity satisfies condition b).

From **Remark 3.2** it directly follows that even very high energies arising on unification of all the interaction types $M_{GUT} \approx 10^{14}GeV - \sim 10^{16}GeV$,(except of gravitational), Satisfy the condition (4).
At the same time, it is clear that the requirement of the Lorentz-invariant QFT, due to the action of Lorentz boost (or same hyperbolic rotations) (formula (3) in [8]), results in however high momenta and energies. But it has been demonstrated that unlimited growth of the momenta and energies is impossible because in this case we fall within the energy region, where the conventional quantum field theory [24]– [26] is invalid.
Note that at the present time there are experimental indications that Lorentz-invariance is violated in QFT on passage to higher energies (for example, [30]).
Proceeding from the above, the requirement for Lorentz-invariance of QFT is possible only within the scope of the condition (4).

4 Measurability Notion.Some Clarifications and Additions

In this Section we briefly consider some of the results from [1]–[6] which are necessary for further studies. Without detriment to further consideration, in the initial definitions we lift some unnecessary restrictions and make important specifications.
Presently, many researchers are of the opinion that at very high energies (Plank's or trans-Planck's) the ultraviolet cutoff exists that is determined by some maximal momentum.
Therefore, it is further assumed that there is a maximal bound for the measurement momenta $p = p_{max}$ represented as follows:

$$p_{max} \doteq p_{\ell} = \hbar/\ell, \qquad (10)$$

where ℓ is some small length and $\tau = \ell/c$ is the corresponding time. Let us call ℓ the *primary* length and τ the *primary* time.
Without loss of generality, we can consider ℓ and τ at Plank's level, i.e. $\ell \propto l_p, \tau = \kappa t_p$, where the numerical constant κ is on the order of 1. Consequently, we have $E_\ell \propto E_p$ with the corresponding proportionality factor, where $E_\ell \doteq p_\ell c$.

Explanation. In the theory under study it is not assumed from the start that there exists some minimal length l_{min} and that ℓ is such. In fact, the minimal length is defined with the use of Heisenberg's Uncertainty Principle (HUP) $\Delta x \cdot \Delta p \geq \frac{1}{2}\hbar$ or of its generalization to high (Planck) energies – Generalized Uncertainty Principle (GUP) [32]–[40], for example, of the form [32]

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' l_p^2 \frac{\Delta p}{\hbar}, \tag{11}$$

where α' is a constant on the order of 1. Evidently this formula (11) initially leads to the minimal length $\widetilde{\ell}$ on the order of the Planck length $\widetilde{\ell} \doteq 2\sqrt{\alpha'} l_p$. Besides, other forms of GUP [40] also lead to the minimal length.
But, as is currently known, HUP has been verified and operates well only at low energies $E \ll E_p$. Moreover, there are some serious arguments against GUP as demonstrated in Section IX of review [40].Because of this, in the present work validity of this principle is not implied from the start. GUP it is given merely as an example. As p_{max} (10) is taken at Planck's level, it is clear that HUP is inapplicable. Taking this into consideration, the existence of a certain minimal length $\widetilde{\ell}$ is not mandatory. So, we start from the *primary* length ℓ and the *primary* time τ. The whole formalism, developed in [1]–[6] on condition that ℓ is the minimal length, is valid for the case when ℓ is the *primary* length but now we can lift the formal requirement for involvement of l_{min} in the theory from the start.

4.1. The **primarily measurable** space-time quantities (variations) are understood as the quantities Δx_i and Δt taking the form

$$\Delta x_i = N_{\Delta x_i} \ell, \Delta t = N_{\Delta t} \tau, \tag{12}$$

where $N_{\Delta x_i}, N_{\Delta t}$ are integer numbers. Further in the text we use both $N_{\Delta x_i}, N_{\Delta t}$ and the equivalent N_{x_i}, N_t.

4.2. Similarly, the **primarily measurable** momenta are considered as a subset of the momenta characterized by the property

$$p_{x_i} \doteq p_{N_{x_i}} = \frac{\hbar}{N_{x_i} \ell}, \tag{13}$$

where N_{x_i} is a nonzero integer number and p_{x_i} is the momentum corresponding to the coordinate x_i.

4.3. Finally, let us define any physical quantity as **primarily or elementary measurable** when its value is consistent with point **4.1**,**4.2** and formulae (12), (13).
Then we consider formula (13) with the addition of the momenta $p_{x_0} \doteq p_{N_0} = \frac{\hbar}{N_{x_0}\ell}$, where N_{x_0} is an integer number corresponding to the time coordinate ($N_{\Delta t}$ in formula (12)).
For convenience, we denote **Primarily Measurable Quantities** satisfying **4.1**–**4.3** in the abbreviated form as **PMQ**. Also, for the **Primarily Measurable Momenta** we use the abbreviation **PMM.**

First, we consider the case of **Low Energies**, i.e. $E \ll E_\ell$ (same $E \ll E_p$.
It is obvious that all the nonzero integer numbers N_{x_i}, N_t (or same $N_{x_\mu}; \mu = 0, ..., 3$) from formulae (12),(13) should satisfy the condition $|N_{x_\mu}| \gg 1$. It is clear that all the momenta p_i at **low energies** $E \ll E_p$ meet the condition $p_i = \hbar/(N_i\ell)$, where $|N_i| \gg 1$ but is not necessarily an integer. With regard for smallness of ℓ and for the condition $|N_i| \gg 1$, we can easily show that the difference $1/(N_i\ell) - 1/([N_i]\ell), (\hbar/(N_i\ell) - \hbar/([N_i]\ell))$ is negligible and in this way all momenta in the region of low energies $E \ll E_p$ may be taken as **PMM** with a high accuracy.
It is obviously that the case of **Low Energies** in this section is coincident with the "low energies" condition from **Remark 3.2.**
It is assumed that a theory we are trying to resolve is a deformation of the initial continuous theory.

Definition 4.1
The deformation is understood as an extension of a particular theory by inclusion of one or several additional parameters in such a way that the initial theory appears in the limiting transition [31].

Then it should be noted that **PMQ** is inadequate for studies of the physical processes. In fact, among **PMQ**, we have no quantities capable to give the infinitesimal quantities $dx_\mu, \mu = 0, ..., 3$ in the limiting transition in a continuous theory.
Therefore, it is reasonable to use notion of **Generalized Measurability**
We define any physical quantity at all energy scales as **generalized measurable** *or, for simplicity,* **measurable** *if any of its values may be obtained in terms of* **PMQ** *specified by points* **4.1**–**4.3**.
The **generalized measurable** quantities will be denoted as **GMQ**.

Note that the space-time quantities

$$\frac{\tau}{N_t}=p_{N_tc}\frac{\ell^2}{c\hbar}$$
$$\frac{\ell}{N_i}=p_{N_i}\frac{\ell^2}{\hbar},1=1,...,3, \tag{14}$$

where p_{N_i},p_{N_tc} are **Primarily Measurable** momenta, up to the fundamental constants, are coincident with p_{N_i},p_{N_tc} and they may be involved at any stage of the calculations but, evidently, they are not **PMQ**, but they are **GMQ**.
So, in the proposed paradigm at low energies $E\ll E_p$ a set of the **PMM** is discrete, and in every measurement of $\mu=0,...,3$ there is the discrete subset $\mathbf{P}_{\mathbf{x}_\mu}\subset$ **PMM**:

$$\mathbf{P}_{\mathbf{x}_\mu}\doteq\{...,p_{N_{x_\mu}-1},p_{N_{x_\mu}},p_{N_{x_\mu}+1},...\}. \tag{15}$$

In this case, as compared to the canonical quantum theory, in continuous space-time we have the following substitution:

$$\mathbf{\Delta p}_\mu\mapsto dp_\mu,\mathbf{\Delta p}_{\mathbf{N}_{\mathbf{x}_\mu}}=\mathbf{p}_{\mathbf{N}_{\mathbf{x}_\mu}}-\mathbf{p}_{\mathbf{N}_{\mathbf{x}_\mu}+\mathbf{1}}=\mathbf{p}_{\mathbf{N}_{\mathbf{x}_\mu}(\mathbf{N}_{\mathbf{x}_\mu}+\mathbf{1})};$$
$$\frac{\mathbf{\Delta}}{\mathbf{\Delta p}_\mu}\mapsto\frac{\partial}{\partial\mathbf{p}_\mu};\frac{\mathbf{\Delta F}(\mathbf{p}_{\mathbf{N}_{\mathbf{x}_\mu}})}{\mathbf{\Delta p}_\mu}=\frac{\mathbf{F}(\mathbf{p}_{\mathbf{N}_{\mathbf{x}_\mu}})-\mathbf{F}(\mathbf{p}_{\mathbf{N}_{\mathbf{x}_\mu}+\mathbf{1}})}{\mathbf{p}_{\mathbf{N}_{\mathbf{x}_\mu}}-\mathbf{p}_{\mathbf{N}_{\mathbf{x}_\mu}+\mathbf{1}}}=\frac{\mathbf{F}(\mathbf{p}_{\mathbf{N}_{\mathbf{x}_\mu}})-\mathbf{F}(\mathbf{p}_{\mathbf{N}_{\mathbf{x}_\mu}+\mathbf{1}})}{\mathbf{p}_{\mathbf{N}_{\mathbf{x}_\mu}(\mathbf{N}_{\mathbf{x}_\mu}+\mathbf{1})}}. \tag{16}$$

And

$$\frac{\ell}{\mathbf{N}_{\mathbf{x}_\mu}}\mapsto dx_\mu;$$
$$\frac{\mathbf{\Delta}}{\mathbf{\Delta}_{\mathbf{N}_{\mathbf{x}_\mu}}}\mapsto\frac{\partial}{\partial x_\mu},\frac{\mathbf{\Delta F}(\mathbf{x}_\mu)}{\mathbf{\Delta}_{\mathbf{N}_{\mathbf{x}_\mu}}}=\frac{\mathbf{F}(\mathbf{x}_\mu+\ell/\mathbf{N}_{\mathbf{x}_\mu})-\mathbf{F}(\mathbf{x}_\mu)}{\ell/\mathbf{N}_{\mathbf{x}_\mu}}. \tag{17}$$

It is clear that for sufficiently high integer values of $|N_{x_\mu}|$, formulae (16),(17) reproduce a continuous paradigm in the momentum space to any preassigned accuracy. However, at low energies $E\ll E_\ell$ a set of **PMM** clearly is not a space. Considering this, the formulae at low energies offer **the Correspondence to Continuous Theory (CCT).**

It is important to make the following remarks in medias res:

Remark 4.1.
In this way any point $\{x_\mu\}\in\mathcal{M}\subset\mathbf{R^4}$ and any set of integer numbers high in absolute values $\{N_{x_\mu}\}$ are correlated with a system of the neighborhoods for this point $(x_\mu\pm\ell/N_{x_\mu})$. It is clear that, with an increase in $|N_{x_\mu}|$, the indicated system converges to the point $\{x_\mu\}$. In this case all the ingredients of the initial (continuous) theory the partial derivatives including are replaced

by the corresponding finite differences.

Remark 4.2.
It is further assumed that at low energies $E \ll E_\ell$ (same $E \ll E_p$) all the *observable quantities* **are PMQ**.
Because of this, values of the length ℓ/N_i and of the time ℓ/N_t from formula (14) could not appear in expressions for *observable quantities*, being involved only in intermediate calculations, especially at the summation for replacement of the infinitesimal quantities $dt, dx_i; i = 1, 2, 3$ on passage from a continuous theory to its measurable variant.

Further it is assumed that at **High Energies**, $E \approx E_p$, **PMQ** are *inadequate* for studies of the theory at these energies. The assumption follows quite naturally. For example, if GUP (11) is valid and if $\ell = \widetilde{\ell}$, then at high energies formula (11) creates the momenta $\Delta p(N_{\Delta x}, GUP)$ which are not **primarily measurable** [4] –[6]:

$$\Delta p \doteq \Delta p(N_{\Delta x}, GUP) = \frac{\hbar}{1/2(N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1})\ell}. \tag{18}$$

Naturally, formula (18) represents only a particular case of variations in the **generalized measurable** momenta at high energies $E \approx E_p$. Suppose, we know that in the general case at high energies $E \approx E_p$ minimal variations of momenta are given by a set of the **generalized measurable** quantities $p_{N_{x_\mu}}$, where we have the integer numbers $N_{x_\mu}, |N_{x_\mu}| \approx 1$. Then it is reasonable to assume that minimal variations of "coordinates" at high energies are given by the following formula:

$$l_H(p_{N_{x_\mu}}) \doteq \frac{\ell^2}{\hbar} p_{N_{x_\mu}}, \tag{19}$$

where $p_{N_{x_\mu}}$ are the above-mentioned **generalized measurable** momenta at high energies.

Remark 4.3
When at low energies $E \ll E_p$ we lift restrictions on integrality of N_{x_μ}, from formulae (16),(17) it directly follows that in this case we have a continuous analog of the well-known theory with the only difference: all the used small quantities become dependent on the existent energies and we can correlate them. In this way formula (17) may be written as

$$\begin{gathered} dx_\mu \leftrightarrow \frac{\ell}{\mathbf{N_{x_\mu}}} \to \frac{\ell}{[\mathbf{N_{x_\mu}}]}, \\ \frac{\partial}{\partial x_\mu} \leftrightarrow \frac{\mathbf{\Delta}}{\mathbf{\Delta_{N_{x_\mu}}}} \to \frac{\mathbf{\Delta}}{\mathbf{\Delta_{[N_{x_\mu}]}}} \end{gathered} \tag{20}$$

where $|N_{x_\mu}| \gg 1$ is a sufficiently large number that varies continuously. It is clear that in formula (20) the first arrow corresponds to the continuous theory with a specific selection of values of the infinitesimal quantities dx_μ. As noted above, the difference $\ell/N_{x_\mu} - \ell/[N_{x_\mu}]$ is negligible and hence the second arrow corresponds to passage from the initial continuous theory to a similar discrete theory. Of course, formula (16) may be rewritten in the like manner. In what follows, formula (20) plays a crucial part in derivation of the results and is greatly important for their understanding.
The main target of the author is to form a quantum theory and gravity only in terms of **PMQ**.

5 QFT in Measurable Form.Origin

Considerations of Section 4 point to the fact that the Least Action Principle at low energies $E \ll E_\ell$ are valid in a **measurable** form with substitution of the measurable analogs defined in foregoing Section for all the components involved in proof of these arguments. For the canonical (continuous) case we use the notation of Section 3 in [24].
Let φ be a set of all the considered fields $\varphi \doteq (\varphi_1, \varphi_2, ...)$. Then the action S in the continuous case taking the form

$$S = \int \mathcal{L}(\varphi, \partial_\mu \varphi) d^4x \tag{21}$$

is replaced by the **measurable** action $S_{meas,N}$

$$S_{meas,\{N\}} = \sum \mathcal{L}_{meas,\{N\}}(\varphi, \frac{\mathbf{\Delta}\varphi}{\mathbf{\Delta}_{\mathbf{N}_{\mathbf{x}_\mu}}}) \prod \frac{\ell}{N_{x_\mu}}, \tag{22}$$

where N_{x_μ} – integers with the property $|N_{x_\mu}| \gg 1$,$\mathcal{L}_{meas,N}$–Lagrangian density of the **measurable** fields φ and of their **measurable** analogs for partial derivatives in formula (17) $\frac{\mathbf{\Delta}\varphi}{\mathbf{\Delta}_{\mathbf{N}_{\mathbf{x}_\mu}}}$. This means that all variations of these functions are expressed in terms of only **measurable** quantities. In the product $\prod$ the index μ takes the values $\mu = 0, ..., 3$, and $\{N\}$–collection of all N_{x_μ},i.e. $\{N\} \doteq \{N_{x_\mu}\}$. Further, where this causes no confusion, for the **measurable** quantities corresponding to the set $\{N\}$ we can equally use both the lower index $\{N\}$ and N.
According to **Remarks 4.1.,4.3.** for the integer numbers N_{x_μ} sufficiently high in absolute value we, to a high accuracy, have

$$S = S_{meas,\{N\}}. \tag{23}$$

Then it is assumed that all the considered functions are **measurable**, i.e. all variations of these functions are expressed in terms of only **measurable**

quantities. The paper [6] presents in detail a measurable form of the Least Action Principle.
It is clear that in all the formulae, similar to formula (22), on passage from QFT in continuous consideration to the **measurable** form of QFT, in accordance with (16) and (17), the substitution is performed

$$\int \mapsto \sum; \partial_\mu \mapsto \frac{\mathbf{\Delta}}{\mathbf{\Delta}_{\mathbf{N}_{\mathbf{x}_\mu}}}; d^4x \mapsto \prod \frac{\ell}{N_{x_\mu}}, ... \tag{24}$$

Now we suppose that condition (4) (or equivalently $E \ll E_\ell$) is satisfied, i.e. the existent energies are low.
Then in general case as be noted in Sections 2,3, EP is valid and in the well-known Quantum Field Theory (QFT) [24]– [26] and, specifically, in its part used for the collider computations, space-time is assumed to be locally flat, i.e. to be locally Minkowskian.
Besides, as noted in **Remark 3.2**, actually all the energies considered experimentally meet the condition $E \ll E_\ell$,(or same $E \ll E_p$) and hence (see the end of **Remark 4.2**) in **measurable** consideration all *observable quantities* are **PMQ**.
In this case in **measurable** picture we have a *discrete* QFT that is *almost-continuous* due to **Remark 4.3**. As such a theory in the momentum representation has the upper limit cut-off, it is not Lorentz-invariant from the start. As distinct from other works in the proposed approach the wave function is considered separately at high energies $E \approx E_p$ and at low energies $E \ll E_p$, with the imposed restriction that the first function is a high-energy deformation of the second function [2]. In other works (for example, in [39]) the wave function is common for all the energy scales. But according to the **Assumption 3.1**, this is impossible because the indicated functions belong to spaces of different geometries: curved and flat.
It is clear that the above-mentioned *discrete* (*almost-continuous*) (QFT), with a cut-off at a certain upper limit of the momenta which are considerably much lower than the Planck, should be ultraviolet-finite. In this case passage to higher energies means going from the momenta $p_N, |N| \gg 1$ to the momenta $p_{N'}, |N| > |N^{'}| \gg 1$ and, vice versa, passage to lower energies is going in the last inequality from the integers $N^{'}$ to the integers N.
For further resolution of the indicated QFT, along with formula (24), we should "translate" correctly the mathematical apparatus of the Fourier transform and Dirac δ-function into the **measurable** form. As already noted in Section 3 there is the experimental indication that Lorentz-invariance is violated on passage to higher energies even for the canonical QFT, i.e. in the continuous space-time paradigm.
A more detailed presentation of this section results was presented in a recently published paper [41].

6 Conclusion

The following conclusions may be drawn:

6.1 As shown in Sections 2 and 3, Einstein's Equivalence Principle (EP) has natural applicability boundaries. In this case the Planck scales $E \approx E_p$ form the upper (rough) boundary. However, a finer boundary for the specific processes in high-energy physics is always considerably lower than the Planck's, lying within the energy region $E \ll E_p$. This boundary is the natural applicability boundary for the well-known Quantum Field Theory (QFT) [24]– [26] *only* in flat space-time.

6.2 In this way we predetermine the two energy scales: $E \approx E_p$ and $E \ll E_p$ corresponding to the Early Universe and the Modern Universe. The remaining (intermediate) energy scales are not yet considered because it is assumed that, due to **Remark 3.2** in Section 3, their influence on the processes under study is minor.

6.3 In the proposed approach the mathematical apparatus of well-known QFT [24]– [26] in continuous space-time based on the use of the **abstract** infinitesimal quantities dx_μ, dp_i, dE is replaced by the apparatus based on the **measurability** notion and involving the ordered small quantities dependent on the existent energies. All small space-time variations in the indicated theories are generated by the momenta, (**primarily measurable** at low energies and **generalized measurable** at high energies). Considering the involvement of the *primary* length $\ell \propto l_p$, in this case the initial theory becomes *discrete* but at low energies, far from the Planck energy $E \ll E_p$, it is **very close to** the initial theory in continuous space-time. Real *discreteness* is revealed at high energies $E \ll E_p$. Such an approach enables one to study the QFT in the same terms at all the energy scales and in principle,considering the content of the item **6.1**, to construct this theory without ultraviolet divergences.

6.4 Evidently that for the correctness of the theory it is necessary that at low energies $E \ll E_p$ all results should not depend on the choice p_{max}.

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