Nuclear Structure of $^{40}$Ca and $^{48}$Ca within a Self-Consistent Skyrme HF-RPA Calculations

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Abstract

The nuclear structure of $^{40}$Ca and $^{48}$Ca nuclei are studied using the self-consistent Hartree-Fock HF and random phase approximation RPA calculations with Skyrme-type interactions SK255, SKI3, SKM*, SKMP and SKP. To verify this method the calculated results of the ground state examined by comparing the binding energy, root mean square and charge density distribution with the available experimental data. Regarding multipole excitations, the strength functions $S(E)$ and transition density of the isoscalar giant monopole resonance (ISGMR), the isovector giant dipole resonance (IVGDR), and isoscalar giant quadrupole resonance (ISGQR) were compared with the available experimental data.

Keywords: Nuclear structure; Collective excitations; Skyrme Hartree Fock. Random Phase Approximation

1. Introduction

Various theoretical approximations exist to deal with the nuclear structure. The self-consistent Hartree-Fock (HF) is reducing the many interacting particles problem to non-interacting particles in a field. The excited states can be described by time-dependent HF with taking into account the long-range or field producing part of the residual interaction. This theory is also expressed in other languages as the random-phase approximation (RPA) [1-4].

The RPA is a theory of excited states of the nucleus which allow the possibility that the excited and ground-state are not of purely independent particle character
but may contain correlations. These correlations are responsible for considerable enhancement of some electromagnetic transition rates, as well as, the spectra of nuclear excitation, which is very important for understanding the nuclear structure [5-9].

One of the best methods is to study the excitation properties of the atomic nucleus by examining its response to a weak external perturbation like the scattering of particle or absorption of a photon. Up to 10 MeV the nucleus responds through the excitation states. Between 10 and 30 MeV, the system response appears on a wide resonance. These are giant resonances GR.

The RPA gives a successful description of nuclear excitation properties GR. To make the calculations of the GR more effective the strength function $S(E)$ technique for the factorized self-consistent Skyrme RPA had been derived. The $S(E)$ peak of low-lying energies has been studied for years and the study of collective models in nuclei provide very important information for understanding the structural and bulk properties of nuclear systems [10].

In the present work, the structure of $^{40,48}$Ca nuclei has been studied in the framework of the self-consistent HF-RPA based on the Skyrme-type interactions SK255, SKI3, SKM*, SKMP and SKP. To verify the method, the ground and the excited properties of the investigated nuclei were compared with the available experimental data.

2. Theory

The Skyrme-type effective nucleon-nucleon interaction is given by [5, 11, 12]:

$$
\langle \vec{k}V_{12}\vec{k}'\rangle = t_0 (1 + x_0 P_{\sigma}) \delta (r) \quad \text{central term}
$$

$$
+ \frac{1}{2} t_1 (1 + x_1 P_{\sigma}) [\delta (r) k^2 + k'^2 \delta (r)]
$$

$$
+ t_2 (1 + x_2 P_{\sigma}) \vec{k}' \cdot \delta (r) \vec{k}
$$

$$
+ \frac{1}{6} t_3 (1 + x_3 P_{\sigma}) \rho^{a} (R) \delta (r)
$$

$$
+ i W_0 (\vec{\sigma}_1 + \vec{\sigma}_2) [\vec{k}' \times \delta (r) \vec{k}]
$$

spin–orbit term

where $r = r_1 - r_2$, $R = (r_1 + r_2)/2$, $P_{\sigma} = (1 + \sigma_1, \sigma_2)/2$ is the spin-exchange operator, $\sigma$ is the Pauli spin matrices and $k' = -(\vec{\nabla}_1 - \vec{\nabla}_2)/2i$ is the Hermitian conjugate of $k = (\vec{\nabla}_1 - \vec{\nabla}_2)/2i$. The parameters $t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3$ and $W_0$ are experimentally adjustable parameters.

The iterative diagonalization method of particle-hole $ph$ RPA starts by adopting an effective Skyrme interaction within the HF equation. Then, the familiar $ph$ RPA eigenvalue equation are solved [13, 14].
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\[
\begin{pmatrix}
A & B \\
-A & -B
\end{pmatrix}
\begin{pmatrix}
X^v \\
Y^v
\end{pmatrix}
= E_v
\begin{pmatrix}
X^v \\
Y^v
\end{pmatrix}
\]

(2)

With

\[
A_{mnij} = (\varepsilon_m - \varepsilon_i) \delta_{mn} \delta_{ij} + V_{mnij}^{ph}
\]

(3)

\[
B_{mnij} = V_{mnij}^{ph}
\]

where $\varepsilon_m$ is the single particle energy. The residual interactions should be in particle-particle $pp$ channel,

\[
V_{mnij}^{ph} = -\sum_{J'}(2J' + 1)\begin{pmatrix}
j_m & j_n & J' \\
j_i & j_j & J
\end{pmatrix}V_{mnij}^{pp}
\]

(4)

The residual interaction can be built from the self-consistent Skyrme-HF energy density functional,

\[
V_{mnij}^{ph} = \frac{\delta^2 E^{HF}}{\delta \rho_{m} \delta \rho_{nj}}
\]

(5)

The strength function of the low-lying energies can be obtained as follows,

\[
S(E) = \sum_{\nu} \left| \langle \nu | \hat{F}_J | 0 \rangle \right|^2 \delta(E - E_\nu)
\]

(6)

Where $\hat{F}_J$ is the nuclear multipole operator between the RPA ground $|0\rangle$ and excited states $|\nu\rangle$ with the corresponding excitation energy $E_\nu$. For plotting purpose, the strength functions is approximated as follows,

\[
S(E) = \sum_{\nu} \left| \langle \nu | \hat{F}_J | 0 \rangle \right|^2 \rho(\Gamma)(E - E_\nu)
\]

(7)

where the Lorentzian function is defined as in the following:

\[
\rho(\Gamma)(E - E_\nu) = \frac{1}{2\pi} \frac{1}{(E - E_\nu)^2 + (\Gamma / 2)^2}
\]

(8)

with $\Gamma$ is the smearing parameter.

The radial transition density of state $|\nu\rangle$ is defined as follows [15],

\[
\delta \rho_{\nu}(r) = \frac{1}{\sqrt{2J + 1}} \sum_{mi} \left[ X_{mi}^{(\nu)} + Y_{mi}^{(\nu)} \right] \left| \langle m | \nu | i \rangle \right|^2 \frac{u_m(r)u_i(r)}{r^2}
\]

(9)
where $X$ and $Y$ are RPA amplitudes. The isoscalar IS (T=0) and isovector IV (T=1) densities defined,

$$
\delta\rho_v^{(IS)}(r) \equiv \delta\rho_{v_s}(r) + \delta\rho_{v_v}(r)
$$

$$
\delta\rho_v^{(IV)}(r) \equiv \delta\rho_{v_v}(r) - \delta\rho_{v_s}(r)
$$

3- Result and discussion

In this paper, the nuclear structure of $^{40,48}$Ca was studied using the fully self-consistent HF-RPA with SK255 [16], SKI3 [17], SKM* [18], SKMP [19], SKP [20] Skyrme-type interactions. The symmetric nuclear matter properties at saturation density $\rho_0$, the incompressibility modulus $K_{NM}$, isoscalar effective mass $m^*/m$, and the enhancement factor $\kappa$ of the investigated interactions are given in Table 1.

The HF calculation of the ground state properties including the binding energy, root mean square and charge density distribution, were examined and compared with the available experimental data. The calculated binding energies per nucleon for the investigated nuclei are listed in Table 2 with the corresponding experimental data [21]. The result are in good agreement with available experimental data with a simple difference by the type of Skyrme parameterization.

The nuclear charge densities give us a picture of the internal structure of nuclei. Figures 3 and 4 shows the calculated HF charge distribution for $^{40}$Ca and $^{48}$Ca, respectively. The used interactions in good agreement with the experimental data [22] at the surface and interior regions.

Table 1. Symmetric nuclear matter properties at saturation density $\rho_0$ [fm$^3$], the incompressibility modulus $K_{NM}$ (MeV), isoscalar effective mass $m^*/m$, and the enhancement factor $\kappa$ of the investigated interactions.

<table>
<thead>
<tr>
<th>Force</th>
<th>$\rho_0$</th>
<th>$K_{NM}$</th>
<th>$m^*/m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SK255</td>
<td>0.157</td>
<td>225.0</td>
<td>0.80</td>
</tr>
<tr>
<td>SKI3</td>
<td>0.158</td>
<td>258.1</td>
<td>0.58</td>
</tr>
<tr>
<td>SKM*</td>
<td>0.160</td>
<td>216.7</td>
<td>0.79</td>
</tr>
<tr>
<td>SKMP</td>
<td>0.157</td>
<td>230.9</td>
<td>0.65</td>
</tr>
<tr>
<td>SKP</td>
<td>0.162</td>
<td>200.8</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Table 2. Ground state properties of the investigated.

<table>
<thead>
<tr>
<th>Skyrme Type</th>
<th>E/A (MeV)</th>
<th>$\langle r_n^2 \rangle^{1/2}$ (fm)</th>
<th>$\langle r_p^2 \rangle^{1/2}$ (fm)</th>
<th>$\langle r_{ch}^2 \rangle^{1/2}$ (fm)</th>
<th>E/A (MeV)</th>
<th>$\langle r_n^2 \rangle^{1/2}$ (fm)</th>
<th>$\langle r_p^2 \rangle^{1/2}$ (fm)</th>
<th>$\langle r_{ch}^2 \rangle^{1/2}$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SK255</td>
<td>-8.94</td>
<td>3.35</td>
<td>3.40</td>
<td>3.49</td>
<td>-8.98</td>
<td>3.63</td>
<td>3.43</td>
<td>3.52</td>
</tr>
<tr>
<td>SKM*</td>
<td>-8.53</td>
<td>3.38</td>
<td>3.43</td>
<td>3.52</td>
<td>-8.76</td>
<td>3.60</td>
<td>3.45</td>
<td>3.54</td>
</tr>
<tr>
<td>SKMP</td>
<td>-8.48</td>
<td>3.37</td>
<td>3.42</td>
<td>3.50</td>
<td>-8.84</td>
<td>3.59</td>
<td>3.43</td>
<td>3.52</td>
</tr>
<tr>
<td>SKP</td>
<td>-8.58</td>
<td>3.40</td>
<td>3.45</td>
<td>3.54</td>
<td>-8.68</td>
<td>3.63</td>
<td>3.48</td>
<td>3.57</td>
</tr>
<tr>
<td>Exp.</td>
<td>-8.55</td>
<td>3.49</td>
<td></td>
<td></td>
<td>-8.67</td>
<td>3.49</td>
<td></td>
<td>3.47</td>
</tr>
</tbody>
</table>

Figure 1. The calculated charge density distribution of $^{40}$Ca compared with the experimental data [22].
The strength distribution (fraction of EWSR) of the isoscalar monopole $E_0$ and the quadrupole $E_2$, and (photo absorption cross section) of the isovector dipole $E_1$ are shown in figures 4 and 5 for $^{40}\text{Ca}$ and $^{48}\text{Ca}$, respectively. The Lorenzian smearing $\Gamma$ of 3 MeV width was used in the calculations. The results were compared with the experimental data [23-25]. Most of interactions work best and agree with data concerning centroid energy ($m_1/m_0$), widths and profiles of strength. The form of the calculated strength distribution for $^{40}\text{Ca}$ are in good agreement with the experimental but the calculated strength distribution peak were 2-6 MeV higher than the experimental. In $^{48}\text{Ca}$, the form of the strength distribution is like Gaussian distribution in the low excitation region but with a large tailing on the high energy extending to 40 MeV. The calculated $m_1/m_0$ were overestimate the data several MeV and the calculations do not reproduce a large tailing seem at higher excitations.

The peak energy is clearly related to $K_{NM}$, therefore high $K_{NM}$ shift the peak to upper energy while the low $K_{NM}$ shift in less [26-28], like SKI3 interaction with $K_{NM} = 258.1$ MeV give the energy equal to 24.96 and 24.38 in $^{40}\text{Ca}$ and $^{48}\text{Ca}$ respectively. The interactions (SKM$^*$ and SKP) with $K_{NM} = 216.7$ and 200.8 respectively, produce the GMR at the same energy but the simple difference in place.

The experimental data show that the ISGMR in $^{48}\text{Ca}$ is 0.7 MeV higher than in $^{40}\text{Ca}$ while four of the investigated Skyrme interactions show the centroid energies of ISGMR for $^{40}\text{Ca}$ were higher than the experimental value and only one of the interaction which is (SKMP) showed the ISGMR at higher energy in $^{48}\text{Ca}$. The centroid energies of ISGMR in $^{48}\text{Ca}$, were more consistent with the experimental value like SKP interaction has the same energy with experimental value $\approx 19.88$ MeV.

The E2 strength (experimental) observed corresponds to $108 \pm 12\%$ and $83^{+10}_{-16}\%$ of E2 EWSR with a centroid of $17.84\pm0.43$ and $18.61\pm0.24$ [23, 24] for

![Figure 2. The calculated charge density distribution of $^{48}\text{Ca}$ compared with the experimental data [22].](image-url)
$^{40}$Ca and $^{48}$Ca respectively. In $^{48}$Ca, the form of the strength distribution is almost Gaussian distribution below 25 MeV, but between 25 and 40 MeV it is distributed roughly uniformly.

A correlation exists between the ISGQR centroid energy of the investigated nuclei with $m'/m$. It is found that the interactions SK255, SKM* having $m'/m=0.80, 0.79$ respectively in $^{40}$Ca and the interaction SKI3 with $m'/m= 0.58$ in $^{48}$Ca, reproduce the experimental data for ISGQR.

The calculated transition densities of proton and neutron for $^{40}$Ca and $^{48}$Ca are presented in Figures 5 and 6 as a function of the radial coordinates of the states using SK255, SKI3, SKM*, SKMP and SKP Skyrme-type interactions for the ISGMR, IVGDR, and ISGQR. In the ISGMR mode, the surface of the nucleus is confirmed for the centroid energy range 19-25 MeV for $^{40,48}$Ca. Clearly, protons and neutrons are oscillated in the same trend, in all cases, the surface has a dominant isoscalar character. The same behavior is illustrated for peak energy in ISGQR, i.e., the IS character of the surface of the nucleus is confirmed, for the energy range 15-22 MeV for $^{40}$Ca and 13-19 MeV for $^{48}$Ca.

The total transition in IVGDR mode is dominated by IV component for the main strength peaks around 18-20 MeV for $^{40,48}$Ca. Obviously, both protons and neutrons contribute to the transition and oscillate in opposite directions.

4. Conclusion

The results of binding energies per nucleon and charge density distribution are in good agreement with the available experimental data. The interactions (SKM* and SKP) with $K_{NM} = (216.7$ and $200.8$ MeV) respectively, produce the ISGMR at the same energy but with simple difference in place. For ISGMR, the strong correlation was obtained for the centroid energy with the NM incompressibility coefficient $K_{NM}$. But, for ISGQR, The correlation between the centroid energy and the effective mass $m^*/m$ was obtained.

References


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Figure 3. The calculated strength distribution for the fraction of EWSR (a) isoscalar monopole (E0), (b) quadrupole (E2), and (c) photo absorption dipole cross section (E1) in $^{40}$Ca, were obtained using different Skyrme-type interactions. A Lorenzian smearing $\Gamma$ of 3 MeV was used in the calculation. Experimental data are from [23, 24] for ISGMR and ISGQR and [25] for IVGDR, are shown as red-solid lines.
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Figure 4. Our calculations of the strength distribution for the fraction of EWSR (a) isoscalar monopole (E0), (b) quadrupole (E2), and (c) photo absorption dipole cross section (E1) in $^{48}\text{Ca}$, were obtained using different Skyrme-type interactions. A Lorenzian smearing $\Gamma$ of 3 MeV was used in the calculation. Experimental data are from [23, 24] for ISGMR and ISGQR and [25] for IVGDR, are shown as red-solid lines.
Figure 5. The calculated transition densities of proton and neutron for $^{40}$Ca, a) ISGMR, b) IVGDR and c) ISGQR. The calculations were done with different Skyrme-type interactions.
Figure 6. The calculated transition densities of proton and neutron for $^{48}$Ca, a) ISGMR, b) IVGDR and c) ISGQR. The calculations were done with different Skyrme-type interactions.

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