Advanced Studies in Theoretical Physics
Vol. 12, 2018, no. 1, 37 - 55
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/astp.2018.71264

The Relationship between the Cosmological Inverse Gravity Assertion and the Cosmological Constant Including an Alternative Possibility to the Graviton

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Abstract

A review of the published paper titled "Cosmology: The theoretical possibility of inverse gravity as a cause of cosmological inflation in an isotropic and homogeneous universe and its relationship to weakly interacting massive particles" is conducted. The original paper introduced the concept of cosmological expansions as described by a parameterized inversion of Newtonian gravitational force which is then applied to the mathematics of an isotropic and homogeneous F-L-R-W universe which inherently incorporates general relativity. Thus, after a review of the seminal paper, the cosmological inverse gravity assertion introduced in the original paper is applied to the cosmological constant of Einstein's field equation, which defines cosmological expansion in terms of the new theoretical assertion. Lastly, it is shown that the mathematical structure of the theoretical concept gives to the notion of a mediating particle that conducts both the attractive forces of gravity and the repulsive forces of cosmological expansion and thus dark energy. This gives to an alternative theoretical notion to the graviton particle.

Keywords: isotropic, homogeneous, expansion, scale factors, cosmological constant, Mediating particle, graviton

Introduction

A recently published paper titled "Cosmology: The theoretical possibility of inverse gravity as a cause of cosmological inflation in an isotropic and homogeneous universe and its relationship to weakly interacting massive particles" [10] introduced a theoretical assertion in cosmology where parameterized inverse variations in Newtonian gravitational force are postulated to mathematically describe cosmological expansion. In the seminal paper, the Cosmological inverse gravity assertion was applied to established aspects of cosmology that showed consistency with observed astronomical phenomena. More specifically, the theoretical assertion was applied to an isotropic and homogeneous F-L-R-W universe incorporating the aspects of gravitational redshift of a photon's wavelength, the Robertson-Walker scale factors, Hubble's law, the Friedman-Walker-Robertson metric, and the Einstein tensor in terms of an F-L-R-W universe which incorporates the description of curved space-time and thus general relativity. Additionally, the assertion was applied to weakly interacting massive particles (WIMPS) which required the use of quantum field theory (which is not reviewed in this paper). Therefore, this paper reviews the seminal paper and expounds on its assertion. This paper applies the cosmological inverse gravity assertion to Einstein's cosmological constant and introduces an alternative to the theoretical notion of a graviton which does contradict the concept's application to weakly interacting massive particles featured in the original paper.

Section 1 reviews the core concepts and mathematical formulations of the original paper. The objective of the original paper was to introduce and show the assertion's mathematical compatibility to fundamental and observed aspects of cosmology (e.g. the Einstein tensor in terms of the F-L-R-W), hence, the objective to section 2 is to apply the theoretical assertion to Einstein's cosmological constant which effectively solidifies and expands the theoretical assertions as a technically correct postulate. Lastly, the mathematical structure of the cosmological inverse gravity assertion, permits a description of a quantum particle that mediates gravity as well as inverse gravity (or expansion), thus implying an alternative notion to the theoretical graviton. Therefore, section 3 introduces the formulations that describe the proposed theoretical quantum particle.

1. A review of the seminal paper

As previously expressed, the objective of section 1 is to review the mathematics and assertion of the seminal paper titled "Cosmology: The theoretical possibility of inverse gravity as a cause of cosmological inflation in an isotropic and homogeneous universe and its relationship to weakly interacting massive particles" [10]. Cosmological Inflation refers to accelerated expansion in the nascent universe within the early epochs shortly after the big bang while expansion refers to the expanding universe at the present time. One flaw with the seminal paper is the lack of clarity between cosmological inflation and expansion.

The cosmological inverse gravity assertion can be applied to inflation, however, it is more conducive to expansion. The original paper referred to the assertion as the inverse gravity inflationary assertion (IGIA). Thus, due to the fact that the theoretical assertion refers to the description of expansion as opposed to inflation, the assertion will be stated as the inverse gravity expansionary assertion (IGEA). Therefore, in commencing with the review, astronomical observations show that cosmological expansion progresses in opposition to gravitational force and thus accelerates cosmological mass and radiation (and space-time) in an inverse direction to gravity [10]. This fact allows one to hypothesize and deduce that the mathematical description of the structure of the universe could inherently incorporate a mathematical term that is inversely proportional to classical Newtonian gravitational force [10]. Due to the fact that cosmological expansion is only detectable at cosmological distances, logically, this implies that the inverse force is parameterized. Thus, a mathematical term that is inversely proportional to gravity must have a parameter that pertains to spatial or astronomical distance [10]. This parameter will permit the inverse term to have substantial effects beyond a given distance and minimal affects below a given distance which is consistent with the description of cosmological expansion. Force value $F_g(r)$ denotes the classical Newtonian gravitational force equation while $F_g'(r)$ denotes the inverse gravity term shown by Eq.1 displayed below [10] [11].

$$F_g(r) = \frac{Gm_1\, m_2}{r^2} \qquad F_g'(r) = \left[\frac{1}{r_0^2}\right]\left[\frac{r^2}{Gm_1\, m_2}\right] \quad ; \qquad 1 > \left[\frac{1}{r_0^2}\right] \tag{1}$$

The constant distance r_0 within the coefficient of the inverse term $F_g'(r)$ is referred to as the <u>spatial cosmological parameter</u> in reference to the IGEA[10]. The variations in the classical gravitational force equation $F_g(r)$ and the inverse gravity force term $F_g'(r)$ combine to form total gravitational force $F_T(r)$ via Eq.2 (below) which is referred to as the <u>Newtonian correction</u> below [10].

$$F_T(r) = F_g'(r) - F_g(r) = \left[\frac{1}{r_0^2}\right]\left[\frac{r^2}{Gm_1\, m_2}\right] - \frac{Gm_1\, m_2}{r^2} \quad ; \qquad 1 > \left[\frac{1}{r_0^2}\right] \tag{2}$$

The direction (+ or -) of the value of total force $F_T(r)$ has relationships defined by the <u>inequalities</u> of radius r to distance r_0 given by the conditions expressed below [10].

$$1.\quad For\ \ r > r_0\,;\ +F_T(r) \qquad\qquad 2.\ For\ \ r < r_0\,;\ -F_T(r) \tag{3}$$

Condition (1) (of Eq.3) describes cosmological expansion or force $+F_T(r)$ away from the gravitational force center (i.e. the center of the universe) for distances $r > r_0$ [10]. Conversely, for condition (2) (of Eq. 3), the inverse gravity term $F_g'(r)$ in total force $F_T(r)$ is negligible at distance $r < r_0$ causing force direction $-F_T(r)$ toward the center of gravitational force (i.e. gravity over takes the inverse term $F_g'(r)$ on $r < r_0$ which represents local scales) [10].This clearly conveys that the core concept of the IGEA is consistent with the <u>general description</u> of the phenomenon of universal or cosmological expansion.

The value of the spatial cosmological parameter of distance r_0 is determined where total force value $F_T(r)$ (of Eq.2) equals zero and radius r equals cosmological parameter r_0 (i.e. $F_T(r_0) = 0$ and $r = r_0$) [10]. Furthermore, this implies that for the condition of $F_T(r_0) = 0$, the inverse force terms $F_g'(r_0)$ and the classical gravitational force term $F_g(r_0)$ of total force $F_T(r_0)$ (of Eq.2) at parametric distance r_0 are equal ($F_g'(r_0) = F_g(r_0)$) in $F_T(r_0)$ [10]. Therefore, total force $F_T(r_0)$ can be presented such that [10]:

$$F_T(r_0) = F_g'(r_0) - F_g(r_0) = \left[\frac{1}{r_0^2}\right]\left[\frac{(r_0)^2}{Gm_1\, m_2}\right] - \frac{Gm_1\, m_2}{(r_0)^2} = 0 \tag{4}$$

Solving Eq.4 for the spatial cosmological parameter of distance r_0 gives a value such that [10]:

$$r_0 = Gm_1\, m_2 \tag{5}$$

The <u>uniform distribution</u> of cosmological masses m_1 and m_2 over a <u>spherically symmetric</u> volume describing a <u>homogeneous isotropic universe</u> separated by a <u>diameter</u> of distance r_0 mathematically require the mass interaction $m_1\, m_2$ to be expressed as a triple integral such that [11]:

$$r_0 = Gm_1\, m_2 = G\left[\int_0^{m_u} \int_0^{\pi} \int_0^{\pi} m(m'_{\text{ц}} m'_{\text{ц}})\, dm d\theta d\phi\right] \tag{6}$$

Where mass m_u (highlighted above) which is the upper limit of the integral with respect to variations in mass is the mass of the universe and thus constitutes all matter of the universe[11]. Thus mass m_u is approximately 27% of the <u>critical density</u> of the universe commonly denoted as ρ_c [10] [11].The critical density of matter denoted $(.27)\rho_c$relates to the mass of the universe m_u and energy E via the relativistic energy equation of $E = m_u c^2 = (.27)\rho_c c^2$, therefore, this implies that universal mass m_u has a value such that [10] [11]:

$$m_u = (.27)\rho_c \equiv (.27)\frac{3H_0^2}{8\pi G} \tag{7}$$

The value of critical density ρ_c is given as $3H_0^2/8\pi G$ (e.g. $\rho_c = 3H_0^2/8\pi G$) in terms of Hubble's constant H_0 and gravitational constant G [10][11]. Variable mass $m'_{\text{ц}}$ within Eq.6 is a function of spherical coordinates at $\theta$and ϕ and mass m (which is a variation in mass in reference to integration) such that [10]:

$$m'_{\text{ц}} = [(mcos\theta sin\phi)^2 + (msin\theta sin\phi)^2 + (mcos\phi)^2]^{1/2} \tag{8}$$

Where variable mass m'_u correspond to the symmetric variations in mass values m_1 and m_2 evenly distributed about the spherically symmetric volume, this implies that mass interaction $m_1\, m_2$ over a spherically symmetric volume is expressed by the triple integral (of Eq. 6) such that [10]:

$$m_1\, m_2 = \int_0^{m_u} \int_0^{\pi} \int_0^{\pi} m^3[(cos\theta sin\phi)^2 + (sin\theta sin\phi)^2 + (cos\phi)^2]\, dmd\theta d\phi \quad (9)$$

As implied by Eq. 5 and Eq.6, masses m_1 and m_2 are spatially located on opposite sides of (and are separated by) parameter distance r_0, thus the continuous sums (or integration) progress as a rotation where the mass values on opposite sides of distance r_0 rotate and sum up to universal mass value m_u [10]. Therefore, the rotation of continuous sums over the spherical coordinates at $\theta$and ϕ that transpire from zero to π encompass the entire spherical volume due to the fact that the rotation on one side of distance r_0 sum up to half the spherical volume (or π) and the other side of distance r_0 rotates and sum up to the other half (or π) of the spherical volume [10]; which equal the whole. Thus, Eq.9 above gives the mass interaction $m_1\, m_2$ corresponding to the gravitational interaction of cosmological masses over distance r_0 [10].
Thus in continuing the review, gravitational potential energy in terms of the IGEA, is the integrations of the Newtonian correction force $F_T(r)$ in respect to radius r as shown below[10][11].

$$U_T(r) = \int F_T(r)dr = \int \left[\left[\frac{1}{r_0^2}\right]\left[\frac{r^2}{Gm_1\, m_2}\right] - \frac{Gm_1\, m_2}{r^2}\right] dr \quad (10)$$

Thus after evaluating the integral of Eq. 10 above, one obtains a value of potential energy $U_T(r)$ in terms of the IGEA such that [10] [11]:

$$U_T(r) = \left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\, m_2}\right] + \frac{Gm_1\, m_2}{r} \quad (11)$$

As a photon propagates over a distance r of the region of expansion, the amount of work enacted on the photon by energy IGEA energy $U_T(r)$ is given by hc/λ_g [10][11]. Hence, gravitational potential energy $U_T(r)$ defined in terms of the IGEA is set equal to the photonic energy of wave length $\lambda_g$such that [10] [11]:

$$\frac{hc}{\lambda_g} = U_T(r) \quad (12)$$

Energy E_0 is the initial energy of the photon ($E_0 = (hc/\lambda_0)$) prior to propagating through the region of expansion [10] [11]. Conclusively, the value of the photonic energy affected by the IGEA potential energy field of $U_T(r)$ can be alternatively expressed such that [10] [11]:

$$\frac{hc}{\lambda_g} = \left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\, m_2}\right] + \frac{Gm_1\, m_2}{r} \quad (13)$$

We briefly express red shift z given below [7] [10].

$$z = \frac{U_T(r)-E_0}{E_0} = \frac{\left(\frac{hc}{\lambda_g}\right)-E_0}{E_0} \equiv \frac{\lambda_0}{\lambda_g} - 1 \tag{14}$$

Eq.14 relates the theoretical potential energy $U_T(r)$ to redshift z which will relates to the scale factors of Eq.17. Now substituting the value of Eq.13 into the gravitational red shift equation of Eq. 14 above gives [7] [9] [10]:

$$z = \frac{1}{E_0}\left(\left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\, m_2}\right] + \frac{Gm_1\, m_2}{r}\right) - 1 \tag{15}$$

Prior to continuing the derivation, an **important clarification** must be expressed. It is paramount to justify the use of gravitational redshift z of Eq. 14-15 as opposed to using the *Schwarzchild expression of gravitational redshift* shown below [9] [10].

$$\frac{\lambda_R}{\lambda_E} - 1 = \sqrt{\frac{1-2GM/r_R}{1-2GM/r_E}} - 1 \tag{16}$$

Where λ_E and λ_R denote the initial and final wavelength values of a photon propagating through a gravitational potential and distance r_E from the gravitational body's center of mass M and distance r_R from the gravitational center of the body to the propagating photon [9][10]. There are two purposes for not using the Schwarzchild equation of gravitational redshift above in the IGEA. The first is that in an FRW universe describing an isotropic and homogeneous model, cosmological expansion and thus redshift (z) is described in relation to the scale factors $a(t)$ (e.g. a_0 and $a(t_{em})$) [9] [10]. Whereas cosmological expansion is a result of the expansion of space-time itself versus gravitational red shift due to the gravitational curving space-time corresponding to a body of mass M [10]. Additionally, it is believed that new volumes of space-time are created as the universe expands. The second purpose is that the Schwarzchild description of gravitational redshift is not mathematically compatible to the IGEA, being that a point mass m_0 (as in GMm_0/r) does not conveniently cancel out the equation when the energy values are set equal to one another [10].

In continuing with the formulation, red shift value z is related to the scale factor of the past (or the time of emission) denoted $a(t_{em})$ and the present denoted a_0 such that [7] [9] [10]:

$$1 + z = \frac{a_0}{a(t_{em})} \tag{17}$$

Substituting the IGEA value of redshift z of Eq. 15 into Eq. 17 above gives [10]:

$$1 + \left(\left(\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\, m_2}\right] + \frac{Gm_1\, m_2}{r}\right]\right) - 1\right) = \frac{a_0}{a(t_{em})} \tag{18}$$

This reduces to [10]:

$$\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{r}\right]=\frac{a_0}{a(t_{em})} \tag{19}$$

If follows that the value of the scale factor $a(t_{em})$ at the time t_{em} of the photon's emission is expressed such that [10]:

$$a(t_{em})=a_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{r}\right]\right]^{-1} \tag{20}$$

Where Eq.20 is of the form $a(t)=1/(1+z)$ [7] which implies that scale factor a_0 equals 1 ($a_0=1$) [10]. Recall that a_0 is the scale factor of the universe as it is presently and $a(t_{em})$ is the scale factor at the emission time t_{em} of the photon (or a scale factor of the universe as it was in the past as some authors state it)[7] [10]. The Friedman-Lemaitre-Walker-Robertson metric is shown below [9] [10].

$$d\Sigma^2=-dt^2+a^2(t)\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2sin^2\theta d\phi^2\right] \tag{21}$$

Substituting the value of scale factor $a(t_{em})$ (Eq.20) into Eq. 21 above gives the <u>Friedman-Walker-Robertson metric</u> in terms of the IGEA such that [10]:

$$d\Sigma^2=-dt^2+\left[a_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{r}\right]\right]^{-1}\right]^2\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2sin^2\theta d\phi^2\right] \tag{22}$$

Observe that Eq. 20 or the IGEA scale factor is a function of radius r, which relates to the Minkowski coordinates (t,x,y,z) for signature (- + + +)such that [9] [10]:

$$r=[-t^2+x^2+y^2+z^2]^{1/2} \qquad \rightarrow \qquad t^2\leq x^2+y^2+z^2 \tag{23}$$

On a cosmological level, the initial points $(t_0=0,x_0=0,y_0=0,z_0=0)$ in radial distance r above reside at an origin point of the singularity of the big bang. Substituting the value of Eq. 23 into Eq. 20 gives the IGEA scale factor as a function of the Minkowski coordinates (denoted $a(t,x,y,z)$) such that [9] [10]:

$$a(t_{em})=a(t,x,y,z)=a_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(-t^2+x^2+y^2+z^2)^{3/2}}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{[-t^2+x^2+y^2+z^2]^{1/2}}\right]\right]^{-1} \tag{24}$$

The time derivative of scale factor $a(t_{em})$ is denoted $a(t,\dot{x},y,z)$ (where $a(t,x,y,z)$ is simply scale factor (t_{em})) in respect to the time coordinate t (keep in mind that $t = t_{em}$) of the Minkowski coordinates and can be expressed such that [9] [10]:

$$a(t,\dot{x},y,z) = \frac{da(t,x,y,z)}{dt} = \frac{\partial}{\partial t}\left[a_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(-t^2+x^2+y^2+z^2)^{3/2}}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{[-t^2+x^2+y^2+z^2]^{1/2}}\right]\right]^{-1}\right] \quad (25)$$

In implementing the chain rule, the first time derivative $(da(t,x,y,z))/dt)$ of the IGIA scale factor denoted $a(t,\dot{x},y,z)$ gives a value such that [10]:

$$a(t,\dot{x},y,z) = 2ta_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(-t^2+x^2+y^2+z^2)^{1/2}}{2Gm_1\,m_2}\right] + \frac{Gm_1\,m_2}{2[-t^2+x^2+y^2+z^2]^{3/2}}\right]\right]\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(-t^2+x^2+y^2+z^2)^{3/2}}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{[-t^2+x^2+y^2+z^2]^{1/2}}\right]\right]^{-2} \quad (26)$$

Eq.25 and thus Eq. 26 afford the opportunity to briefly present Hubble's constant in terms of the IGEA such that [9] [10] [11]:

$$H(t) = \frac{\dot{a}}{a} = \left[\frac{1}{a(t,x,y,z)}\right]\frac{da(t,x,y,z)}{dt} \quad (27)$$

Therefore, Hubble's constant takes on a value in terms of the IGEA such that [10] [11]:

$$H(t) = 2ta_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(-t^2+x^2+y^2+z^2)^{1/2}}{2Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{2[-t^2+x^2+y^2+z^2]^{3/2}}\right]\right]\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(-t^2+x^2+y^2+z^2)^{3/2}}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{[-t^2+x^2+y^2+z^2]^{1/2}}\right]\right]^{-1} \quad (28)$$

Thus in an isotropic and homogeneous universe, the $V(t)$ velocity of cosmological expansion is expressed in reference to *Hubble's Law* such that [9] [10] [11]:

$$V(t) = \frac{R\dot{a}(t)}{a(t)} = \left[\frac{R}{a(t)}\right]\frac{da(t)}{dt} \quad (29)$$

Where R is the distance from the observer, the velocity of expansion $V(t)$ in terms of the IGEA is given such that [10] [11]:

$$V(t) = 2tRa_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(-t^2+x^2+y^2+z^2)^{1/2}}{2Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{2[-t^2+x^2+y^2+z^2]^{3/2}}\right]\right]\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(-t^2+x^2+y^2+z^2)^{3/2}}{3Gm_1\,m_2}\right]+\frac{Gm_1\,m_2}{[-t^2+x^2+y^2+z^2]^{1/2}}\right]\right]^{-1} \quad (30)$$

According to Wald [9], the Ricci tensor values of R_{tt} and R_{**} are then related to the scale factor $a(t,x,y,z)$ in <u>terms of the IGEA</u> by the equations of (where $\ddot{a} = d^2(a(t,x,y,z))/dt^2$) [9] [10]:

$$R_{tt} = -3a(t,\ddot{x},y,z)/a(t,x,y,z) \tag{31}$$

$$R_{**} = a(t,x,y,z)^{-2}R_{xx} = \frac{a(t,\ddot{x},y,z)}{a(t,x,y,z)} + 2\,\frac{(a(t,\dot{x},y,z))^2}{a(t,x,y,z)^2} \tag{32}$$

As stated by Wald, the general value of Ricci tensor R_{xx} in Eq.31-32 above relates to the christoffel symbol via the form [9] [10]:

$$R_{xx} = \sum_c \frac{\partial y}{\partial x^c}\Gamma^c_{xx} - \frac{\partial}{\partial x^x}\left(\sum_c \Gamma^c_{cx}\right) + \sum_{d,c}\left(\Gamma^d_{xx}\Gamma^c_{dc} - \Gamma^d_{cx}\Gamma^c_{dx}\right) \tag{33}$$

Thus we acknowledge that the value of the symmetric Christoffel symbols Γ^a_{bc} are of the form [9] [10]:

$$\Gamma^a_{bc} = \frac{1}{2}\sum_d g^{ad}\left\{\frac{\partial g_{cb}}{\partial x^b} + \frac{\partial g_{ca}}{\partial x^c} - \frac{\partial g_{bc}}{\partial x^c}\right\} \tag{34}$$

Which correspond to the expressions of the Ricci tensor $R_{tt}$and R_{**} of Eq.31-32. It is important to state that the value of the scalar curvature R is given such that [9] [10]:

$$R = -R_{tt} + 3R_{**} \tag{35}$$

Substituting the value of Eq.31 and Eq.32 into Eq.35 give a value such that [9] [10]:

$$R = -R_{tt} + 3R_{**} = 6\left(\frac{a(t,\ddot{x},y,z)}{a(t,x,y,z)} + \frac{(a(t,\dot{x},y,z))^2}{a(t,x,y,z)^2}\right) \tag{36}$$

Conclusively as expressed in the seminal paper and by Wald [9], the values of the Einstein tensor denoted G_{tt} and G_{**} in terms of the IGEA scale factor $a(t,x,y,z)$ as given such that [10]:

$$G_{tt} = \frac{3(a(t,\dot{x},y,z))^2}{a(t,x,y,z)^2} = R_{tt} + \frac{1}{2}R = 8\pi\rho \tag{37}$$

$$G_{**} = -2\,\frac{a(t,\ddot{x},y,z)}{a(t,x,y,z)} - \frac{\left(a(t,\dot{x},y,z)\right)^2}{a(t,x,y,z)^2} = R_{**} - \frac{1}{2}R = 8\pi P \tag{38}$$

Due to the fact that the description of the IGEA is defined in reference to a homogeneous and isotropic universe, the general evolutions for a isotropic and homogeneous universe as defined by Wald [9] and in respect to the IGEA scale factors are given such that [1] [9] [10]:

$$\frac{3(a(t,\dot{x},y,z))^2}{a(t,x,y,z)^2} = 8\pi\rho - \frac{3k}{a(t,x,y,z)^2} \tag{39}$$

$$\frac{3a(t,\ddot{x},y,z)}{a(t,x,y,z)} = -4\pi(\rho + 3P\ ) \tag{40}$$

Where P denotes pressure corresponding to thermal radiation pressure and ρ is the average mass density [9] [10], the scale factors $a(t,x,y,z)$ and their corresponding time derivatives ($a(t,\dot{x},y,z)$ and $a(t,\ddot{x},y,z)$) can be defined in terms of the IGEA scale factor of Eq. 38); constant k is equal to +1 ($k = +1$) and $r > r_0$ for positive spherical curvature describing the expansion of the cosmological fluid in a homogeneous isotropic universe [9] [10]. This shows the complete mathematical incorporation of the IGEA to the mathematical description of a homogeneous and isotropic universe and achieves the goal of defining the IGEA in terms of curved space-time which is consistent with General relativity.

2. Einstein's Cosmological constant defined in terms of the IGEA

The objective of the seminal paper [10] was to introduce the cosmological inverse gravity assertion and its correlation to observed cosmological phenomena and established fundamental aspects of cosmology. Thus, in the seminal paper, the cosmological inverse gravity assertion introduced fundamental mathematical correlations to an F-L-R-W universe that excluded Einstein's cosmological constant. Therefore, Section 2 will introduce Einstein's cosmological constant in terms of the IGEA. The IGEA is described in terms of the F-L-R-W Universe, thus, we begin with Einstein field equations including the cosmological constant Λ in terms of the Einstein field equation define in an F-L-R-W universe for thermal pressure P and the average mass density ρ shown below (curvature constant k is equal to +1 ($k = +1$) for $r > r_0$) [8].

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \frac{\Lambda c^2}{3} = \frac{8\pi G}{3}\rho \tag{41}$$

$$2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \Lambda c^2 = -\frac{8\pi G}{3}P \tag{42}$$

Where G is the gravitational constant and c the velocity of light, the values of the cosmological constant Λ can be expressed such that:

$$\Lambda = \frac{3}{c^3}\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} - \frac{8\pi G}{3}\rho\right] = \frac{1}{c^2}\left[2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} + \frac{8\pi G}{3}P\right] \tag{43}$$

Recall that the scale factor a in terms of the IGEA is expressed such that:

$$a(t,x,y,z) = a_0\left[\frac{1}{E_0}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(-t^2+x^2+y^2+z^2)^{3/2}}{3Gm_1\,m_2}\right] + \frac{Gm_1\,m_2}{[-t^2+x^2+y^2+z^2]^{1/2}}\right]\right]^{-1} \tag{44}$$

Setting scale factor a in Eq.41-43 equal to the IGEA scale factor $a(t,x,y,z)$ ($a = a(t,x,y,z)$), the cosmological constant in terms of the IGEA denoted Λ_{IG}, have values in terms of thermal pressure P and the average mass density ρ such that:

$$\Lambda_{IG} = \frac{3}{c^3}\left[\left(\frac{a(t,\dot{x},y,z)}{a(t,x,y,z)}\right)^2 + \frac{kc^2}{\left(a(t,x,y,z)\right)^2} - \frac{8\pi G}{3}\rho\right] \tag{45}$$

$$\Lambda_{IG} = \frac{1}{c^2}\left[2\left(\frac{a(t,\ddot{x},y,z)}{a(t,x,y,z)}\right) + \left(\frac{a(t,\dot{x},y,z)}{a(t,x,y,z)}\right)^2 + \frac{kc^2}{\left(a(t,x,y,z)\right)^2} + \frac{8\pi G}{3}P\right] \tag{46}$$

Where
$a(t,\dot{x},y,z) = da(t,x,y,z))/dt \quad and \quad a(t,\ddot{x},y,z) = d^2(a(t,x,y,z))/dt^2$.
Thus, concluding the derivation and the mathematical incorporation of the IGEA to the cosmological constant.

3. An alternative to the graviton particle as derived by the IGEA

The IGEA heralds an alternative theoretical proposal to the nature of the speculated mediating force particle of gravity i.e. the graviton. The IGEA postulates that the expansion of the universe in opposition to gravity at cosmological scales implies that a particle mediating the force of gravity must also incorporate the inverse variations of dark energy and thus the force of expansion. In commencing with the mathematical description of the theoretical IGEA mediating particle, gravitational potential energy on a cosmological level was defined in terms of the IGEA in Eq. 10 such that:

$$U_T(r) = \left[\frac{1}{r_0^2}\right]\left[\frac{r^3}{3Gm_1\,m_2}\right] + \frac{Gm_1\,m_2}{r} \tag{47}$$

As expressed by Eq. 9, the gravitational mass interaction (or mass product) m_1m_2 denotes the cosmological masses of the universe which are evenly and uniformly distributed over a spherical volume, the mass product m_1m_2 was expressed as the triple integral such that:

$$m_1m_2 = \int_0^{m_u}\int_0^{\pi}\int_0^{\pi} m^3[(cos\theta sin\phi)^2 + (sin\theta sin\phi)^2 + (cos\phi)^2]\,dmd\theta d\phi \tag{48}$$

Therefore, for gravitational interactions between masses that are less than cosmological masses m_1 and m_2 (of mass product m_1m_2 above), mass values m_{a1} and m_{a2} are any two mass values that are within and less than the total cosmological mass value. Thus, the mass product of masses m_{a1} and m_{a2} ($m_{a1}m_{a2}$) are required to be less than the mass product of cosmological masses m_1 and m_2 (m_1m_2) as expressed by the inequality below.

$$m_1m_2 > m_{a1}m_{a2} \tag{49}$$

Here, we define energy E_a as the potential energy of gravity and inverse gravity between two arbitrary masses m_{a1} and m_{a2} which are separated by a distance γ_a ($r \geq \gamma_a$). This implies that values of energy E_a are <u>less than</u> the cosmological level energy $U_T(r)$ of Eq. 47 ($U_T(r) > E_a$). The value of energy E_a is given in terms of masses m_{a1} and m_{a2} and distance $\gamma_a$such that:

$$E_a = \left[\frac{1}{r_0^2}\right]\left[\frac{(\gamma_a)^3}{3Gm_{a1}m_{a2}}\right] + \frac{Gm_{a1}m_{a2}}{\gamma_a} \tag{50}$$

Observe that the parameter of distance r_0 in Eq.50 above has a value that remains in terms cosmological mass product m_1m_2 (of Eq.6) as shown below.

$$r_0 = Gm_1\, m_2 \equiv G\int_0^{m_u}\int_0^{\pi}\int_0^{\pi} m^3[(cos\theta sin\phi)^2 + (sin\theta sin\phi)^2 + (cos\phi)^2]\, dmd\theta d\phi \tag{51}$$

The parameterization of the inverse term of gravity (i.e.$1/r_0^2$) is contingent on the cosmological mass (of Eq.48) of the universe and remains constant at any level including the subatomic and quantum realms. The value of parameter distance r_0 is determined by the value of the matter of the universe m_u defined in Eq.7, along with variations in coordinates at θ and ϕ. Recall that m_u of Eq.7 has a value such that:

$$m_u = (.27)\rho_c \equiv (.27)\frac{3H_0^2}{8\pi G} \tag{52}$$

The value of universe mass m_u within parameter r_0 is substantial and effectively parameterizes the inverse gravity variations of masses m_{a1} and m_{a2} separated by distance γ_a within energy E_a on <u>both</u> macroscopic and subatomic levels. Thus, the objective of this section is to show that gravitational interactions and thus force between any arbitrary mass m_{a1} and m_{a2} which are separated by a distance γ_a ($\gamma_a > 0$) are mediated by bosons of energy E_a, thus energy E_a is set equal to relativistic energy pc as shown below (where p denotes momentum and c the velocity of light).

$$E_a = pc \tag{53}$$

Eq.53 can alternatively be expressed such that [10]:

$$pc = \left[\frac{1}{r_0^2}\right]\left[\frac{(\gamma_a)^3}{3Gm_{a1}m_{a2}}\right] + \frac{Gm_{a1}m_{a2}}{\gamma_a} \tag{54}$$

Like the theoretical graviton, the IGEA mediating particle does not have a length limit or range as with the mediating particles of the other three fundamental forces. However, as with the other mediating particles, the IGEA mediating particle is governed by the uncertainty principle ($\Delta E\Delta t \geq \hbar$), thus the IGEA energy value E_a is set equal uncertainty in energy of ΔE (i.e.$\Delta E = E_\mu$) [10]. The product of energy (ΔE) and time (Δt) ($\Delta E\Delta t = (E_a)\Delta t$) can be expressed such that [10]:

$$\Delta E \Delta t = \left(\left[\frac{1}{r_0^2}\right]\left[\frac{(\gamma_a)^3}{3Gm_{a1}m_{\mu 2}}\right] + \frac{Gm_{a1}m_{a2}}{\gamma_a}\right)\Delta t \tag{55}$$

The Heisenberg uncertainty principle ($\Delta E \Delta t \geq \hbar$) can be expressed in terms of the IGIA such that:

$$\left(\left[\frac{1}{r_0^2}\right]\left[\frac{(\gamma_a)^3}{3Gm_{a1}m_{a2}}\right] + \frac{Gm_{a1}m_{a2}}{\gamma_a}\right)\Delta t \geq \hbar \tag{56}$$

Thus the values of energy E_a and the uncertainty in time Δt must satisfy the inequality of Eq.56 above. In continuing the mathematical description of the theoretical particle, we solve Eq.54 for momentum p giving a value of momentum such that:

$$p = \frac{1}{c}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(\gamma_a)^3}{3Gm_{a1}m_{a2}}\right] + \frac{Gm_{a1}m_{a2}}{\gamma_a}\right] \tag{57}$$

Thus, we observe that the theoretical particle has the property of both repulsive and attractive terms within the momentum value p as shown below.

$$repulsive\text{:}\quad \left[\frac{1}{cr_0^2}\right]\left[\frac{(\gamma_a)^3}{3Gm_{a1}m_{a2}}\right] \qquad attractive\text{:}\quad \frac{Gm_{a1}m_{a2}}{c\gamma_a} \tag{58}$$

Therefore, for sufficient distance values γ_a, momentum p is *repulsive*, thus for lesser distance values of γ_a, momentum p is $attractive$. An <u>important dilemma</u> that must be addressed is embodied in the hypothetical question of “what if masses m_{a1} and m_{a2} of mass product $m_{a1}m_{a2}$ of Eq.58 become infinitesimally small (i.e. $m_{a1} \to 0$ and $m_{a2} \to 0$ which implies that $m_{a1}m_{a2} \to 0$)?”. This would imply that the repulsive or inverse gravity term in Eq.57-58 would become infinitely large, which means that the momentum, force, and energy value between the particles of infinitesimally small mass values m_{a1} and m_{a2} would become excessively large which <u>has not be observed in nature</u>. Thus, this implies that there is a minimal limit for masses m_{a1} and m_{a2} in the quantum and/or subatomic realm. This size is determined by the Heisenberg uncertainty principle ($\Delta E \Delta t \geq \hbar$); the uncertainty in energy ΔE is set equal to $m_a c^2$ ($\Delta E = m_a c^2$), where mass m_a represents mass values m_{a1} and m_{a2}. Measuring from a unit of time, the uncertainty in time Δt assumes a value of 1 second ($\Delta t = 1s$), therefore, one obtains the expression:$(m_a c^2)(1s) \geq \hbar$. Solving the inequality for mass m_a gives an inequality such that:

$$m_a \geq \frac{\hbar}{c^2} \tag{59}$$

Conclusively, the minimum value permitted for each individual value of masses m_{a1} and m_{a2} of mass product $m_{a1}m_{a2}$ is expressed by Eq.59 above which has a calculated value of approximately $7.36 \times 10^{-33} eV/c^2$ ($m_a \geq (7.36 \times 10^{-33} eV/c^2)$). Thus, in continuing with the theoretical description, for an observer spatially measuring the phenomenon in R^{μ} degrees of freedom, the unit vector u in R^{μ} ($u \in R^{\mu}$, $|u| = 1$) is applied to momentum value p of Eq.57 such that:

$$p_{ц} = pu \equiv \frac{u}{c}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(\gamma_a)^3}{3Gm_{a1}m_{a2}}\right] + \frac{Gm_{a1}m_{a2}}{\gamma_a}\right] \tag{60}$$

Therefore, the momentum $p_{ц}$ in ц spatial dimensions ($p_{ц} \in R^{\mu}$) is given by Eq. 60 above. Due to the wave-like characteristics of the quantum world, the wave number k_{μ} ($k_{ц} \in R^{\mu}$) can be expressed in terms of the IGEA such that [11]:

$$k_{\mu} = \frac{p_{ц}}{\hbar} \equiv \frac{u}{\hbar c}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(\gamma_a)^3}{3Gm_{a1}m_{\mu2}}\right] + \frac{Gm_{a1}a}{\gamma_a}\right] \tag{61}$$

Eq.61 above, allows the expression of dispersion relations between angular velocity ω and wave number k_{μ} ($\left|k_{\mu}c\right|^2 = |\omega|^2$) [6]. The mediating particle constitutes a free particle in space, hence, we acknowledge the complex plane wave equation and it conjugate at momentum value $p_{ц}$ and position x_{μ} ($x_{\mu} \in R^{\mu}, such\ that \left|x_{\mu}\right| \leq \gamma_a$) such that [11]:

$$\Psi = e^{i\frac{p_{ц}x_{ц}}{\hbar}} \qquad\qquad \Psi^{*} = e^{-i\frac{p_{ц}x_{ц}}{\hbar}} \tag{62}$$

Substituting the value of momentum $p_{ц}$ of Eq.60 in wave functions $\Psi$and Ψ^{*} of Eq.62 above give:

$$\Psi = exp\frac{ix_{\mu}}{\hbar}\left[\frac{u}{c}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(\gamma_a)^3}{3Gm_{a1}m_{a2}}\right] + \frac{Gm_{a1}m_{a2}}{\gamma_{\mu}}\right]\right] \tag{63}$$

$$\Psi^{*} = exp\left[-\frac{ix_{\mu}}{\hbar}\left[\frac{u}{c}\left[\left[\frac{1}{r_0^2}\right]\left[\frac{(\gamma_a)^3}{3Gm_{a1}m_{a2}}\right] + \frac{Gm_{a1}m_{a2}}{\gamma_a}\right]\right]\right] \tag{64}$$

At this juncture, we are mathematically prepared to give a complete quantum expression of the IGEA mediating particle which appropriately includes field theory. The goal is to describe a gravitational interaction within the quantum realm. The IGEA mediating particle describes gravitational interactions that include the inverse variations of dark energy which is the energy of expansion, hence, the interaction Langrangian L_{int} describing the IGEA mediating particle is the same as the one formulated for the theoretical graviton. The interaction Langrangian formulated to describe the theoretical graviton as expressed by Holstein [3] is given such that:

$$L_{int} = -\frac{1}{2}\kappa T_{\mu v}h^{\mu v} \tag{65}$$

Where κ is the coupling constant which relates to Newton's gravitational constant G such that: $\kappa^2 = 32\pi G$ [3]. The gravitational field tensor $h^{\mu v}$ [2] is typically used in describing the variations in the geometry of space-time due to gravitational waves, therefore, this indicates that the IGEA mediating particle is the quantum vibration of space-time between masses m_{a1} and m_{a2}. The gravitation field tensor $h_{\mu v}$ is expressed in terms of tensor $A_{\mu v}$ and the IGEA wave function Ψ of Eq. 62 such that [3]:

$$h_{\mu v} = A_{\mu v}\Psi \tag{66}$$

Gravitational interactions are typically described within the Traverse traceless gauge (The TT gauge) [2]. Hence the matrix $A_{\mu v}$ of Eq.66 has a value associated with the TT gauge given such that [2]:

$$A_{\mu v} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{xx} & A_{xy} & 0 \\ 0 & A_{xy} & -A_{xx} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{67}$$

As expressed in the interaction [9],

$$h^{\mu v} = \eta_{\mu v} h_{\mu v} \tag{68}$$

Where $\eta_{\mu v}$ is the Minkowski flat metric[9], we observe the stress-energy-momentum tensor $T_{\mu v}$ of graviton interaction Lagrangian L_{int} of Eq.65 expressed <u>in terms of a scalar field</u> (e.g. $\phi(x_ц)$) has a value such that [3]:

$$T_{\mu v} = \partial_\mu \phi(x_ц)\, \partial_v \phi^\dagger(x_ц) + \partial_\mu \phi^\dagger(x_ц)\, \partial_v \phi(x_ц) - g_{\mu v}[\partial_\mu \phi(x_ц) \partial^v \phi^\dagger(x_ц) - \phi(x_ц)\phi^\dagger(x_ц) m^2] \tag{69}$$

The stress-energy- momentum tensor $T_{\mu v}$ is defined on <u>linear manifold</u> M that is a subspace to a complex Hilbert space ђ ($M \subset$ ђ $\cup$ C; C *being the set of complex numbers*) [1], therefore, the scalar fields $\phi(x_ц)$ and $\phi^\dagger(x_ц)$ constitute complex quantum fields which serve as a differentiable manifold of space-time. In terms of quantum field theory, the scalar fields $\phi(x_ц)$ and $\phi^\dagger(x_ц)$ are the particle and anti-particle fields which have a form [4]:

$$\phi(x_ц) = \int \frac{d^3p}{\sqrt{2(2\pi)^3}} \bar{u}_f(p)\hat{a}^\dagger \Psi + \int \frac{d^3p}{\sqrt{2(2\pi)^3}} \bar{v}_f(p)\hat{b}\Psi^* \tag{70}$$

Or Alternatively,

$$\phi = \phi^+ + \phi^- \tag{71}$$

And [4],

$$\phi^{\dagger}\left(x_{\text{ц}}\right)=\int \frac{d^3p}{\sqrt{2(2\pi)^3}}\bar{u}_f(p)\hat{b}^{\dagger}\Psi+\int \frac{d^3p}{\sqrt{2(2\pi)^3}}\bar{v}_f(p)\hat{a}\Psi^{*} \tag{72}$$

Or Alternatively,

$$\phi^{\dagger}=\phi^{\dagger -}+\phi^{\dagger +} \tag{73}$$

The wave function Ψ and its complex conjugate Ψ^{*} within field functions $\phi\left(x_{\text{ц}}\right)$ and $\phi^{\dagger}\left(x_{\text{ц}}\right)$ of Eq. 70 and Eq.72 are of the form of the IGEA wave functions of Eq. 63-64 [10]. Eq. 70 and Eq.72 show that the IGEA mediating particle is a field with wave-like variations. Where the notation ∂_a in Eq.69 is the vector valued partial derivative or gradient vector of the form:

$$\partial_a=\frac{\partial}{\partial x_a} \tag{74}$$

As shown in the Klein-Gordon term $(\partial_{\mu}\phi(x_{\text{ц}})\partial^{v}\phi^{\dagger}(x_{\text{ц}})-\phi(x_{\text{ц}})\phi^{\dagger}(x_{\text{ц}})m^{2})$ of the stress-energy-momentum tensor $T_{\mu v}$ of Eq.69, the contravariant vector $\partial^{v}\phi^{\dagger}\left(x_{\text{ц}}\right)$ relates to the covariant vector $\partial_{v}\phi^{\dagger}\left(x_{\text{ц}}\right)$ via the Minkowski metric $\eta_{\mu v}$such that [9]:

$$\partial^{v}\phi^{\dagger}\left(x_{\text{ц}}\right)=\eta_{\mu v}\partial_{v}\phi^{\dagger}\left(x_{\text{ц}}\right) \tag{75}$$

The spinors of $\bar{u}_f(p)$ and $\bar{v}_f(p)$ corresponds to the integer spin of bosons (or masseless particles) and their corresponding antiparticles expressed in the scalar fields $\phi\left(x_{\text{ц}}\right)$ and $\phi^{\dagger}\left(x_{\text{ц}}\right)$ of Eq. 69-72 [4] [10] [11]. Thus the values of spinors $\bar{u}_f(p)$ and $\bar{v}_f(p)$ correspond to the solution set of [4] [10]:

$$\bar{u}_1(p)=\sqrt{\frac{E+m_p}{2m_p}}\begin{bmatrix}1\\0\\\frac{p_3}{E+m_p}\\\frac{p_1+ip_2}{E+m_p}\end{bmatrix}\qquad \bar{u}_2(p)=\sqrt{\frac{E+m_p}{2m_p}}\begin{bmatrix}0\\1\\\frac{p_1-ip_2}{E+m_p}\\-\frac{p_3}{E+m_p}\end{bmatrix} \tag{76}$$

$$\bar{v}_1(p) = \sqrt{\frac{E+m_p}{2m_p}} \begin{bmatrix} \frac{p_3}{E+m_p} \\ \frac{p_1+ip_2}{E+m_p} \\ 1 \\ 0 \end{bmatrix} \qquad \bar{v}_2(p) = \sqrt{\frac{E+m_p}{2m_p}} \begin{bmatrix} \frac{p_1-ip_2}{E+m_p} \\ -\frac{p_3}{E+m_p} \\ 1 \\ 0 \end{bmatrix} \tag{77}$$

Therefore, in continuing to define the components of the scalar field functions $\phi(x_{ц})$ and $\phi^{\dagger}\,(x_{ц})$, the creation operators ($\hat{a}^{\dagger}, \hat{b}^{\dagger}$) and the annihilation operators ($\hat{a}, \hat{b}$) of field $\phi(x_{ц})$ satisfy the properties of bosons [5] [8] [10]. The creation and annihilation operators have the properties such that [5] [10]:

$$[\hat{a}, \hat{a}^{\dagger}] = \hat{a}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a} = \delta \tag{78}$$

$$[\hat{b}, \hat{b}^{\dagger}] = \hat{b}\hat{b}^{\dagger} - \hat{b}^{\dagger}\hat{b} = \delta \tag{79}$$

$$[\hat{a}^{\dagger}, \hat{a}^{\dagger}] = [\hat{a}, \hat{a}] = 0 \qquad [\hat{b}^{\dagger}, \hat{b}^{\dagger}] = [\hat{b}, \hat{b}] = 0 \tag{80}$$

And [5],

$$\hat{a}^{\dagger}\hat{a} = \hat{b}^{\dagger}\hat{b} = n \tag{81}$$

Where n denotes an arbitrary number. Lastly, the metric tensor $g_{\mu v}$ presented in the stress-energy-momentum tensor $T_{\mu v}$ of Eq. 69 is the sum of the Minkowski metric $\eta_{\mu v}$ and the gravitational tensor field $h_{\mu v}$ of Eq.66 multiplied by a coupling constant κ such that[3]:

$$g_{\mu v} = \eta_{\mu v} + \kappa h_{\mu v} \tag{82}$$

Which constitutes a 4 by 4 matrix. The interaction Lagrangian L_{int} of Eq. 65 encompasses the dynamics within a complete quantum description of the IGEA alternative to the Graviton.

Conclusion

The advantage of the IGEA that is conveyed in both papers is that the concept can be applied on the cosmological level as well as the subatomic. This advantage is warranted by the assertion's ability to show consistency and compatibility on an observational and mathematical basis in reference to known phenomena.

Acknowledgements. I would like to thank my lord and savior Jesus Christ for this achievement. Moreover, I would like to thank my father Edward Walker, my mother Juanita Walker, my sister Brittani Shingles, my nephew Robert Shingles, my brother Jonathan Oliver, and Mr. Eugene Thompson for their support.

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Received: January 9, 2018; Published: January 30, 2018