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The Relationship between Burst Regularity and Spike-Generating Sodium Conductance in a Mathematical Model of Snail RPa1 Neurons

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Abstract

A previously reported mathematical model of snail RPa1 neurons comprises a system of nonlinear ordinary differential equations. This study focuses on the $g_{Na(TTX)}$ parameter of this model that describes the spike-generating sodium conductance and performs a numerical simulation to investigate how variations of $g_{Na(TTX)}$ affect the dynamical states of the model. The results demonstrate that $g_{Na(TTX)}$ determines whether the bursting state of the model is regular or chaotic.

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1 Introduction

Electrophysiological recordings have revealed that snail RPa1 neurons exhibit various dynamical states (e.g., regular repetitive spiking state, chaotic spiking state, regular bursting state, and chaotic bursting state) [1]. Previous studies have proposed a mathematical model that reproduces these dynamical states [1, 2]. This snail RPa1 neuron model, formulated using the Hodgkin–Huxley concept, comprises a system of nonlinear ordinary differential equations (ODEs). From a viewpoint of nonlinear dynamics, it will be interesting to investigate how the chaotic states of the RPa1 neuron model are modulated by different parameters. In fact, a previous study has shown that varying a certain parameter changes the dynamical state of the model from a regular repetitive spiking state → a chaotic spiking state → a regular bursting state [1]. However, this previous study did not investigate the sensitivity of the chaotic bursting state to varying parameters. Spike-generating sodium conductance is one of the parameters of the model. A previous study demonstrated the important role of spike-generating sodium conductance in regulating the dynamical states of a mathematical model of vibrissa motoneuron, which is different from the snail RPa1 neuron model [3]. Inspired by this previous study, we aimed to explore the role of spike-generating sodium conductance in regulating the dynamical states of the snail RPa1 neuron model and used numerical simulation to investigate the sensitivity of the chaotic bursting state of the snail RPa1 neuron model to the variations of the spike-generating sodium conductance.

2 The Model

The previously described mathematical model of snail RPa1 neurons comprises a system of eight coupled nonlinear ODEs [1]. The time evolution of state variables is described by Equations (1)–(8) as follows:

$$\begin{aligned}\frac{dV}{dt} = \frac{1}{0.02}\Bigg(&-0.13\left(\frac{1}{1+e^{-0.2(V+45)}}\right)(V-40)-0.18m_B h_B(V+58)\\ &-0.02(V-40)-0.25(V+70)\\ &-g_{Na(TTX)}m^3h(V-40)-10n^4(V+70)\\ &-m_{Ca}{}^2(V-150)-0.01\left(\frac{1}{1+e^{-0.06(V+45)}}\right)\left(\frac{1}{1+e^{15000([\mathrm{Ca}]-0.00004)}}\right)(V-150)\Bigg)\end{aligned} \quad (1)$$

$$\frac{dm_B}{dt} = \frac{1}{0.05}\left(\frac{1}{1+e^{0.4(V+34)}}-m_B\right) \quad (2)$$

$$\frac{dh_B}{dt} = \frac{1}{1.5}\left(\frac{1}{1+e^{-0.55(V+43)}}-h_B\right) \quad (3)$$

$$\frac{dm}{dt} = \frac{1}{0.0005}\left(\frac{1}{1+e^{-0.4(V+31)}}-m\right) \quad (4)$$

$$\frac{dh}{dt} = \frac{1}{0.01}\left(\frac{1}{1+e^{0.25(V+45)}}-h\right) \quad (5)$$

$$\frac{dn}{dt} = \frac{1}{0.015}\left(\frac{1}{1+e^{-0.18(V+25)}} - n\right) \tag{6}$$

$$\frac{dm_{Ca}}{dt} = \frac{1}{0.01}\left(\frac{1}{1+e^{-0.2V}} - m_{Ca}\right) \tag{7}$$

$$\frac{d[\mathrm{Ca}]}{dt} = 0.002\left(-\frac{{m_{Ca}}^{2}(V-150)}{2F\left(\frac{4}{3}\pi 0.1^{3}\right)} - 50[\mathrm{Ca}]\right), \tag{8}$$

where V is the membrane potential of RPa1 neurons (mV); m_{B}, h_{B}, m, h, n, and m_{Ca} are gating variables (unitless); [Ca] is the concentration of intracellular calcium (mM); $g_{Na(TTX)}$ is a system parameter describing the spike-generating sodium conductance (μS); and F (= 96485 C/mol) is the Faraday constant. Using initial conditions (V = –42 mV, m_B = 0.95, h_B = 0.77, m = 0.14, h = 0.1, n = 0.048, m_{Ca} = 0.0002, and [Ca] = 6.5×10^{-5} mM), Equations (1)–(8) were numerically solved using the free and open source software Scilab (http://www.scilab.org/).

3 Numerical Results

The present study focused on the variations of the $g_{Na(TTX)}$ parameter by altering its default value of 400 μS by 97%–103%. Figure 1 demonstrates the time course of the membrane potential of the RPa1 neuron model under varying conditions of $g_{Na(TTX)}$. At a $g_{Na(TTX)}$ of 97% of the default value, the model exhibits a regular bursting state: a type of burst (marked “a1” in Figure 1a) appearing periodically. At 98% and 99% of the default $g_{Na(TTX)}$, the dynamics of the model were found to be similar to those at 97% of the default $g_{Na(TTX)}$: a type of burst similar to “a1” (marked “b1” and “c1” in Figure 1b and 1c, respectively) appearing periodically. At 100% of the default $g_{Na(TTX)}$, the model exhibits a chaotic bursting state similar to that reported in the previous study [1]: various types of bursts that are different from “a1,” “b1,” and “c1” (marked “d1,” “d2,” “d3,” “d4,” and “d5” in Figure 1d) appear to be very irregular. At 101% of the default $g_{Na(TTX)}$, the model shows a regular bursting state: a type of burst that is different from “a1,” “b1,” and “c1” but is similar to “d2” (marked “e1” in Figure 1e) periodically appears. At 102% and 103% of the default $g_{Na(TTX)}$, the dynamics of the model are similar to those at 97%, 98%, and 99% of the default $g_{Na(TTX)}$: a type of burst similar to “a1,” “b1,” and “c1” (marked “f1” and “g1” in Figure 1f and 1g, respectively) periodically appears.

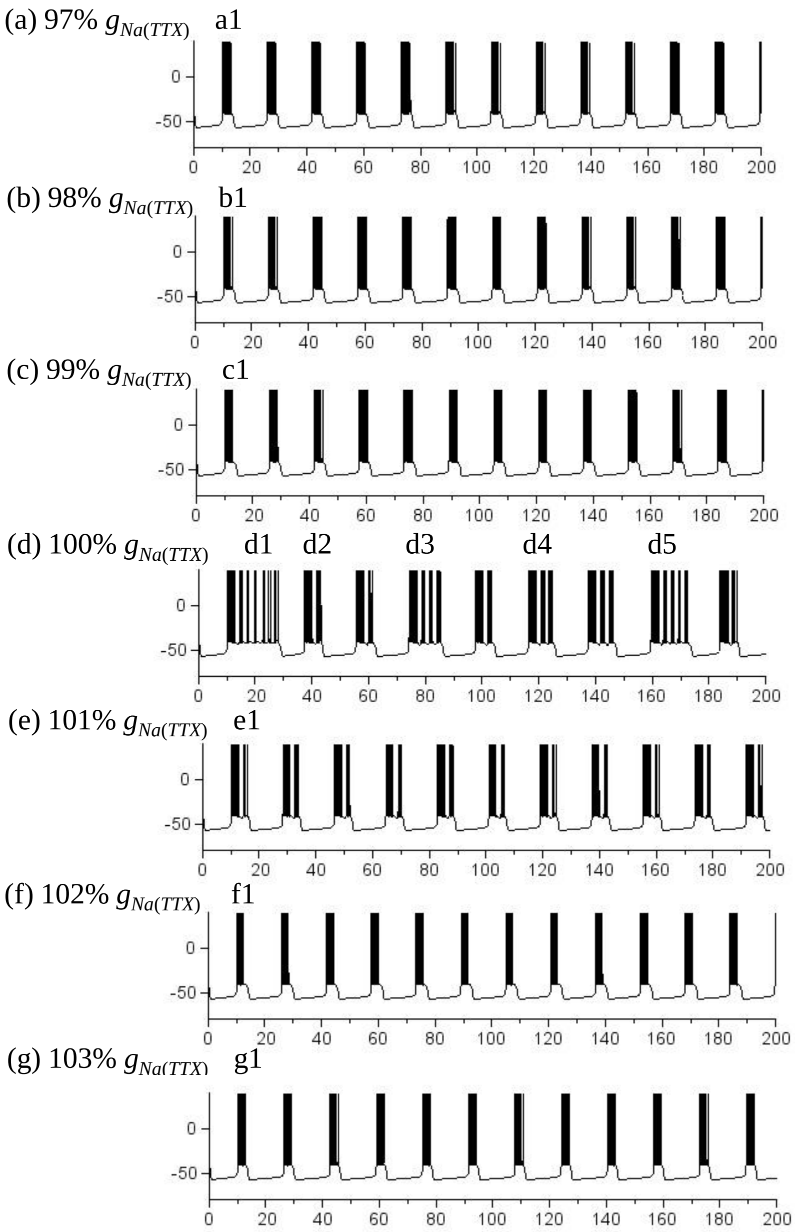

Figure 1. Time course of the membrane potential of the snail RPa1 neuron model. (a) 97% $g_{Na(TTX)}$, (b) 98% $g_{Na(TTX)}$, (c) 99% $g_{Na(TTX)}$, (d) 100% $g_{Na(TTX)}$, (e) 101% $g_{Na(TTX)}$, (f) 102% $g_{Na(TTX)}$, and (g) 103% $g_{Na(TTX)}$. Across all panels, the horizontal axis indicates time (s) and the vertical axis indicates the membrane potential (mV). a1, b1, c1, d1, d2, d3, d4, d5, e1, f1, and g1 denote bursting patterns observed under varying $g_{Na(TTX)}$ conditions.

4 Discussion and Conclusion

In a previous study of the vibrissa motoneuron model, a relationship between repetitive spiking states of the neuron model and the spike-generating sodium conductance has been demonstrated [3]. However, a bursting state cannot be studied in this study. The present study demonstrates the relationship between the spike-generating sodium conductance and the bursting states of the neuron model. A previous study has proposed two possible mechanisms that could underlie the chaotic bursting state: an intrinsic mechanism and a synaptic mechanism [1]. Evidence for the latter mechanism has been reported in a previous study stating that the appearance of a regular bursting state or a chaotic bursting state depends on the synaptic conductance [4]. However, the influence of intrinsic mechanism on the bursting states has not been studied previously. The present study demonstrates the capability of an intrinsic mechanism in modulating the regularity of bursting and reveals that the appearance of a regular bursting state or a chaotic bursting state depends on the spike-generating sodium conductance. In conclusion, the present study elucidates the following relationship between the spike-generating sodium conductance and the bursting state of the snail RPa1 neuron model: 1) the value of the spike-generating sodium conductance determines the regularity of bursting, 2) the range of the spike-generating sodium conductance that induces a chaotic bursting state is smaller than that which induces a regular bursting state, and 3) increasing or decreasing the spike-generating sodium conductance relative to the default value (400 μS) achieves a transition from the chaotic bursting state to the regular bursting state.

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