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Minimal Quantities and Measurable Variant of Gravity in the General Form

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Abstract

In this paper gravity is studied within the scope of the **measurability** notion introduced by the author in his previous works. The **measurable** variant of General Relativity (GR) is constructed and it is shown that this variant represents its *deformation*. In the general form it is demonstrated that all the basic ingredients of GR have their **measurable** analogs, the way to derive every term in a **measurable** variant of the Einstein equations is presented. Passage of the **measurable** analog of GR to the ultraviolet (Planck) region is considered, showing that it is quite natural from the viewpoint of the methods and approaches developed in this work. The results obtained are discussed; a further course of studies by the author is indicated.

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1 Introduction

This paper is a continuation of the author's works [1]–[9]. The target of the indicated works is to construct a correct quantum theory and gravity in terms of the variations (increments) dependent on the existent energies.
It is clear that such a theory should not involve the infinitesimal space-time

variations $dt, dx_i, i = 1, ..., 3$ and, in general, any abstract small quantities $\delta t, \delta x$.
The main instrument specified in the above mentioned articles was the **measurability** concept introduced proceeding from the existence in a theory of the minimal (fundamental) length ℓ.
Within the framework of the concept, the theory becomes discrete but at low energy levels E, distant from the Plank energies $E \ll E_P$, it becomes very close to the initial theory in the continuous space-time paradigm.
This paper presents a study of gravity within the scope of the **measurability** notion. A **measurable** variant of General Relativity (GR) is constructed and it is shown that this variant presents its *deformation* according to the definition given in the well-known paper [10]. In the general form it is demonstrated that all the basic ingredients of GR have their **measurable** analogs; the way to derive every term in the **measurable** variant of the Einstein equations is presented. Passage of the **measurable** analog of GR to the ultraviolet (Planks) region is considered, showing that it is quite natural from the viewpoint of the notions and approaches developed in the work.
This paper is based on the findings of the work [3].
The structure of this paper is as follows. Section 2 presents the information necessary for extension of the findings earlier obtained in [1]–[9]. Then in Section 3 the principal mathematical apparatus required to study **measurability** in gravity is considered and, specifically, the **measurable** variant (approximation) of the coordinate transformations. Section 4 presents the main results. Finally, in Section 5 the author considers his further course of studies.

2 Measurability Concept

2.1 Primary and Generalized Measurability in General Case

We begin with a particular minimal (universal) unit for measurement of the length ℓ corresponding to some maximal energy $E_\ell = \frac{\hbar c}{\ell}$ and a universal unit for measurement of time $\tau = \ell/c$. Without loss of generality, we can consider ℓ and τ at Planks level, i.e. $\ell = \kappa l_p, \tau = \kappa t_p$, where the numerical constant κ is on the order of 1. Consequently, we have $E_\ell \propto E_p$ with the corresponding proportionality factor.
Note that ℓ and τ are referred to as "minimal" and "universal" units of measurement because in our case this is actually true.
Now consider in the space of momenta $\mathbf{P}$ defined by the conditions

$$\mathbf{p} = \{p_{x_i}\}, i = 1, .., 3; |p_{x_i}| \neq 0, \tag{1}$$

Then we can easily calculate the numerical coefficients N_i as follows:

$$N_i = \frac{\hbar}{p_{x_i}\ell}, or \qquad (2)$$
$$p_{x_i} \doteq p_{N_i} = \frac{\hbar}{N_i\ell}$$

Here p_{x_i} is the momentum corresponding to the coordinate x_i.

Definition 1. Primary Measurability
1.1 The momenta **p** given by the formula (1) are called the **primary measurable** momenta when all N_i from equation (2) are integer numbers.
1.2. Any variation in Δx_i for the coordinates x_i and Δt of the time t is considered **primarily measurable** if

$$\Delta x_i = N_{\Delta x_i}\ell, \Delta t = N_{\Delta t}\tau, \qquad (3)$$

where $N_{\Delta x_i} \neq 0$ and $N_{\Delta t} \neq 0$ are integer numbers.
1.3 Let us define any physical quantity as **primary or elementary measurable** when its value is consistent with points **1.1** and **1.2** of this Definition.

We consider two different cases.

A) **Low Energies,** $E \ll E_\ell$.
In **P** we consider the domain $\mathbf{P}_{LE} \subset \mathbf{P}$ (LE is abbreviation of "Low Energies") defined by the conditions

$$\mathbf{p} = \{p_{x_i}\}, i = 1,..,3; P_\ell \gg |p_{x_i}| \neq 0, \qquad (4)$$

where $P_\ell = E_\ell/c$–maximal momentum.
In this case the formula of (2) takes the form

$$N_i = \frac{\hbar}{p_{x_i}\ell}, or \qquad (5)$$
$$p_{x_i} \doteq p_{N_i} = \frac{\hbar}{N_i\ell}$$
$$|N_i| \gg 1,$$

where the last row of the formula (5) is given by the requirement (4).
As the energies $E \ll E_\ell$ are low, i.e. ($|N_i| \gg 1$), **primary measurable** momenta are sufficient to specify the whole domain of the momenta to a high accuracy $\mathbf{P}_{LE}$.
It is clear that

$$[N_i] \leq N_i \leq [N_i] + 1, \qquad (6)$$

where $[\aleph]$ defines the integer part of $\aleph$. Then $|N_i|^{-1}$ falls within the interval with the finite points $|[N_i]|^{-1}$ and $|[N_i]+1|^{-1}$ (which of the numbers is greater or smaller, depends on a sign of N_i). In any case we have $|N_i^{-1}-[N_i^{-1}]| \leq |([N_i]+1)^{-1}-[N_i]^{-1}| = |([N_i]+1)[N_i]|^{-1}$.
Thus, the difference between p_{N_i} and $p_{[N_i]}$ is negligibly small. Therefore, the **primary measurable** momenta p_{N_i}, ($|N_i| \gg 1$) are sufficient to specify the whole domain of the momenta to a high accuracy $\mathbf{P}_{LE}$.
This means that in the indicated domain a discrete set of **primary measurable** momenta $p_{N_i}, (i=1,...,3)$ from formula (5) varies almost continuously, practically covering the whole domain.
That is why further $\mathbf{P}_{LE}$ is associated with the domain of **primary measurable** momenta, satisfying the conditions of the formula (4) (or (5)).
Then boundaries of the domain $\mathbf{P}_{LE}$ are determined for each coordinate by the condition

$$\mathbf{N}^* \geq |N_i| \geq \mathbf{N}_* \gg 1,$$

where high natural numbers $\mathbf{N}^*, \mathbf{N}_*$ are determined by the problem at hand.
The choice of the number $\mathbf{N}^*$ is of particular importance. If $\mathbf{N}^* < \infty$, then it is clear that the studied momenta fall within the domain $\mathbf{P}_{LE}$. Assuming the condition $\mathbf{N}^* = \infty$, to $\mathbf{P}_{LE}$ for every coordinate x_i we should add "improper" (or "singular") point $p_{x_i}=0$ (these cases are called **degenerate**).
In any case, for each coordinate x_i, the boundaries of $\mathbf{P}_{LE}$ are of the form:

$$p_{\mathbf{N}^*} \leq |p_{N_i}| \leq p_{\mathbf{N}_*} \qquad (7)$$

For definiteness, we denote $\mathbf{P}_{LE}$, having the boundaries specified by the formula (7), in terms of $\mathbf{P}_{LE}[\mathbf{N}^*, \mathbf{N}_*]$.
It is obvious that in this formalism **small** increments for any component p_{N_i} of the momentum $\mathbf{p} \in \mathbf{P}_{LE}$ are values of the momentum $p_{N_i'}$, so that $|N_i'| > |N_i|$. And then, incrementing $|N_i'|$, we can obtain **arbitrary small** increments for the momenta $\mathbf{p} \in \mathbf{P}_{LE}$.

A) **High Energies,** $E \approx E_\ell$.
In this case formula (2) takes the form

$$N_i = \frac{\hbar}{p_{x_i}\ell}, or \qquad (8)$$
$$p_{x_i} \doteq p_{N_i} = \frac{\hbar}{N_i\ell}$$
$$|N_i| \approx 1.$$

And the discrete set p_{N_i} is introduced as **primary measurable** momenta.

It is clear that **primarily measurable** variations in Δx_i for the coordinates x_i and in Δt for the time t given in point **1.2** of **Definition 1** could hardly play a role of small spatial and temporal variations.
However, space-time quantities

$$\frac{\tau}{N_t} = p_{N_t c}\frac{\ell^2}{c\hbar}$$
$$\frac{\ell}{N_i} = p_{N_i}\frac{\ell^2}{\hbar}, 1 = 1, ..., 3 \tag{9}$$

for $|N_i| \gg 1, |N_t| \gg 1$ are small and they may be arbitrary small for sufficiently high values of $|N_i|, |N_t|$:

$$\frac{\tau}{N_t} = p_{N_t c}\frac{\ell^2}{c\hbar} \stackrel{N_t \to \infty}{\longrightarrow} 0,$$
$$\frac{\ell}{N_i} = p_{N_i}\frac{\ell^2}{\hbar} \stackrel{N_i \to \infty}{\longrightarrow} 0, 1 = 1, ..., 3, \tag{10}$$

Here $p_{N_i}, p_{N_t c}$–corresponding **primarily measurable** momenta.
Of course, due to point 1.2 of Definition 1, the space and time quantities $\tau/N_t, \ell/N_i$ are not **primary measurable** despite the fact that they, to within a constant factor, are equal to **primarily measurable** momenta.
Therefore, it seems expedient to introduce the following definition:

Definition 2. Generalized Measurability
We define any physical quantity at all energy scales $E \leq E_\ell$ as **generalized measurable** or, for simplicity, **measurable** if any of its values may be obtained in terms of **Primarily Measurable Quantities** specified by points **1.1–1.3** of **Definition 1**.

It is evident that any **primarily measurable quantity (PMQ)** is **measurable**. Generally speaking, the contrary is not correct, as indicated by formula (9).
The "improper" points associated with $|N_i| = \infty$ and $|N_t| = \infty$ may be introduced into **Definition 2.**, respectively, as in the case of low energies.
It has been shown that the **Primary Measurable Momenta** nearly cover the whole momenta domain $\mathbf{P}_{LE}$ at low energies $E \ll E_\ell$. However, this is no longer the case at **all the energy scales** $E \leq E_\ell$.
Therefore, the main target of the author is to construct a quantum theory at all energy scales $E \leq E_\ell$ in terms of **measurable** quantities.
In this theory the values of the physical quantity $\mathcal{G}$ may be represented by the numerical function $\mathcal{F}$ in the following way:

$$\mathcal{G} = \mathcal{F}(N_i, N_t, \ell) = \mathcal{F}(N_i, N_t, G, \hbar, c, \kappa), \tag{11}$$

where N_i, N_t–integers from the formulae (2),(9) and $G, \hbar, c$ are fundamental constants. The last equality in (11) is determined by the fact that $\ell = \kappa l_p$ and $l_p = \sqrt{G\hbar/c^3}$.
If $N_i \neq 0, N_t \neq 0$ (**nondegenerate** case), then it is clear that (11) can be rewritten as follows:

$$\mathcal{G} = \mathcal{F}(N_i, N_t, \ell) = \widetilde{\mathcal{F}}((N_i)^{-1}, (N_t)^{-1}, \ell) \tag{12}$$

Then at low energies $E \ll E_\ell$, i.e. at $|N_i| \gg 1, |N_t| \gg 1$, the function $\widetilde{\mathcal{F}}$ is a function of the variables changing practically continuously, though these variables cover a discrete set of values. Naturally, it is assumed that $\widetilde{\mathcal{F}}$ varies smoothly (i.e. practically continuously). As a result, we get a model, discrete in nature, capable to reproduce the well-known theory in continuous space-time to a high accuracy, as it has been stated above.
Obviously, at low energies $E \ll E_\ell$ the formula (12) is as follows:

$$\mathcal{G} = \mathcal{F}(N_i, N_t, \ell) = \widetilde{\mathcal{F}}((N_i)^{-1}, (N_t)^{-1}, \ell) = \\ = \widetilde{\mathcal{F}}_p(p_{N_i}, p_{N_{tc}}, \ell), \tag{13}$$

where $p_{N_i}, p_{N_{tc}}$ are **primary measurable** momenta.

2.2 Generalized Measurability and Generalized Uncertainty Principle

Basic results of this Subsection are contained in [3] and [9].
Further it is convenient to use the deformation parameter α_a. This parameter has been introduced earlier in the papers [15],[16],[17]–[20] as a *deformation parameter* (in terms of paper [10]) on going from the canonical quantum mechanics to the quantum mechanics at Planck's scales (Early Universe) that is considered to be the quantum mechanics with the minimal length (QMML):

$$\alpha_a = \ell^2/a^2, \tag{14}$$

where a is the primarily measuring scale. It is easily seen that the parameter α_a from Equation (14) is discrete as it is nothing else but

$$\alpha_a \doteq \alpha_{N_a} = \ell^2/a^2 = \frac{\ell^2}{N_a^2\ell^2} = \frac{1}{N_a^2}. \tag{15}$$

for primarily measurable $a = N_a\ell$.
It should be noted that Heisenberg's Uncertainty Principle (HUP) [12] is fair at low energies $E \ll E_P$. However it was shown that at the Planck scale a high-energy term must appear:

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' l_p^2 \frac{\triangle p}{\hbar} \tag{16}$$

where l_p is the Planck length $l_p^2 = G\hbar/c^3 \simeq 1,6\ 10^{-35}m$ and α' is a constant. In [21] this term is derived from the string theory, in [22] it follows from the simple estimates of Newtonian gravity and quantum mechanics, in [23] it comes from the black hole physics, other methods can also be used [25],[24],[30]. Relation (16) is quadratic in Δp

$$\alpha' l_p^2\,(\Delta p)^2 - \hbar\,\Delta x \Delta p + \hbar^2 \leq 0 \tag{17}$$

and therefore leads to the minimal length

$$\Delta x_{min} = 2\sqrt{\alpha'} l_p \doteq \ell \tag{18}$$

Inequality (16) is called the Generalized Uncertainty Principle (GUP) in Quantum Theory.
Let us show that the **generalized-measurable** quantities are appeared from the **Generalized Uncertainty Principle (GUP)** [21]–[32] (formula (16)) that naturally leads to the minimal length ℓ (18).
Really solving inequality (16), in the case of equality we obtain the apparent formula

$$\Delta p_{\pm} = \frac{(\Delta x \pm \sqrt{(\Delta x)^2 - 4\alpha' l_p^2})\hbar}{2\alpha' l_p^2}. \tag{19}$$

Next, into this formula we substitute the right-hand part of formula (3) for **primarily measurable** $\Delta x = N_{\Delta x}\ell$. Considering (18), we can derive the following:

$$\Delta p_{\pm} = \frac{(N_{\Delta x} \pm \sqrt{(N_{\Delta x})^2 - 1})\hbar\ell}{\frac{1}{2}\ell^2} = \\ = \frac{2(N_{\Delta x} \pm \sqrt{(N_{\Delta x})^2 - 1})\hbar}{\ell}. \tag{20}$$

But it is evident that at low energies $E \ll E_\ell; N_{\Delta x} \gg 1$ the plus sign in the nominator (20) leads to the contradiction as it results in very high (much greater than the Plancks) values of Δp. Because of this, it is necessary to select the minus sign in the numerator (20). Then, multiplying the left and right sides of (20) by the same number $N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1}$, we get

$$\Delta p = \frac{2\hbar}{(N_{\Delta x} + \sqrt{N_{\Delta x}^2 - 1})\ell}. \tag{21}$$

Δp from formula (21) is the **generalized-measurable** quantity in the sense of **Definition 2.** However, it is clear that at low energies $E \ll E_\ell$, i.e. for $N_{\Delta x} \gg 1$, we have $\sqrt{N_{\Delta x}^2 - 1} \approx N_{\Delta x}$. Moreover, we have

$$\lim_{N_{\Delta x}\rightarrow\infty} \sqrt{N_{\Delta x}^2 - 1} = N_{\Delta x}. \tag{22}$$

Therefore, in this case (21) may be written as follows:

$$\Delta p \doteq \Delta p(N_{\Delta x}, HUP) = \frac{\hbar}{1/2(N_{\Delta x}+\sqrt{N_{\Delta x}^2-1})\ell} \approx \frac{\hbar}{N_{\Delta x}\ell} = \frac{\hbar}{\Delta x}; N_{\Delta x} \gg 1, \quad (23)$$

in complete conformity with HUP. Besides, $\Delta p \doteq \Delta p(N_{\Delta x}, HUP)$, to a high accuracy, is a **primarily measurable** quantity in the sense of **Definition 1**. And vice versa it is obvious that at high energies $E \approx E_{\ell}$, i.e. for $N_{\Delta x} \approx 1$, there is no way to transform formula (21) and we can write

$$\Delta p \doteq \Delta p(N_{\Delta x}, GUP) = \frac{\hbar}{1/2(N_{\Delta x}+\sqrt{N_{\Delta x}^2-1})\ell}; N_{\Delta x} \approx 1. \quad (24)$$

At the same time, $\Delta p \doteq \Delta p(N_{\Delta x}, GUP)$ is a **Generalized Measurable** quantity in the sense of **Definition 2**.
Thus, we have

$$GUP \rightarrow HUP \quad (25)$$

for

$$(N_{\Delta x} \approx 1) \rightarrow (N_{\Delta x} \gg 1). \quad (26)$$

Also, we have

$$\Delta p(N_{\Delta x}, GUP) \rightarrow \Delta p(N_{\Delta x}, HUP), \quad (27)$$

where $\Delta p(N_{\Delta x}, GUP)$ is taken from formula (24), whereas $\Delta p(N_{\Delta x}, HUP)$ from formula (23).

Comment 2.*
From the above formulae it follows that, within GUP, the **primarily measurable** *variations (quantities) are derived to a high accuracy from the* **generalized-measurable** *variations (quantities) only in the low-energy limit $E \ll E_P$*

Next, within the scope of GUP, we can correct a value of the parameter α_a from formula (15) substituting a for Δx in the expression $1/2(N_{\Delta x}+\sqrt{N_{\Delta x}^2-1})\ell$. Then at low energies $E \ll E_{\ell}$ we have the **primarily measurable** quantity $\alpha_a(HUP)$

$$\alpha_a \doteq \alpha_a(HUP) = \frac{1}{[1/2(N_a+\sqrt{N_a^2-1})]^2} \approx \frac{1}{N_a^2}; N_a \gg 1, \quad (28)$$

that corresponds, to a high accuracy, to the value from formula (15).
Accordingly, at high energies we have $E \approx E_{\ell}$

$$\alpha_a \doteq \alpha_a(GUP) = \frac{1}{[1/2(N_a+\sqrt{N_a^2-1})]^2}; N_a \approx 1. \quad (29)$$

When going from high energies $E \approx E_\ell$ to low energies $E \ll E_\ell$, we can write

$$\alpha_a(GUP) \stackrel{(N_a\approx 1)\rightarrow(N_a\gg 1)}{\longrightarrow} \alpha_a(HUP) \tag{30}$$

in complete conformity to *Comment 2**.
Remark 2.1 What is the main difference between **Primarily Measurable Quantities (PMQ)** and **Generalized Measurable Quantities (GMQ)**? **PMQ** defines variables which may be obtained as a result of an immediate experiment. **GMQ** defines the variables which may be *calculated* based on **PMQ**, i.e. based on the data obtained in previous clause.

Remark 2.2. It is readily seen that a minimal value of $N_a = 1$ is *unattainable* because in formula (24) we can obtain a value of the length l that is below the minimum $l < \ell$ for the momenta and energies above the maximal ones, and that is impossible. Thus, we always have $N_a \geq 2$. This fact was indicated in [15],[16], however, based on the other approach.

Let us for three space coordinates $x_i; i = 1, 2, 3$ we introduce the following notation:

$$\begin{gathered}\Delta(x_i) \doteq \widetilde{\Delta}[\alpha_{N_{\Delta x_i}}] = \alpha_{N_{\Delta x_i}\ell}(N_{\Delta x_i}\ell) = \ell/N_{\Delta x_i};\\ \frac{\Delta_{N_{\Delta x_i}}[F(x_i)]}{\Delta(x_i)} \equiv \frac{F(x_i+\Delta(x_i))-F(x_i)}{\Delta(x_i)},\end{gathered} \tag{31}$$

where $F(x_i)$ is **"measurable"** function, i.e function represented in terms of **measurable** quantities.
Then function $\Delta_{N_{\Delta x_i}}[F(x_i)]/\Delta(x_i)$ is **"measurable"** function too.
It's evident that

$$\lim_{|N_{\Delta x_i}|\rightarrow\infty} \frac{\Delta_{N_{\Delta x_i}}[F(x_i)]}{\Delta(x_i)} = \lim_{\Delta(x_i)\rightarrow 0} \frac{\Delta_{N_{\Delta x_i}}[F(x_i)]}{\Delta(x_i)} = \frac{\partial F}{\partial x_i}. \tag{32}$$

Thus, we can define a **measurable** analog of a vectorial gradient ∇

$$\nabla_{\mathbf{N_{\Delta x_i}}} \equiv \{\frac{\Delta_{N_{\Delta x_i}}}{\Delta(x_i)}\} \tag{33}$$

and a **measurable** analog of the Laplace operator

$$\boldsymbol{\Delta}_{(N_{\Delta x_i})} \equiv \nabla_{\mathbf{N_{\Delta x_i}}}\nabla_{\mathbf{N_{\Delta x_i}}} \equiv \sum_i \frac{\Delta^2_{N_{\Delta x_i}}}{\Delta(x_i)^2} \tag{34}$$

Respectively, for time $x_0 = t$ we have:

$$\begin{gathered}\Delta(t) \doteq \widetilde{\Delta}[\alpha_{N_{\Delta t}}] = \alpha_{N_{\Delta t}\tau}(N_{\Delta t}\tau) = \tau/N_{\Delta t};\\ \frac{\Delta_{N_{\Delta t}}[F(t)]}{\Delta(t)} \equiv \frac{F(t+\Delta(t))-F(t)}{\Delta(t)},\end{gathered} \tag{35}$$

then

$$\lim_{|N_{\Delta t}|\to\infty}\frac{\Delta_{N_{\Delta t}}[F(t)]}{\Delta(t)}=\lim_{\Delta(t)\to 0}\frac{\Delta_{N_{\Delta t}}[F(t)]}{\Delta(t)}=\frac{dF}{dt}. \qquad (36)$$

We shall designate for momenta $p_i; i=1,2,3$

$$\Delta p_i=\frac{\hbar}{N_{\Delta x_i}\ell};$$
$$\frac{\Delta_{p_i}F(p_i)}{\Delta p_i}\equiv\frac{F(p_i+\Delta p_i)-F(p_i)}{\Delta p_i}=\frac{F(p_i+\frac{\hbar}{N_{\Delta x_i}\ell})-F(p_i)}{\frac{\hbar}{N_{\Delta x_i}\ell}}. \qquad (37)$$

¿From where similarly (32) we get

$$\lim_{|N_{\Delta x_i}|\to\infty}\frac{F(p_i+\Delta p_i)-F(p_i)}{\Delta p_i}=\lim_{|N_{\Delta x_i}|\to\infty}\frac{F(p_i+\frac{\hbar}{N_{\Delta x_i}\ell})-F(p_i)}{\frac{\hbar}{N_{\Delta x_i}\ell}}=$$
$$=\lim_{\Delta p_i\to 0}\frac{F(p_i+\Delta p_i)-F(p_i)}{\Delta p_i}=\frac{\partial F}{\partial p_i}. \qquad (38)$$

Therefore, in low energies $E\ll E_\ell$, i.e. at $|N_{\Delta x_i}|\gg 1; |N_{\Delta t}|\gg 1, i=1,...,3$ in passages to the limit (32),(36),(38) it's possible to obtain from **"measurable"** functions partial derivatives like in case of continuous space-time. That is, the partial derivatives of from **"measurable"** functions can be considered as **"measurable"** functions with any given precision.
In this case the infinitesimal space-time variations $dt, dx_i, i=1,...,3$ are appearing from formula (10) in the limit from **measurable** quantities.

3 Measurability. The Main Instruments

The calculations from Section 2 (formula (6) and the fragment directly following this formula) indicate that, to a high accuracy, the momentum (p_{N_μ}), where N_μ is any (but not only integer) number having the property $|N_\mu|\gg 1$, may be thought to be equal to the momentum $(p_{[N_\mu]})$.
Therefore, it is assumed that we are in the domain of low energies $E\ll E_\ell$, and we start from the **primary measurable** momenta $(p_{N_i}, p_{N_{tc}})$ in the left-hand side of formula (10) to have

$$|N_\mu|\gg 1 \qquad (39)$$

for all the elements of the set $\{N_\mu\}\doteq\{N_{tc},N_i\}, i=1,2,3; N_{tc}=N_t=N_0$.
Further we equivalently use both the designation N_μ and $N_{\Delta x_\mu}$. In the latter case it is assumed that the selected point has the coordinates x_μ.

To construct the **measurable** variant of a theory, we formulate the following principle.

Principle of Correspondence to Continuous Theory.

The infinitesimal space-time quantities $dx_\mu; \mu = 0, ..., 3$ and also infinitesimal values of the momenta $dp_i, i = 1, 2, 3$ and of the energies dE form the basic instruments (construction materials) for any theory in continuous space-time. Because of this, to construct the **measurable** variant of such a theory, we should find the adequate substitutes for these quantities.
It is obvious that in the first case the substitute is represented by the quantities ℓ/N_μ, where $|N_\mu|$ – no matter how large (but finite!) integer, whereas in the second case by $p_{N_i} = \frac{\hbar}{N_i \ell}; i = 1, 2, 3; \mathcal{E}_{N_0} = \frac{c\hbar}{N_0 \ell}$, where N_μ – integer with the above properties.

Remark 3.1. In this way in the proposed approach all the **primary measurable** momenta at low energies $E \ll E_\ell$, $p_{N_\mu}, |N_\mu| \gg 1$ are small quantities, the **primary measurable** momenta p_{N_μ} with no matter how large $|N_\mu| \gg 1$ being analogous to *infinitesimal* quantities of a continuous theory.
It is clear that in this case we consider the whole set of the momenta (formula (4)), not imposing the restrictions from formula (7). These restrictions may naturally appear when solving a particular problem for the processes in the preset bounds of the energy scales.
It should be noted that, as all the experimentally involved energies E are low, they meet the condition $E \ll E_\ell$, specifically for LHC the maximal energies are $\approx 10 TeV = 10^4 GeV$, that is by 15 orders of magnitude lower than the Planck energy $\approx 10^{19} GeV$. But since the energy E_ℓ is on the order of the Planck energy $E_\ell \propto E_p$, in this case all the numbers N_i for the corresponding momenta will meet the condition $min|N_i| \approx 10^{15}$,i.e., the formula of(5). So, all the experimentally involved momenta are considered to be **primary measurable** momenta at low energies $E \ll E_\ell$.

Let us consider any coordinate transformation $x^\mu = x^\mu(\bar{x}^\nu)$ of the space–time coordinates in continuous spacetime. Then we have

$$dx^\mu = \frac{\partial x^\mu}{\partial \bar{x}^\nu} d\bar{x}^\nu. \tag{40}$$

As mentioned at the beginning of this section, in terms of **measurable** quantities we have the substitution

$$dx^\mu \mapsto \frac{\ell}{N_{\Delta x_\mu}}; d\bar{x}^\nu \mapsto \frac{\ell}{\bar{N}_{\Delta \bar{x}_\nu}}, \tag{41}$$

where $N_{\Delta x_\mu}, \bar{N}_{\Delta \bar{x}_\nu}$ – integers ($|N_{\Delta x_\mu}| \gg 1, |\bar{N}_{\Delta \bar{x}_\nu}| \gg 1$) sufficiently high in absolute value, and hence in the **measurable** case (40) is replaced by

$$\frac{\ell}{N_{\Delta x_\mu}} = \Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})\,\frac{\ell}{\bar{N}_{\Delta \bar{x}_\nu}}. \qquad (42)$$

Equivalently, in terms of the **primary measurable** momenta we have

$$p_{N_{\Delta x_\mu}} = \Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})\,p_{\bar{N}_{\Delta \bar{x}_\nu}}, \qquad (43)$$

where $\Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu}) \doteq \Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, p_{N_{\Delta x_\mu}}, p_{\bar{N}_{\Delta \bar{x}_\nu}})$ – corresponding matrix represented in terms of **measurable** quantities.
It is clear that, in accordance with formula (10), in passage to the limit we get

$$\lim_{|N_{\Delta x_\mu}|\to\infty} \frac{\ell}{N_{\Delta x_\mu}} = dx^\mu =$$
$$= \lim_{|\bar{N}_{\Delta \bar{x}_\nu}|\to\infty} \Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})\,\frac{\ell}{\bar{N}_{\Delta \bar{x}_\nu}} = \frac{\partial \bar{x}^\mu}{\partial x^\nu}\,dx^\nu. \qquad (44)$$

Equivalently, passage to the limit (44) may be written in terms of the **primary measurable** momenta $p_{N_{\Delta x_\mu}}, p_{\bar{N}_{\Delta \bar{x}_\nu}}$ multiplied by the constant $\ell^2/\hbar$.
How we understand formulae (41)–(44)?
The initial (continuous) coordinate transformations $x^\mu = x^\mu(\bar{x}^\nu)$ gives the matrix $\frac{\partial x^\mu}{\partial \bar{x}^\nu}$. Then, for the integers sufficiently high in absolute value $\bar{N}_{\Delta \bar{x}_\nu}, |\bar{N}_{\Delta \bar{x}_\nu}| \gg 1$, we can derive

$$\frac{\ell}{N_{\Delta x_\mu}} = \frac{\partial x^\mu}{\partial \bar{x}^\nu}\,\frac{\ell}{\bar{N}_{\Delta \bar{x}_\nu}}, \qquad (45)$$

where $|N_{\Delta x_\mu}| \gg 1$ but the numbers for $N_{\Delta x_\mu}$ are not necessarily integer. Still, as noted above, the difference between $\ell/N_{\Delta x_\mu}$ and $\ell/[N_{\Delta x_\mu}]$ (and hence between $p_{N_{\Delta x_\mu}}$ and $p_{[N_{\Delta x_\mu}]}$) is negligible.
Then substitution of $[N_{\Delta x_\mu}]$ for $N_{\Delta x_\mu}$ in the left-hand side of (45) leads to replacement of the initial matrix $\frac{\partial x^\mu}{\partial \bar{x}^\nu}$ by the matrix $\Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})$ represented in terms of **measurable** quantities and, finally, to the formula (42). Clearly, for sufficiently high $|N_{\Delta x_\mu}|, |\bar{N}_{\Delta \bar{x}_\nu}|$, the matrix $\Delta_{\mu\nu}(x^\mu, \bar{x}^\nu, 1/N_{\Delta x_\mu}, 1/\bar{N}_{\Delta \bar{x}_\nu})$ may be selected no matter how close to $\frac{\partial x^\mu}{\partial \bar{x}^\nu}$.
Similarly, in the **measurable** format we can get the formula

$$d\bar{x}^\mu = \frac{\partial \bar{x}^\mu}{\partial x^\nu}\,dx^\nu. \qquad (46)$$

Thus, any coordinate transformation may be represented, to however high accuracy, by the **measurable** transformation (i.e., written in terms of **measurable** quantities), where the principal components are the **measurable** quantities $\ell/N_{\Delta x_\mu}$ or the **primary measurable** momenta $p_{N_{\Delta x_\mu}}$.

4 Measurability in Gravity in the General Form

According to the results from the previous section, the **measurable** variant of gravity should be formulated in terms of the **measurable** space-time quantities $\ell/N_{\Delta x_\mu}$ or **primary measurable** momenta $p_{N_{\Delta x_\mu}}$.
Let us consider the case of the random metric $g_{\mu\nu} = g_{\mu\nu}(x)$ [33],[34], where $x \in R^4$ is some point of the four-dimensional space R^4 defined in **measurable** terms. Now, any such point $x \doteq \{x_\chi\} \in R^4$ and any set of integer numbers $\{N_{x_\chi}\}$ dependent on the point $\{x_\chi\}$ with the property $|N_{x_\chi}| \gg 1$ may be correlated to the bundle with the base R^4 as follows:

$$B_{N_{x_\chi}} \doteq \{x_\chi + \frac{\ell}{N_{x_\chi}}\} \mapsto \{x_\chi\}. \tag{47}$$

It is clear that $\lim\limits_{|N_{x_\chi}|\to\infty} B_{N_{x_\chi}} = R^4$.
Then as a *canonically measurable prototype* of the infinitesimal space-time interval square [33],[34]

$$ds^2(x) = g_{\mu\nu}(x)dx^\mu dx^\nu \tag{48}$$

we take the expression

$$\Delta s^2_{\{N_{x_\chi}\}}(x) \doteq g_{\mu\nu}(x, \{N_{x_\chi}\})\frac{\ell^2}{N_{x_\mu}N_{x_\nu}}. \tag{49}$$

Here $g_{\mu\nu}(x, \{N_{x_\chi}\})$ – metric $g_{\mu\nu}(x)$ from formula (48) with the property that minimal **measurable** variation of metric $g_{\mu\nu}(x)$ in point x has form

$$\Delta g_{\mu\nu}(x, \{N_{x_\chi}\})_\chi = g_{\mu\nu}(x + \ell/N_{x_\chi}, \{N_{x_\chi}\}) - g_{\mu\nu}(x, \{N_{x_\chi}\}), \tag{50}$$

Let us denote by $\Delta_\chi g_{\mu\nu}(x, \{N_{x_\chi}\})$ quantity

$$\Delta_\chi g_{\mu\nu}(x, \{N_{x_\chi}\}) = \frac{\Delta g_{\mu\nu}(x, \{N_{x_\chi}\})_\chi}{\ell/N_{x_\chi}}. \tag{51}$$

It is obvious that in the case under study the quantity $\Delta g_{\mu\nu}(x, \{N_{x_\chi}\})_\chi$ is a **measurable** analog for the infinitesimal increment $dg_{\mu\nu}(x)$ of the χ-th component $(dg_{\mu\nu}(x))_\chi$ in a continuous theory, whereas the quantity $\Delta_\chi g_{\mu\nu}(x, \{N_{x_\chi}\})$ is a **measurable** analog of the partial derivative $\partial_\chi g_{\mu\nu}(x)$.
In this manner we obtain the (47)-formula induced bundle over the metric manifold $g_{\mu\nu}(x)$:

$$B_{g,N_{x_\chi}} \doteq g_{\mu\nu}(x, \{N_{x_\chi}\}\} \mapsto g_{\mu\nu}(x). \tag{52}$$

Referring to formula (10), we can see that (49 may be written in terms of the **primary measurable** momenta $(p_{N_i}, p_{N_t}) \doteq p_{N_\mu}$ as follows:

$$\Delta s^2_{N_{x_\mu}}(x) = \frac{\ell^4}{\hbar^2} g_{\mu\nu}(x, \{N_{x_\chi}\}) p_{N_{x_\mu}} p_{N_{x_\nu}} = g_{\mu\nu}(x, \{N_{x_\chi}\}) \ell^2 (\alpha_{N_{x_\mu}} \alpha_{N_{x_\nu}})^{1/2}, \quad (53)$$

where $\alpha_{N_{x_\mu}} \alpha_{N_{x_\nu}}$ in the last equality is taken from formula (15) of Section 2. Considering that $\ell \propto l_P$ (i.e., $\ell = \kappa l_P$), where $\kappa = const$ is on the order of 1, in the general case (53), to within the constant $\ell^4/\hbar^2$, we have

$$\Delta s^2_{N_{x_\mu}}(x) = g_{\mu\nu}(x, \{N_{x_\chi}\}) p_{N_{x_\mu}} p_{N_{x_\nu}}. \quad (54)$$

As follows from the previous formulae, the **measurable** variant of General Relativity should be defined in the bundle $B_{g,N_{x_\chi}}$.
As the base operators used to construct General Relativity in a continuous theory have the corresponding measurable analogs, the base quantities of General Relativity also have their **measurable** analogs.
In particular, the Christoffel symbols [33],[34]

$$\Gamma^\alpha_{\mu\nu}(x) = \frac{1}{2} g^{\alpha\beta}(x) \left(\partial_\nu g_{\beta\mu}(x) + \partial_\mu g_{\nu\beta}(x) - \partial_\beta g_{\mu\nu}(x)\right) \quad (55)$$

have the **measurable** analog

$$\Gamma^\alpha_{\mu\nu}(x, N_{x_\chi}) = \frac{1}{2} g^{\alpha\beta}(x, N_{x_\chi}) \left(\Delta_\nu g_{\beta\mu}(x, N_{x_\chi}) + \Delta_\mu g_{\nu\beta}(x, N_{x_\chi}) - \Delta_\beta g_{\mu\nu}(x, N_{x_\chi})\right). (56)$$

Similarly, for the *Riemann tensor* in a continuous theory we have [33],[34]:

$$R^\mu{}_{\nu\alpha\beta}(x) \equiv \partial_\alpha \Gamma^\mu_{\nu\beta}(x) - \partial_\beta \Gamma^\mu_{\nu\alpha}(x) + \Gamma^\mu_{\gamma\alpha}(x)\, \Gamma^\gamma_{\nu\beta}(x) - \Gamma^\mu_{\gamma\beta}(x)\, \Gamma^\gamma_{\nu\alpha}(x). \quad (57)$$

With the use of formula (56), we can get the corresponding **measurable** analog, i.e. the quantity $R^\mu{}_{\nu\alpha\beta}(x, N_{x_\chi})$.
In a similar way we can obtain the **measurable** variant of *Ricci tensor*, $R_{\mu\nu}(x, N_{x_\chi}) \equiv R^\alpha{}_{\mu\alpha\nu}(x, N_{x_\chi})$, and the **measurable** variant of *Ricci scalar*: $R(x, N_{x_\chi}) \equiv R_{\mu\nu}(x, N_{x_\chi})\, g^{\mu\nu}(x, N_{x_\chi})$.
So, for the *Einstein equations* (EU) in a continuous theory [33],[34]

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} - \frac{1}{2} \Lambda\, g_{\mu\nu} = 8\,\pi\, G\, T_{\mu\nu} \quad (58)$$

we can derive their **measurable** analog, for short denoted as (EUM):

$$R_{\mu\nu}(x, N_{x_\chi}) - \frac{1}{2} R(x, N_{x_\chi})\, g^{\mu\nu}(x, N_{x_\chi}) - \frac{1}{2} \Lambda(x, N_{x_\chi})\, g^{\mu\nu}(x, N_{x_\chi}) = 8\,\pi\, G\, T_{\mu\nu}(x, N_{x_\chi}), (59)$$

where G – Newtons gravitational constant.
For correspondence with a continuous theory, the following passage to the limit must take place for all the points x:

$$\lim_{|N_{x_\chi}|\to\infty} \Lambda(x, N_{x_\chi}) = \Lambda, \tag{60}$$

where the cosmological constant Λ is taken from formula(58).
Moreover, for high $|N_{x_\chi}|$, the quantity $\Lambda(x, N_{x_\chi})$ should be practically independent of the point x, and we have

$$\Lambda(x, N_{x_\chi}) \approx \Lambda(x^{'}, N^{'}_{x^{'}_\chi}) \approx \Lambda, \tag{61}$$

where $x \neq x^{'}$ and $|N_{x_\chi}| \gg 1, |N^{'}_{x^{'}_\chi}| \gg 1$.
Actually, it is clear that formula (60) reflects the fact that (EUM) given by formula (59) represents deformation of the Einstein equations (EU) (58) in the sense of the Definition given in [10] with the deformation parameter N_{x_χ}, and we have

$$\lim_{|N_{x_\chi}|\to\infty} (EUM) = (EU). \tag{62}$$

We denote this deformation as $(EUM)[N_{x_\chi}]$. Since at low energies $E \ll E_P$ and to within the known constants we have $\ell/N_{x_\chi} = p_{N_{x_\chi}} = \alpha^{1/2}_{N_{x_\chi}}$, the following deformations of (EU) are equivalent to

$$(EUM)[N_{x_\chi}] \equiv (EUM)[p_{N_{x_\chi}}] \equiv (EUM)[\alpha^{1/2}_{N_{x_\chi}}]. \tag{63}$$

So, on passage from (EU) to the **measurable** deformation of $(EUM)[N_{x_\chi}]$ (or identically $(EUM)[p_{N_{x_\chi}}], (EUM)[\alpha^{1/2}_{N_{x_\chi}}]$) we can find solutions for the gravitational equations on the metric bundle $B_{g,N_{x_\chi}} \doteq g_{\mu\nu}(x, \{N_{x_\chi}\})$ (formula (52))given by formula (49).
What are the advantages of this approach?

4.1 First, as $|N_{x_\chi}| \gg 1$, from the above formulae it follows that the metric $g_{\mu\nu}(x, \{N_{x_\chi}\})$ belonging to $B_{g,N_{x_\chi}}$ and representing a solution for $(EUM)[N_{x_\chi}]$, to a high accuracy, is a solution for the Einstein equations (EU) in a continuous theory.
Besides, formula (62) shows that at sufficiently high $|N_{x_\chi}|$ this accuracy may be however high. In this way the *Correspondence principle* to a continuous theory takes place.

4.2 We replace the abstract infinitesimal quantities dx_μ, incomparable with

each other, by the specific small quantities ℓ/N_{x_μ} which may be made however small at sufficiently high $|N_{x_\chi}|$, still being ordered and comparable. Because of this, we can compare small values of the squared intervals $\Delta s^2_{\{N_{x_\chi}\}}(x)$ from formula (49).
Possibly, this will help to recover the *causality* property for all solutions in $(EUM)[N_{x_\chi}]$ without pathological solutions in the form of the Closed Time-like Curves (CTC), involved in some models of General Relativity [35]–[38], in $(EUM)[N_{x_\chi}]$. This means that, for the metrics $\tilde{g}_{\mu\nu}(x)$ in General Relativity generating the Closed Time-like Curves we have no **prototype** in the mapping(52).

4.3. Finally, this approach from the start is quantum in character due to the fact that the fundamental length ℓ is proportional to the Planck length $\ell \propto l_P$ and includes the whole three fundamental constants, the Planck constant $\hbar$ as well. Besides, it is naturally dependent on the energy scale: sets of the metrics $g_{\mu\nu}(x, \{N_{x_\chi}\})$ with the lowest value $|N_{x_\chi}|$ correspond to higher energies as they correspond to the momenta $\{p_{N_{x_\chi}}\}$ which are higher in absolute value. This is the case for all the energies E.
However, minimal measurable increments for the energies $E \approx E_P$ are not of the form ℓ/N_{x_μ} because the corresponding momenta $\{p_{N_{x_\chi}}\}$ are no longer **primary measurable**, as indicated by the results in Section 2.
So, in the proposed paradigm the problem of the ultraviolet generalization of the low-energy **measurable** gravity $(EUM)[N_{x_\chi}]$ (formula (59)) is actually reduced to the problem: what becomes with the **primary measurable** momenta $\{p_{N_{x_\chi}}\}, |N_{x_\chi}| \gg 1$ at high Plancks energies.
In a relatively simple case of (GUP) in Section 2 we have the answer. And, using the fact that $(EUM)[N_{x_\chi}] \equiv (EUM)[p_{N_{x_\chi}}]$ (63), based on the results of Section 2, we can construct a correct high-energy passage to the Planck energies $E \approx E_p$

$$(EUM)[p_{N_{x_\chi}}, |N_{x_\chi}| \gg 1] \mapsto (EUM)[p_{N_{x_\chi}}(GUP), |N_{x_\chi}| \approx 1], \qquad (64)$$

where $p_{N_{x_\chi}}(GUP) = \Delta p(\Delta x_\chi), GUP)$ according to formula (24) of Section 2.
In this specific case, we can construct the natural ultraviolet generalization $(EUM)[p_{N_{x_\chi}}, |N_{x_\chi}| \gg 1] \doteq (EUM)[p_{N_{x_\chi}}]$.
The theoretical calculations
$(EUM)[p_{N_{x_\chi}}(GUP), |N_{x_\chi}| \approx 1]$ derived at Plancks energies are obviously *discrete*,**measurable**, and represent a high-energy deformation in the sense of the [10] **measurable** gravitational theory $(EUM)[p_{N_{x_\chi}}, |N_{x_\chi}| \gg 1]$.

5 Conclusion Commentaries

In this paper we develop a constructive approach to the derivation of a **measurable** variant (analog) of gravity as a continuation of the studies presented in [3]. Compared to [3], the earlier obtained results have been used to show, how the "**measurable**" metric should look in the general case and how to find all the terms of the corresponding gravitational equations representing a **measurable** analog of the Einstein equations.

Naturally, such a study necessitates correct definition and elucidation of the physical meaning of **measurable** analogs for all the basic ingredients of General Relativity [33],[34]: tensors, covariant differentiation, parallel transport, geodesics, etc.

Of great interest is to establish the exact form of a **measurable** variant of the Einstein-Hilbert action

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\,\sqrt{|g|}\,(R+\Lambda) + S_M\,(g_{\mu\nu}, \text{matter})\,. \tag{65}$$

Proceeding from the results earlier obtained in [3], [8], passage to the **measurable** variant of S_{EH} (65) necessitates the substitution

$$\int d^4x \mapsto \sum \frac{\ell}{|N_{x_\chi}|}, g(x) \mapsto g(x, N_{x_\chi}), R(x) \mapsto R(x, N_{x_\chi}), ... \tag{66}$$

It is obvious that this substitution may be reformulated in terms of the **primary measurable** momenta $p_{|N_{x_\chi}|}$ with the use of the relation $\frac{\ell}{|N_{x_\chi}|} = \frac{\ell^2}{\hbar}p_{|N_{x_\chi}|}$.

It is important to study the details associated, for example, with a number of the summands in formula (66).

By the authors opinion, for the construction of a **measurable** variant of gravity it is required to consider the following problems.

5.1. The behavior of $(EUM)[N_{x_\chi}]$ and its high-energy limit depending on the selected energy scale, i.e., depending on the quantities $|N_{x_\chi}|$. In this context, it is interesting, how the "quantum corrections" at low energies $E \ll E_P$ and semiclassical approximation should look like?

5.2. As in the well-known works by S.Hawking [39] –[41] all the results have been obtained within the scope of the semiclassical approximation, seeking for a solution of the above-mentioned problem is of primary importance. More precisely, we must find, *how to describe thermodynamics and quantum mechanics using the language of the* **measurable** *variant of gravity and what is the difference (if any) from the continuous treatment in this case.*

The author has already started a study of this problem for a simple case of the Schwarzschild black holes [7],[9].

To have a deeper understanding of the problem, we should know about the transformations of the notion of quantum information for the **measurable** variant of gravity and quantum theory at low $E \ll E_P$ and at high $E \approx E_P$ energies. Possibly, a new approach to the solution of the Information Paradox Problem [39] will offer a better insight.

5.3. As at low energies $E \ll E_P$ the **measurable** variant of gravity may be written in terms of the **primary measurable** momenta $p_{N_{x_\chi}}$, an analog of the *equivalence principle* may be also formulated in terms of the **primary measurable** momenta $p_{N_{x_\chi}}$.
Since the *equivalence principle* in a continuous theory reflects its *locality*, the problem is, what are the differences of the *equivalence principle* in the **measurable** variant of gravity from the *equivalence principle* in General Relativity and what are the transformations of this principle in both cases on passage to the quantum domain. Specifically, we should know, what is the correlation with "the quantum equivalence principle" introduces in the preprint [42].

5.4. As noted in point **4.3.**, in a simple case of (GUP) considered in Section 2 passage to **quantum gravity** in the **measurable** variant of General Relativity is represented by formula (64). However, (GUP) may be of a more complex form as considered in the survey work [43]. In this case on passage to **quantum gravity** the formula (64) is still valid.
But in the most general case we should find a correct expression for the momenta.
Using the proposed paradigm, we can denote the **measurable** momenta in the most general case (formulas (11)–(13)) at Plancks energies $E \approx E_P$ as $p_{N_{x_\chi}}(|N_{x_\chi}| \approx 1) \doteq p^{HE}_{N_{x_\chi}}$. Then on passage to **quantum gravity** we have

$$(EUM)[p_{N_{x_\chi}}, |N_{x_\chi}| \gg 1] \mapsto (EUM)[p^{HE}_{N_{x_\chi}}]. \qquad (67)$$

Thus, within the scope of the **measurability** notion, at all the energy scales E we can derive a common (in a sense universal) apparatus and the mathematical form for gravitational equations based on the introduction of **measurable** momenta, the definition of which involves all the three fundamental constants, the Planck constant $\hbar$ in particular. And this means that gravity is a quantum theory by its nature. But, as noted above, this property of gravity is revealed only at the scale of Plancks energies $E \approx E_P$.
By the authors opinion, the proposed approach offers the possibility to combine correctly a quantum theory and gravity in a most simple and natural way.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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