Advanced Studies in Theoretical Physics
Vol. 12, 2018, no. 4, 157 - 172
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/astp.2018.71047

Nonlinear Effects of Gravity in Cosmology

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Abstract

We investigate some nonlinear effects of gravity in cosmology. Possible physically interesting consequences include: non-requirement of dark matter and dark energy, asymmetric gravitational matter-creation, emergent homogeneity/isotropy & asymptotic flatness, resolution of "cosmic coincidence" $\Omega_m \sim \Omega_\Lambda$, effective cutoff of gravitational interaction at the scale of cosmic voids.

1 Introduction

The standard Friedmann-Robertson-Walker(FRW)-model of cosmology is overly simplified as it assumes perfect, and eternal, spatial homogeneity and isotropy - something simply not observed in the semi-local physical universe. The largest known structure observed so far is around 3000 Mpc [1], comparable to the size of the observable universe and clearly in conflict with the assumed "cosmological principle" of FRW. Still, it is almost universally used. The main reason, apart from habit, being that these extreme simplifying assumptions reduce Einstein's equations from a generally intractable set of ten coupled, highly nonlinear, partial differential equations to an analytically solvable set of just two coupled ordinary differential equations (Friedmann's equations), with only one dynamical degree of freedom, the cosmic scalefactor $a(t)$.

As gravitational energy itself gravitates, in the real "lumpy" universe we may, however, get "runaway solutions" where gravity nonlinearly amplifies the effect of matter, the most well-known example being the formation of a black

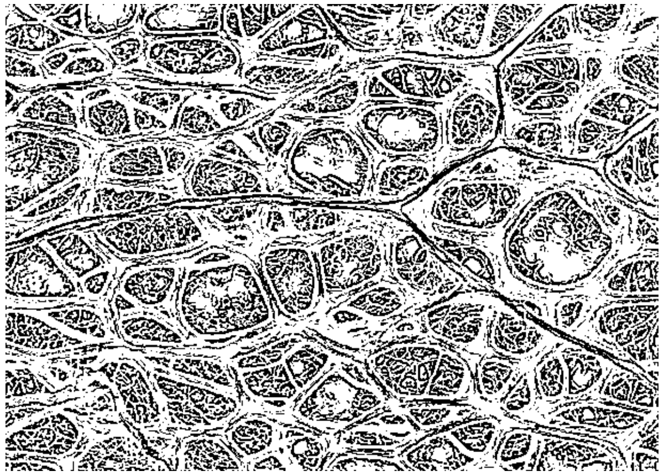

Figure 1: Schematic representation of large-scale spatial structure, with gravitationally bound regions of overdensity surrounding gravitationally unbound, expanding regions of underdensity. Just like the soap in the bubbles in a bubble-bath is distributed in a way to minimize the nonlinear energy, something similar should explain the observed voids and filaments in the cosmos (even though global energy is ill-defined in general relativity).

hole which in the end continues to exist solely due to gravitational nonlinearity. In less extreme cases we should nevertheless expect important, cumulative, corrections to the naive, often linear(ized), approximations to general relativity normally employed, as cosmological observations are very sensitive to the actual large-scale properties of the universe. As it long has been observationally known that there are huge spongy structures in the universe; cosmic "filaments" and "voids" [2], where the former contain all observable matter, whereas the latter seem essentially empty of such, these effects should play a major role for the dynamics and observations of the universe, ever more so as structure formation proceeds.

2 General Relativity

The cosmos, according to the classical theory general relativity, is an unchanging "block" of 4D-spacetime, in which all events in the global universe are embedded. Formally exact, we have for the global scalar curvature invariant (proportional to the Einstein-Hilbert action, and related to the Yamabe invariant of differential geometry)

$$\int_{tot} R\sqrt{|g|}\, d^4x = A, \tag{1}$$

where $R = R^\mu_\mu = g^{\mu\nu} R_{\mu\nu}$ is the scalar curvature, and g the determinant of the metric. For our present classically causal (observable) universe since the big bang the 4-volume is finite. If we imagine its 3D spatial hypersurfaces as non-simply connected, with evolving stringy regions of overdensity, surrounding bubbly regions of underdensity, Fig. 1, for the gravitationally bound regions of structure (observationally known to be almost "fractal-like"/scale-invariant and hierarchical), we have

$$\int_b R\sqrt{|g|}\, d^4x = B, \tag{2}$$

and for the gravitationally unbound regions, using (1), we must have

$$\int_u R\sqrt{|g|}\, d^4x = U = A - B. \tag{3}$$

From Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}, \tag{4}$$

where $T_{\mu\nu}$ encodes all non-gravitational energy-momentum and stress, the 4D scalar curvature always fulfills

$$R^\mu_\mu - \frac{1}{2} g_{\mu\nu} g^{\mu\nu} R = -\frac{8\pi G}{c^4} T^\mu_\mu, \tag{5}$$

i.e.

$$R = \frac{8\pi G}{c^4} T. \tag{6}$$

And as we never have perfect matter-less vacuum in the real universe, only in idealized models, the scalar curvature is generally non-zero, and positive definite in the sign convention we use. So far the discussion has been exact and generally covariant/invariant.

Now approximating, for a perfect fluid in comoving coordinates in homogeneous and isotropic space

$$T = \rho(t)c^2 - 3p(t). \tag{7}$$

Hence, for an eternally expanding global FRW-universe T, and thus R, goes to zero as $t \to \infty$. For pure radiation $p = \rho c^2/3$, giving $R_{rad} = 0$, so radiation does not contribute to the mean spacetime curvature. (Generally, the energy-momentum tensor of a pure electromagnetic radiation field is traceless so $T_{rad} = 0$ always.) For a present matter-dominated epoch $p \simeq 0$ ("dust") and $R_{matter} = 8\pi G\rho(t)/c^2$. If a cosmological constant, Λ, is introduced in (4) via a term $\Lambda g_{\mu\nu}$,

$$R = \frac{8\pi G}{c^4} T - 4\Lambda, \tag{8}$$

then $R \to -4\Lambda = const.$ as $t \to \infty$, making (1) and the Einstein-Hilbert action ill-defined as the 4-volume goes to infinity; another, mathematical, reason to avoid Λ quite apart from the physical reasons: two vastly different Λ's at inflation and now, $\sim 10^{120}$ discrepancy between the theoretically expected value of Λ and the observationally inferred one, no known physical fields can generate Λ, etc.

As all particles are hyper-relativistic in the early universe, they can then be treated as radiation and thus $T_{early} \to T_{rad} = 0 \Rightarrow R_{early} \to 0 \Rightarrow$ (Einstein-Hilbert Action)$_{early} \to 0$, explaining why the universe started out smooth on the average. Furthermore, as $T \to 0$ also as $t \to \infty$, (1) and the global action is non-divergent, provided that $\Lambda = 0$. As a quantum treatment is necessary when the action over a characteristic 4-volume is less than or $\sim \hbar$, we see that a universe containing pure radiation (as in the earliest epochs, and the latest ones if the universe expands forever and black holes evaporate) really would have to be treated quantum gravitationally; only in the intermediate epochs does the classical treatment apply. This also opens the possibility of small (quantum) inhomogeneities/fluctuations in the early universe, whereas the classical treatment would be smooth, precluding "seeds" from which gravitational structures later could grow.We thus see that quantum considerations is a necessity for cosmology, not a luxury.

To study dynamics, *i.e.* evolution, we must make a 3+1-split of spacetime, which at the same time destroys the globally invariant character of the 4D spacetime "block". However, cosmological structure formation introduces a natural "arrow of time" all by itself - the big bang model being isotropic in its spatial part only, not in time, so some time-slices may physically be more natural than others.

To illustrate with a simple explicit example, the scalar curvature in a perfectly homogeneous and isotropic (FRW) universe is just a function of the cosmic scalefactor $a = a(t)$

$$R = R^0_0 + R^1_1 + R^2_2 + R^3_3 = \frac{6}{c^2}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right), \tag{9}$$

and

$$R^0_0 = \frac{3}{c^2}\frac{\ddot{a}}{a}, \tag{10}$$

$$R^1_1 = R^2_2 = R^3_3 = \frac{1}{c^2}\left(\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{2kc^2}{a^2}\right), \tag{11}$$

where $k = +1, 0, -1$ characterizes the spatial curvature of the FRW-universe.

More generally, for gravitationally bound spatial regions, Fig. 1, we have

$$\int_b R^{(3)} \sqrt{|g^{(3)}|}\, d^3x = C, \tag{12}$$

where $R^{(3)}$ is the intrinsic scalar curvature for the spatial hypersurface at a given epoch, and $g^{(3)}$ the determinant of its induced three-metric. We must, by necessity, be situated in a region where gravitationally bound structures have formed, and so do all objects we observe, which is not characteristic of the global physical universe. For the gravitationally unbound, but finite, spatial regions in Fig. 1 we have

$$\int_u R^{(3)} \sqrt{|g^{(3)}|}\, d^3x = D, \tag{13}$$

where in a globally spatially flat universe

$$C + D = \int_{tot,flat} R^{(3)} \sqrt{|g^{(3)}|}\, d^3x. \tag{14}$$

As structure formation proceeds and the universe expands the cosmic "voids" increasingly contribute the bulk of the physical volume[1], cosmologically mimicking the effects of a mysterious, but fictitious, dark energy (see below). We empirically know that the "lumpiness" of the observed universe has increased with time, and that the effect thus monotonously increases, at least until very late epochs far removed from the present. From the reasoning above it then becomes natural that we should observe an (apparent) "acceleration" only in an epoch with appreciable void/filament structure formation ($z \sim 1$ and below). And as humans could not have evolved before this structure formation had taken place, it is also natural that we should live in this epoch, and before all stars have exhausted their energy supplies. In the much simplified FRW standard model of cosmology with non-zero cosmological constant Λ, on the other hand, it is just inexplicable why the "cosmic coincidence" $\Omega_{matter} \sim \Omega_\Lambda$ should occur *now*, as $\Omega_{matter} \gg \Omega_\Lambda$ as $t \to 0$ and $\Omega_{matter} \ll \Omega_\Lambda$ as $t \to \infty$. Also, to obtain the observed large-scale structure in a "mere" 14 billion years, the standard FRW-model must be "doped" by huge amounts of dark matter (ΛCDM-model), much larger than the contribution from known "normal" matter, as newtonian gravity used in N-body simulations to study

[1] As a non-expanding two-dimensional analogy, a flat sheet of metal if punched with small, sharp indentations "compensates" by curving the other way in between. (Even the original "flat" sheet is, if magnified sufficiently, seen to consist of negatively and positively curved regions.) Similarly, the surface of the earth globally has positive curvature, even though locally there are regions of negative curvature such as saddle-shaped mountain passes. If we do not assume global flatness cosmologically, we will have a different right-hand side in Eq.(14). However, in the universe, global *spatial* flatness translates to zero energy in the newtonian (weak-field) setting, making it "cost nothing" to create such a universe, and making it possible for it to exist indefinitely; $\Delta E \Delta t \sim \hbar$, $\Delta E \to 0 \Rightarrow \Delta t \to \infty$, where asymptotically the newtonian limit becomes exact. In that sense a flat universe would not be merely a possibility, requiring fine-tuning of initial conditions, but a necessity, see Eqs. (17) and (19).

supposedly "small" perturbations in the assumed globally valid background FRW-geometry does *not* generate additional gravity. In full, nonlinear general relativity the "doping" may be automatically provided by gravity itself - as gravity now *does* generate additional gravity [3] it will nonlinearly amplify the gravity of the matter present in bound regions. Also, as the field equations are nonlinear, solutions do not superpose, unlike gravitational potentials in the newtonian approximation. The field of two galaxies is *not* the sum of the "individual" fields, let alone the huge number used in (linearized) N-body simulations. In that sense, structure formation in the universe, both in space and time, may be just one more example of spontaneous self-organization, known to require nonequilibrium and nonlinearity [4]. One should keep in mind that the dark matter has never been seen, and is completely hypothetical. There is, in fact, not a single shred of independent evidence for either dark matter or dark energy outside of astronomy/cosmology.

Additionally, if the global universe is flat - plausible through observations of the cosmic microwave background (CMB) - structure formation, where gravity is increasingly attractive, must be balanced by effectively diminishing gravity in the (void) regions in between.

As an explicit, simplified toy-model example, take semi-local FRW-"sub-universe"-regions with their own density ρ, $\Omega = \rho/\rho_{crit}$, k and Hubble parameter $H = \dot{a}/a$. As $\Omega_b > 1$ patches asymptotically shrink (due to physical structure formation processes, including non-gravitational dissipation/friction in the real universe), $\Omega_u < 1$ patches grow to take their place. In a FRW sub-universe at the present epoch, with low to moderate redshift z, $\Omega_b \gg 1$ and increasing, $\Omega_u \ll 1$ and decreasing. For $z \sim 1100$ we should have $\Omega_b \simeq \Omega_u \simeq 1$, due to the high isotropy and homogeneity of the CMB, differing by just one part in $\sim 10^{-5}$. However, as nonlinear structure formation proceeds, the differences become more and more pronounced and important. In this approximation, for the observable universe, where no volume elements are infinite, we have for the intrinsic curvature, which governs if a sub-universe patch expands forever or eventually recollapses,

$$\int_{FRW,b} R^{(3)}_{FRW,b}\sqrt{|g^{(3)}_{FRW,b}|}\, d^3x + \int_{FRW,u} R^{(3)}_{FRW,u}\sqrt{|g^{(3)}_{FRW,u}|}\, d^3x = \int_{FRW,tot} R^{(3)}_{FRW,flat}\sqrt{|g^{(3)}_{FRW,flat}|}\, d^3x, \tag{15}$$

but as $R^{(3)}_{FRW}$ only depends on t, we get

$$R^{(3)}_{FRW,b}V_b + R^{(3)}_{FRW,u}V_u = R^{(3)}_{FRW,flat}V_{tot}, \tag{16}$$

where V is the physical 3-volume of the FRW spatial hypersurface. As V_b either increases slowly, is static or shrinks, from one epoch to the next, while

V_u increases fast, the magnitude of $R^{(3)}_{FRW,u}$ decreases. For such an asymptotically "empty" sub-universe, $\ddot{a} \to 0$ from a *negative* value, and $k = -1$, so from (9), as $R \to 0$ ($\rho \to 0$ as $t \to \infty$ in (7)) gives $\dot{a} \to c$. That is, luminosity distances in the global universe speed up relative to the bound regions, where $\dot{a}$ is monotonously decreasing, eventually becoming negative, $\dot{a} \to -\infty$ in finite time, *i.e.* mimics acceleration completely without dark energy as the unbound regions start to dominate the total volume, *i.e.* when structure formation has become pronounced. At the same time V_b singularly shrinks to zero; sub-universe "crunches" in finite time (if all other, stabilizing, interactions but gravity are neglected - but even if included $V_b \ll V_u$ for large t). The observed motion of the local group of galaxies relative to the Hubble flow (~ 600 kms^{-1}) is one realization of such an effect in the *real* universe. As the bound/unbound regions can be very intricately nested it is possible that the observed hierarchical structure in the real universe that has been produced since $z \leq 1100$ has its origin in a similar effect as in this simplified toy-model.

3 Matter Creation

It is known that the only other strongly nonlinear fundamental interaction with massless force-carrier particles, Quantum Chromodynamics (QCD), supports "string-like" characteristics of the effective force ($\neq 1/r^2$) due to the nonlinearity. If we assume that the same occurs for gravity, as it too couples nonlinearly to itself, and as a first approximation assume that the effect scales with the intrinsic strength between two protons, hydrogen being the main component of the visible universe, and as $F_{QCD}/F_{gravity} \sim 10^{38}$, we get for the present characteristic length-scale of Gravity-string-alterations 10^8 lightyears, which is intriguing, as it is of the same order as the size of cosmic filaments and voids (the effect needing time, *i.e.* space, to become effective). Due to the strongly nonlinear nature, even very small initial deviations may then have large effects, especially over vast regions of space and time.

Gravity, as it self-couples, should thus also get a correction to its "field-lines", see Fig. 2. This could then hypothetically create matter, akin to Hoyle's old steady-state model with continuous creation [5], just like "hadronization" in QCD: when the flux-tube is stretched it eventually becomes energetically favorable to create new matter particles non-adiabatically out of the field energy. The scale of hadronization in QCD is ~ 10 fm, and as the work needed to create proton-pairs is $F_{QCD} \cdot l_{QCD} \simeq F_{gravity} \cdot l_{gravity} = 2m_p c^2$, in cosmology matter creation should take place at a scale of roughly 10^8 lightyears. Gravity then gets an effective cut-off at this scale, just like QCD exponentially disappears outside nuclei (even though gluons are massless), giving an immediate reason as to why the universe should seem statistically homogeneous and isotropic at scales $\gg 10^8$ lightyears.

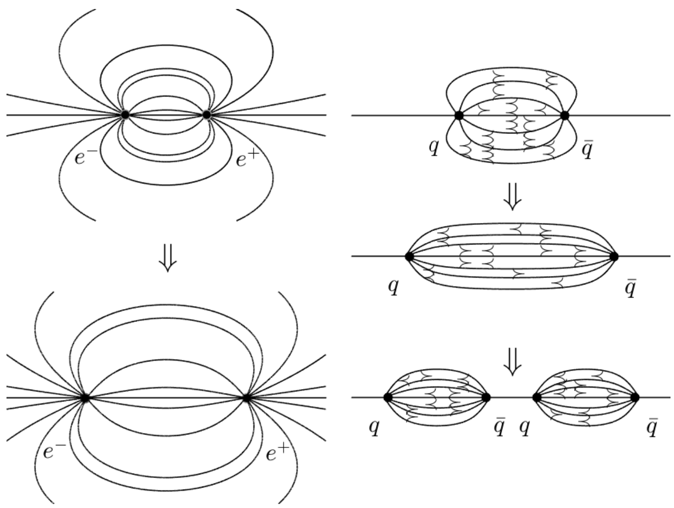

Figure 2: Schematic representation of effect without (QED: left) and with (QCD: right) nonlinear self-interaction. Gravity should behave in a way similar to QCD, as it too is nonlinearly self-interacting. However, at completely different scale. Above a threshold distance the effective force is $\sim$ constant, the potential rising as $\sim r$, until it becomes favorable to create matter, which gives an effective cut-off of the interaction. Heuristically, one can thus immediately appreciate why matter should lie along filaments, surrounding voids, Fig. 1. This should qualitatively be true also in full general relativity, as gravity itself gravitates, but can no longer be described by a simple (single) potential.

As general relativity is globally neither CP nor CPT invariant, the CPT-theorem being valid only for Lorentz-covariance [6], strict particle-antiparticle symmetry need not apply, meaning that CP can be broken, even if T-reversal symmetry is still obeyed. However, it is the T-symmetry that conserves energy, and this too is broken globally in general relativity. (Physically, this is evident in the real universe as *e.g.* gravitational structure formation defines what is to be considered "forwards" in time.) So CP is broken globally, which means that a global asymmetry between particles and antiparticles is theoretically allowed, as also is observed. The notion of particle in *special* relativistic field theory is directly linked to the Poincaré group. Its absence in the case of the gravitational field opens up new possibilities, such as particle production (observed in asymptotically flat regions) by a time-varying $g_{\mu\nu}$. It is an empirical fact that the weaker the interaction, the fewer conservation laws does it obey. So this aspect of gravity, the by far weakest of all known interactions, need not be entirely surprising. "Gravitation conserves everything or nothing depending on how you look at it" [7]. (As the interval between the created particles is spacelike, they could not annihilate even if CP would be conserved.) More concretely, this dissipative effect should enhance distribution of matter along the field lines, *i.e.* between the (initially tiny) anisotropic matter concentrations, making them more and more pronounced, qualitatively explaining why matter is observed to increasingly lie along filamentary "rope-like" structures. At the same time, as gravity effectively concentrates along the filaments it decreases in the voids, making them relatively speed up. For visualization, the familiar 3D "raising dough"-analogy can still be utilized, but with embedded "raisins" (galaxies) replaced by a "flexible web" (filaments) that expands and thins as the voids expand, see Fig. 1. The 2D "inflating balloon"-analogy can also be used, but with "coins" (galaxies) glued to the surface replaced by "rubber bands" crisscrossing the balloon surface. Regions "in between" (voids) then relatively expand increasingly faster than the "flexible web"/"rubber bands" (filaments), giving an apparent relative acceleration. As always, these graphic analogies are not exact, and should not be perceived as such, but only as rough guides for the mind.

The filaments eventually always (re)collapse whereas the voids expand eternally as: i) $\Omega_b > 1 > \Omega_u$ and, ii) some of the kinetic energy in filaments produce new matter. As our semi-local neighborhood is situated in such an overdense region it makes gravity here, and in similar patches elsewhere, appear stronger than naively estimated globally; mimicking cosmological dark matter, and at the same time the relative expansion rate between voids and filaments is monotonously increasing; mimicking dark energy. Roughly, as gravity is increasingly concentrated along the filaments, Fig. 2, it gets "diluted" in the voids. A newtonian way to view it, even without the "stringy" picture, is that in flat ($k = 0$) Euclidean space the field-line-density piercing a spherical shell decreases as r^{-2}, giving the normal newtonian gravitational force $F \propto r^{-2}$. In a positively curved space ($k = +1$) the area of the shell is smaller than $4\pi r^2$, meaning that the gravitational force decreases slower than r^{-2} (*i.e.* gravity is stronger). For a negatively curved space ($k = -1$) the shell-area is larger than $4\pi r^2$ so the gravitational force decreases faster than r^{-2} (*i.e.* gravity is perceived as weaker).

If matter creation does occur along these lines, we get a simple and natural explanation of the "flatness problem", *i.e.* why the universe is perceived to be flat regardless of initial conditions ($\neq$ globally recollapsing). Excess kinetic energy of the matter automatically generates new matter until there is an exact balance between gravitational and kinetic energy, the matter thus automatically approaching critical density globally as the expansion proceeds, the expansion asymptotically going towards zero, just like for a *flat* FRW-universe as $t \to \infty$.

In the newtonian setting, the energy per unit mass anywhere in the universe is

$$E = \frac{v^2}{2} - \frac{GM}{r} = \frac{\dot{a}^2}{2} - \frac{GM}{a}. \tag{17}$$

If $E < 0$ the motion is below escape velocity and the sub-universe recollapses, if $E > 0$ it expands forever with $v \neq 0$, and $E = 0$ is the balanced case where the kinetic energy is exactly cancelled by the gravitational.

The results from FRW are identical,

$$M = \frac{4\pi}{3}\rho a^3, \tag{18}$$

$$\frac{\dot{a}^2}{2} - \frac{4\pi G}{3}\rho a^2 = -\frac{kc^2}{2}, \tag{19}$$

so E and k are directly related[2].

Rewritten as

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} = H^2, \tag{20}$$

where

$$H \equiv \frac{\dot{a}}{a}, \tag{21}$$

is the (measurable) semi-local Hubble parameter, for $k = 0$ this gives

$$\rho_{crit} = \frac{3H^2}{8\pi G}. \tag{22}$$

[2] $E = 0$ has probability measure zero in the allowed interval $-\infty < E < \infty$; this is the "flatness"- or "fine-tuning"-problem in models without matter creation (normally "solved" by primordial inflation).

If there is creation of matter the kinetic term will decrease faster, the balance going into increasing M, speeding up the structure formation process in bound regions.

In QCD, hadronization globally conserves energy and momentum of matter by construction due to the absence of external forces, as in particle physics the, inherently nonlocal, gravitational effect can be (and is) completely neglected, as spacetime is effectively flat to extremely high accuracy in particle physics experiments. This cannot be fulfilled in general relativity, and especially not in cosmology, as: i) global energy is not really defined, ii) the generally covariant four-divergence $T^{\mu\nu}_{;\nu} = 0$ is not a conservation law but describes the local *exchange* of energy and momentum between (local) matter and (nonlocal) gravitation [8], the energy and momentum of matter not being conserved in the presence of dynamic spacetime curvature but changing in response to it.

The equivalence principle itself precludes a local definition of gravitational energy, but if we introduce the gravitational energy-momentum pseudo-tensor $t_{\mu\nu}$ (not generally covariant), then

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + t^{\mu\nu}, \tag{23}$$

locally does give a (merely Lorentz-covariant) conservation law

$$\tilde{T}^{\mu\nu}{}_{,\nu} = 0, \tag{24}$$

where the comma denotes normal partial (not covariant) differentiation. So, there seems to be no fundamental obstacle in *principle* to gravitational matter creation.

4 Chaotic Gravity

Just like the weather on earth is inherently unpredictable after ∼1 week due to the highly nonlinear nature of the system, similarly cosmology should have a limit of detailed predictability, both for predictions and retrodictions, *i.e.* both forwards and backwards in time, the timescale t_0 being dependent on the inhomogeneity/anisotropy and intensity of gravity. Even in idealized regions completely devoid of matter, gravity itself is governed by highly nonlinear equations [3], the Einstein equations (4) permitting spacetime to be curved even if $T_{\mu\nu} = 0$, *i.e.* pure curvature generating curvature, pure gravity generating gravity. This means, among other things, that deterministic chaos for test-particles (both photons and matter) in nearby geodesic motion in the real, semi-local, inhomogeneous/anisotropic universe should make them diverge exponentially

$$\Delta \sim e^{t/t_0}, \tag{25}$$

losing detailed predictability for $t > t_0$ (where t_0^{-1} more or less is a positive Lyapunov exponent), or for spatial length-scales l_0,

$$\Delta \sim e^{ct/l_0}. \tag{26}$$

The geodesic deviation equation, for a set of nearby, initially parallel, geodesics $\gamma_s(\tau)$ parameterized by s and τ (each s giving a geodesic with parameter τ) for non-massless test particles is

$$\frac{D^2 S^\mu}{d\tau^2} = R^\mu_{\nu\rho\sigma} v^\nu v^\rho S^\sigma, \tag{27}$$

where D is the directional covariant derivative of S along τ, $R^\mu_{\nu\rho\sigma}$ the Riemann tensor, v the four-velocities and

$$S^\sigma = \frac{\partial x^\sigma}{\partial s}, \tag{28}$$

the deviation vector. The geodesic deviation equation simply says that the relative acceleration between two neighboring geodesics is proportional to the curvature.

Instead of explicitly trying to invoke the equation of geodesic deviation, as it is impossible in practice to obtain information about $R^\mu_{\nu\rho\sigma}$ everywhere in an inhomogeneous, anisotropic universe, a very simple intuitive physical analogy would be a special "pinball machine" consisting of many rows of pins on an inclined plane. If a ball is released from the top, its final position at the bottom would show the tell-tale chaotic signature: extreme sensitivity to initial conditions. In general relativity/cosmology, the pins would be replaced by local $\tilde{T}^{\mu\nu}$ (Ω_b and Ω_u in the case of the "nested" FRW-subuniverse toy-model) and the balls by test-particles following the local geodesics. Also, unless there is a preferred cosmic proper time, any attempt at detailed calculations would meet with the problems of quantifying chaos in a generally covariant way [9].

5 Emergent Homogeneity and Isotropy

As one expects severe gravitational/metric "turbulence" [3] in the early universe (as opposed to the comparatively "laminar" approximate Hubble flow of today) one would nevertheless expect thorough mixing leading to a high degree of *statistical* homogeneity and isotropy. This would be "turbulence" in space-time itself, so naive causal constraints (global FRW-horizons) do not apply, and is automatically embedded in general relativity needing no *ad hoc* mechanism like inflation. If gravitational "turbulence" is nearly scale-invariant, like turbulence in fluids, it could generate the approximately scale-invariant fluctuations seemingly needed for structure formation in the universe. As the earliest epoch furthermore must be described by some sort of quantum gravity it should exhibit the same kind of nonlocality that normal quantum mechanics is known to do through tests [10], [11], [12] of Bell's theorem [13] from 1972 until the present [14], making classical causal horizons in that epoch lose their meaning and observed global near-isotropy (*e.g.* CMB) natural. This is based on a *known* property of laboratory-tested physics, and does not rely on speculative (and untested) additional hypotheses about what happens $\leq 10^{-32}$ s after time zero in the very early universe. Also, a pure quantum state of the universe would have zero entropy, beginning to increase only as "observations" (collapse of the wavefunction, *i.e.* quantum $\rightarrow$ classical transition) occur, connecting that to the cosmological arrow of time and the 2nd law of thermodynamics. If the energy content of the universe furthermore is zero (the universe arising from a quantum fluctuation) it implies global flatness[1], and a quantum universe created in its groundstate would be isotropic.[3] Lastly, it should suppress the classical chaos into quantum "chaos" at the earliest epoch possibly leading to unique observational signatures.

6 Dark Matter?

The dark matter normally inferred in galaxy clusters, *e.g.* through the gravitational lens effect, as a broadly distributed hump devoid of any visible matter, superposed on "spiky" individual galaxies, may in part or totality be the nonlocal energy ($\neq T_{\mu\nu}$) of the gravitational field itself. It would by definition be "dark", *i.e.* invisible, as it has no other interactions but gravitational. Furthermore, already well below 10^8 lightyears, gravity should get corrections to r^{-2} due to the contracting of field-lines, Fig. 2. This would also mean that in a spiral galaxy the rotational (disk) plane should experience a stronger effective gravity, making deviations from Keplerian motion natural for large r, *without* any dark matter. Using the analogy between gravity and QCD the effective gravitational force, outside some threshold range (and well before matter production, which effectively cuts off gravity), should be $\sim$ constant, *i.e.* the effective gravitational potential being

$$\phi \simeq -\frac{GM}{r} + \alpha M r, \tag{29}$$

resulting in

$$v \simeq \sqrt{\frac{GM}{r} + \alpha M r}, \tag{30}$$

[3] Making inflation superfluous, which is inconsistent anyway as it is based on classical (hypothetical) fields, assuming definite values at each point in spacetime, contributing to the energy-momentum tensor in an era where quantum effects should reign supreme.

the first term giving the normal Keplerian dynamics for most astrophysical scales, while the second term dominates for large r, giving

$$v \propto \sqrt{r}, \tag{31}$$

compatible with observations [15]. As α starts to dominate when $\alpha Mr \geq GM/r$, *i.e.* for $r \geq \sqrt{G/\alpha}$, this could explain galactic dynamics without dark matter if α is of the order 10^{-51} N/kg^2. The ratio G/α should in principle be calculable from a theory of quantum gravity, Fig. 2, and may even be scale-dependent.

Furthermore, the recently empirically discovered, unexpected, direct and seemingly universal relation between the "dark matter" and the normal visible baryonic mass in spiral galaxies [16] could get an automatic explanation as a nonlinear effect of gravity *itself*, without the need for any dark matter. One should keep in mind that such a relation is not at all natural for dark matter models, and was not anticipated or predicted by them.

7 Conclusion

The traditional approach to cosmology is: 1. Construct an overly simplified global model (FRW, with only *one* degree of freedom). 2. Deduce its dynamical consequences through general relativity (Friedmann's equations) and believe blindly in the conclusions, conveniently "forgetting" the severe approximations made. 3. Observe that the consequences do *not* correspond to the real universe, especially in the recent era. 4. Invent make-shift add-on "solutions" (dark matter, dark energy) to save the model - instead of discarding it and constructing a less simplified model.

We, instead, propose that full nonlinear gravity itself has the potential to explain most cosmological observations and currently perceived "enigmas", without the introduction of new *ad hoc* components of the universe. The arguments and model-calculations in this article at least do not preclude such a possibility.

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Received: November 5, 2017; Published: May 16, 2018