Advanced Studies in Theoretical Physics
Vol. 12, 2018, no. 4, 181 - 188
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/astp.2018.8413

Chaos Control for the Lorenz System

Pedro Pablo Cárdenas Alzate

Department of Mathematics and GEDNOL
Universidad Tecnológica de Pereira
Pereira, Colombia

Germán Correa Vélez

Department of Mathematics
Universidad Tecnológica de Pereira
Pereira, Colombia

Fernando Mesa

Department of Mathematics and GEDNOL
Universidad Tecnológica de Pereira
Pereira, Colombia

Abstract

This article presents an analysis of the chaotic dynamics presented by the Lorenz system and how this behavior can be eliminated through the implementation of sliding mode control. It is necessary to know about the theory of stability of Lyapunov to develop the appropriate control that allows to bring the system to the desired point of operation.

Keywords: Lorenz system, stability of Lyapunov, chaos control, sliding modes

1 Introduction

Chaotic behavior has been analyzed in a variety of engineering systems. The main characteristic of these systems is the extreme sensitivity to the initial conditions, that is, a small variation in the initial state will cause a departure from the trajectories in the state space. The chaotic behavior is irregular, complex and undesirable for some engineering systems, as it causes low performance and can be very destructive, for this reason it is necessary to know when the system enters this state of chaos and if it does, how to recover it . Hence, the analysis and control of nonlinear dynamic systems has taken great importance in recent decades [1].

The most analyzed system has been the model of the Lorenz equations, which is considered a paradigm since it is a simple model that has chaotic characteristics depending on its parameters. In the literature there are several studies on the control of chaotic systems. Vicent and Yu [2] developed a Bang-Banga controller that regulates the chaotic system to one of its unstable points. Ott, Gebogi and Yorke [3] proposed techniques that allow converting the movement of the strange attractor into periodic oscillations. Hartley and Mossayebi [4] have proposed classical control models, which lead to undesirable chaotic transients in the system. Álvarez Gallegos [5] obtained interesting results from the linearization of feedback based on the theory of nonlinear geometric control. In 1997, Zeng and Singh, used an adaptive control law to drive the chaotic system to a specific point in the state space [6].

In this paper, it is based on the application of control by sliding planes, to eliminate the chaotic behavior of the Lorenz system and to take the trajectories of the phase space to a particular point, which was taken as the point of equilibrium. To fulfil this objective, section two deals with the description of the Lorenz system and the presence of chaotic characteristics for certain values of its parameters. Section three addresses the issue of the stability of Lyapunov as this methodology leads to the development of the control technique by sliding planes. Finally, section four shows the results obtained and their respective analysis.

2 Chaos in the Lorenz system

The Lorenz equations are a simplified model of an incompressible convective air flow between two horizontal plates with a temperature difference, subject to gravity. The motivation of these equations was to highlight why the climate is unpredictable despite being a deterministic system. The model is the following:

$$\begin{cases} \dot{x} &= \sigma(y - x) \\ \dot{y} &= (\rho - z)x - y \\ \dot{z} &= xy - \beta z, \end{cases} \tag{1}$$

where x is the fluid velocity; y is the horizontal temperature; z is the vertical temperature; σ is the Prandtl number; ρ is the Rayleigh number and β is the geometry factor. Here, we assume that the parameters σ, ρ and β are known. The Lorenz system presents a complex dynamics depending on the values of its parameters, for values $0 < \rho < 1$ the dynamics of the system is stable, while for high values of ρ, the dynamic behavior becomes chaotic. For example, with values of $\sigma = 10$, $\rho = 28$ and $\beta = 8/3$, unpredictable dynamics are obtained, these values being the most common for the analysis.

The visualization of the chaotic behavior in said system is obtained by simulation by Matlab. Figure 1 shows the strange attractor generated by this dynamic and Figure 2 shows the time series of each of the state variables.

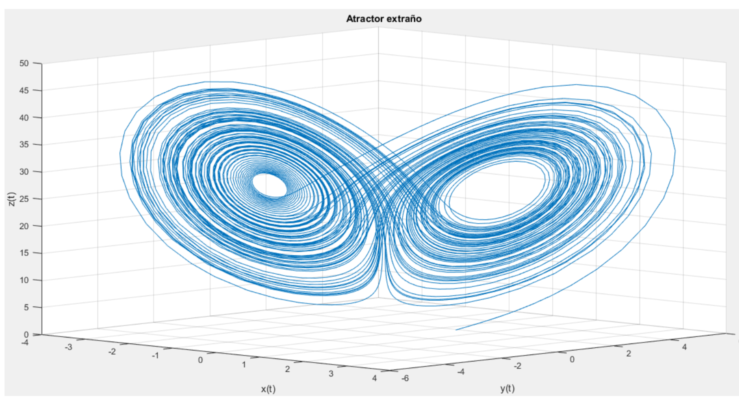

Figure 1: Lorenz attractor.

3 Analysis and control for the Lorenz system

3.1 Determination of the equilibrium point

Now, from the system (1) we obtain

$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} \pm\sqrt{\beta(\rho - 1)} \\ \pm\sqrt{\beta(\rho - 1)} \\ \rho - 1 \end{bmatrix}$$

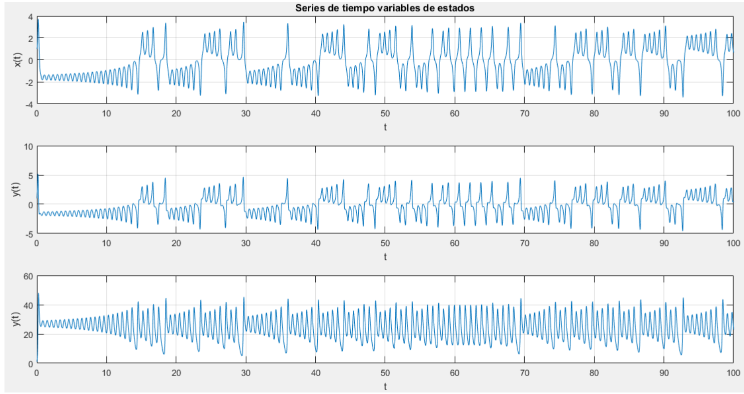

Figure 2: State variables.

The local stability of this point can be determined by linearization around it and calculating the characteristic polynomial in the following way [7]:

$$(D_{\vec{x}}F)\,(\vec{x_e}) = \begin{bmatrix} -\sigma & \sigma & 0 \\ \rho - z_e & -1 & -x_e \\ y_e & x_e & -\beta \end{bmatrix} \tag{2}$$

$$\begin{aligned}\Delta\,(\lambda I - (D_{\vec{x}}F)(\vec{x_e})) = \lambda^3 + (\beta+\sigma+1)\lambda^2 + (\beta + x_e^2 + 1 + (\beta+\rho+z_e)\sigma)\lambda \\ + (\beta - \beta\rho + x_e^2\sigma + x_e y_e \sigma + \beta z_e \sigma)\end{aligned} \tag{3}$$

For the values of the parameters taken in this article, the eigenvalues are $\lambda_1 = -3.381$ and $\lambda_{2,3} = -5.142 \pm 24.312$, that is, the equilibrium point is asymptotically stable.

3.2 Stability by Lyapunov and control by sliding planes

The theory of stability is based mainly on the theory of Lyapunov, which proposes two methodologies for the determination of it. The first of these is the indirect methodology, which is based on finding the transfer function of the system and finding the poles of that function, which must belong to the left complex half-plane so that the system is asymptotically stable.

The direct methodology consists in the construction of a function that fulfills 3 conditions, called the Lyapunov function, these conditions are the following:

- $V\,(\vec{x_e}) = 0$
- $V\,(\vec{x}) > 0 \;\forall\; \vec{x} \neq \vec{x_e}$

- $\dot{V}(\vec{x}) < 0$ along the trajectories of $\dot{x} = f(x)$.

By means of this direct Lyapunov methodology, the sliding surface can be defined to apply the control. This is defined as:

$$\sigma(\vec{x}) = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n \tag{4}$$

A candidate Lyapunov function can be

$$V(\vec{x}) = \frac{1}{2}\sigma(\vec{x})^2 \tag{5}$$

Now, by deriving Equation. (5) the third condition of stability will be shown [8]:

$$\dot{V}(\vec{x}) = \sigma(\vec{x})\dot{\sigma}(\vec{x}). \tag{6}$$

Therefore, realizing the substitution $x = x - x_e$, $y = y - y_e$ and $z = z - z_e$, we have the following function of Lyapunov for the Lorenz system:

$$V = \frac{1}{2}\left(c_1 (x - x_e)^2 + c_2 (y - y_e)^2 + c_3 (z - z_e)^2\right) \tag{7}$$

$$\dot{V} = c_1 (x - x_e)^2 \dot{x} + c_2 (y - y_e)^2 \dot{y} + c_3 (z - z_e)^2 \dot{z}. \tag{8}$$

The new Lorenz system with the application of open loop control for the elimination of chaotic dynamic behavior, will be the following:

$$\begin{cases} \dot{x} &= \sigma (y - x) + u_1(t) \\ \dot{y} &= (\rho - z) x - y + u_2(t) \\ \dot{z} &= xy - \beta z + u_3(t), \end{cases} \tag{9}$$

where $u_1(t)$, $u_2(t)$ and $u_3(t)$ are external control excitations. This control in open loop has certain restrictions regarding the determination of the control parameters, for this reason it is convenient to apply control in closed loop since the number of parameters is reduced, leaving the system:

$$\begin{cases} \dot{x} &= \sigma (y - x) \\ \dot{y} &= (\rho - z) x - y + u_2(x, y, z) \\ \dot{z} &= xy - \beta z. \end{cases} \tag{10}$$

From the system in (10) the control law can be determined, replacing (10) in (8) and forcing $\dot{V}(\vec{x})$ to be less than zero for all cases except at the equilibrium point. Said control law will be:

$$u_2 = -\eta sign(\sigma(\vec{x})) - \dot{y} + \sin(2\pi * t), \tag{11}$$

where $\sigma(\vec{x}) = y - y_e$.

4 Analysis of results

In this section we will show the results obtained when applying the control by sliding planes to the Lorenz system. The parameters used are those mentioned in section two, with which the equilibrium points are obtained:

$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} 8.4853 \\ 8.4853 \\ 27 \end{bmatrix}.$$

It is expected that the trajectories of the system in the phase space tend to equilibrium point. The simulation was implemented in Matlab using the function *ode45* for the solution of the system of non-linear differential equations. The results are shown in Figures 3 and 5, where each of the state variables is shown for a value τ between [100-300].

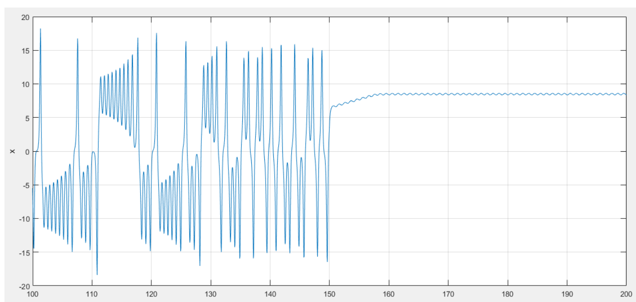

Figure 3: State variable $x(t)$.

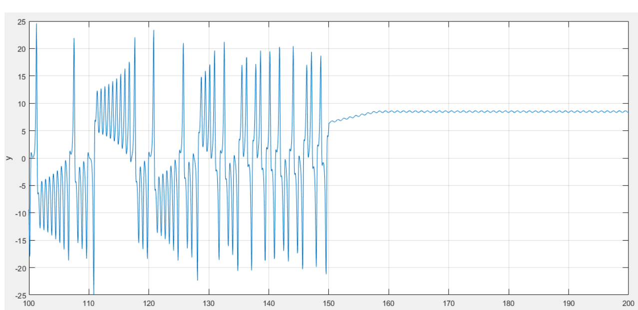

Figure 4: State variable $y(t)$.

Here, it is possible to highlight a time that the system has a high convergence to the expected operation points, besides the response time is good considering that it is a model for the variation of the climate. On the other

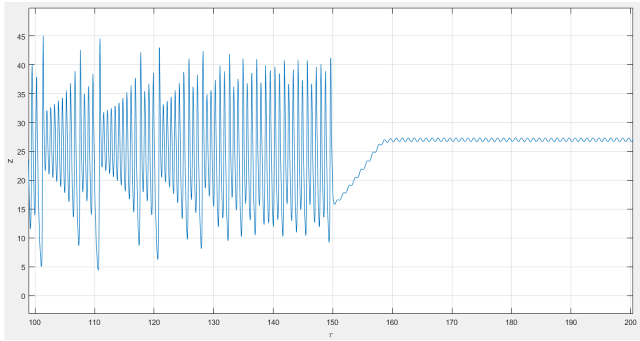

Figure 5: State variable $z(t)$.

hand, the control signal is not very high, therefore, it would not be necessary to have high values of voltage sources if one wanted to implement said simulation by means of an electrical circuit [9]. This signal is shown in Figure 6.

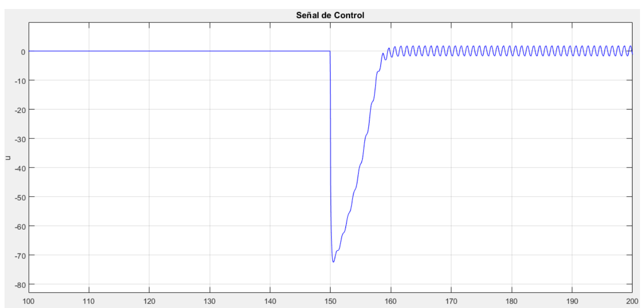

Figure 6: Control signal u_2.

5 Conclusion

The characterization of the dynamics of any system allows the development of adequate control techniques that lead the trajectories of the system in the phase space to the desired point of operation. The technique of sliding mode control is easy to implement for any type of dynamic system, since it is derived from Lyapunov's direct stability methodology. In addition, it provides very good results when compared to classical control techniques.

Acknowledgements. We would like to thank the referee for his valuable suggestions that improved the presentation of this paper and our gratitude to the Department of Mathematics of the Universidad Tecnológica de Pereira

(Colombia) and the group GEDNOL.

References

[1] S. Yang, Chieh-Li Chen, Her-Terng Yau Control of chaos in Lorenz system, *Chaos, Solitons and Fractals*, **13** (2002), 767-780. https://doi.org/10.1016/s0960-0779(01)00052-2

[2] T. Vincent, Jianzu Yu, Control of a chaotic system, *Dynamics and Control*, **1** (1991), 35-52. https://doi.org/10.1007/bf02169423

[3] C. Grebogi, E. Ott, J. York, Controlling chaotic dynamical systems, *SovietAmerican Prespectives on Nonlinear Science*, D.K. Campbell ed., American Institute of Physics, 1990, 153-172.

[4] T. Hartly, F. Mossayebi, Classical control of a chaotic system, *Conference on Control Applications(IEEE),* (1992), 522-526. https://doi.org/10.1109/cca.1992.269819

[5] Troy Shinbrot, Edward Ott, Celso Grebogi, James A. Yorke, Using chaos to direct trajectories to targets, *Phys. Rev. Lett.*, **65** (1990), 3215-3218. https://doi.org/10.1103/physrevlett.65.3215

[6] Y. Zeng et al., Adaptative control of chaos in Lorenz system, *Dynamics Control*, (1994), 97-114.

[7] R. Choroszucha, Control of the Lorenz equations, Department of Naval and Marine Engineering, University of Michigan, (2000).

[8] S. H. Strogatz, Ronald F. Fox, Nonlinear dynamics and chaos: With applications to physics, biology, chemistry and engineering, *Physics Today*, **48** (1995), 93- 94. https://doi.org/10.1063/1.2807947

[9] A. Wolf, Jack B. Swift, Harry L. Swinney, John A. Vastano, Determining lyapunov exponents from a time series, *Physica D: Nonlinear Phenomena*, **16** (1985), no. 3, 285-317. https://doi.org/10.1016/0167-2789(85)90011-9

Received: April 25, 2018; Published: May 31, 2018