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Dependence of the Ghostbursting Model's Dynamical States on the Current Injected into the Dendritic Compartment and the Ratio of Somatic to Total Surface Areas

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Abstract

The ghostbursting model is a mathematical model that describes the dynamics of the electrical excitability of the weakly electric fish *Apteronotnus leptorhynchus*. This model is described by a system of nonlinear ordinary differential equations. The present study focuses on two system parameters of the model (i.e., the current injected into the dendritic compartment and the ratio of somatic to total surface areas); in addition, a computer simulation analysis of the model is performed. By changing these parameter values, the dynamical states of the model in the two-dimensional parameter space are revealed.

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Keywords: Mathematical model, computer simulation, ghostbursting model, current injected into the dendritic compartment, ratio of somatic to total surface areas

1 Introduction

The electrical excitability of electrosensory pyramidal neurons of the weakly electric fish *Apteronotnus leptorhynchus* is described by the ghostbursting model that was developed in a previous study based on the Hodgkin–Huxley concept [1]. The ghostbursting model is a two-compartment mathematical model described by a system of six coupled nonlinear ordinary differential equations. The model's equations are classified into two groups: the first contains two equations describing the dynamics of the somatic compartment and the second contains four equations describing the dynamics of the dendritic compartment. The effect of varying two system parameters of the ghostbursting model (namely, the current injected into the somatic compartment and the ratio of somatic to total surface areas) on the dynamical states of the model has been previously reported [2]. Two-compartment mathematical models similar to the ghostbursting model have been previously proposed [3, 4]. These studies consider the possibility that a current can be injected into the model not only via the somatic compartment but also via the dendritic compartment. When taking this into consideration, it is important to elucidate how the current injected into the dendritic compartment of the ghostbursting model affects the ghostbursting model's dynamical states. However, this issue was not investigated in the previous study [2]. The present study performs a numerical simulation to elucidate the effect of variations in two systems parameters (i.e., the current injected into the dendritic compartment and the ratio of somatic to total surface areas) on the dynamical states of the ghostbursting model.

2 The Ghostbursting Model

The ghostbursting model comprises six state variables: the membrane potential of the somatic compartment [V_s (mV)], the membrane potential of the dendritic compartment [V_d (mV)], and four gating variables of the ionic currents contained in the somatic or dendritic compartment (n_s, h_d, n_d, and p_d). The dynamics of these state variables can be described as follows:

$$\frac{dV_s}{dt} = I_s - \frac{1}{\kappa}\left(V_s - V_d\right) - 55\left(\frac{1}{1+e^{-(V_s+40)/3}}\right)^2 \left(1-n_s\right)\left(V_s-40\right) - 20n_s^2\left(V_s+88.5\right) - 0.18\left(V_s+70\right) \tag{1}$$

$$\frac{dV_d}{dt} = I_d - \frac{1}{1-\kappa}\left(V_d - V_s\right) - 5\left(\frac{1}{1+e^{-(V_d+40)/5}}\right)^2 h_d\left(V_d-40\right) - 15n_d^2 p_d\left(V_d+88.5\right) - 0.18\left(V_d+70\right) \tag{2}$$

$$\frac{dn_s}{dt} = \frac{1}{0.39}\left(\frac{1}{1+e^{-(V_s+40)/3}} - n_s\right) \tag{3}$$

$$\frac{dh_d}{dt} = \frac{1}{1+e^{(V_d+52)/5}} - h_d \tag{4}$$

$$\frac{dn_d}{dt} = \frac{1}{0.9}\left(\frac{1}{1+e^{-(V_d+40)/5}} - n_d\right) \tag{5}$$

$$\frac{dp_d}{dt} = \frac{1}{5.0}\left(\frac{1}{1+e^{(V_d+65)/6}} - p_d\right) \tag{6}.$$

I_s (μA/cm^2), I_d (μA/cm^2), and κ are system parameters of the model: I_s is the current injected into the somatic compartment, I_d (μA/cm^2) is the current injected into the dendritic compartment, and κ is the ratio of somatic to total surface areas. In the previous study, the effect of variations of I_s and κ on the dynamics of the ghostbursting model was investigated under conditions wherein I_d was fixed to be zero [2]. In contrast, in the present study, the effect of variations of I_d and κ under conditions wherein I_s is fixed to be zero was investigated. In the present study, I_d was varied from 2.8 to 5.6 μA/cm^2, and κ was varied from 0.24 to 0.40. A detailed description of equations (1)–(6) has been previously provided [1, 2]. Numerical simulations of equations (1)–(6) were performed using the free and open-source software Scilab (http://www.scilab.org/) under the initial conditions: $V_s = -70$ mV, $V_d = -70$ mV, $n_s = 0.00005$, $h_d = 0.973$, $n_d = 0.002$, and $p_d = 0.697$.

3 Numerical Results and Discussion

Modulation of the dynamical states of the ghostbursting model by I_d and κ is shown in Figure 1. First, we investigated the dynamical states of the model under conditions wherein κ is 0.34 or more. An increase in I_d under such conditions wherein κ is fixed to a certain value changes the dynamical state, for example, from a quiescent state to a repetitive spiking state to a bursting state. An increase in κ, such as 0.34→0.36→0.38→0.40, induces an increase in the I_d threshold for the transition from a quiescent state to a repetitive spiking state, such as 3.6→3.6→3.8→4.0 μA/cm^2. In addition, an increase in κ induces an increase in the I_d threshold for the transition from a repetitive spiking to a bursting state, such as 4.0→4.4→5.0→5.6 μA/cm^2. Next, we investigated the dynamical states of the model under conditions wherein κ is 0.32 or less. An increase in I_d under such conditions wherein κ is fixed to a certain value changes the dynamical state, for example, from the quiescent state to the bursting state. An increase in κ, such as 0.24→0.26→0.28→0.30→0.32, induces an increase in the I_d threshold for the transition from a quiescent state to a bursting state, such as 3.0→3.2→3.2→3.4→3.4 μA/cm^2.

From a biophysical viewpoint, I_s and I_d are similar in that an increase in each parameter's value has an excitatory effect on the ghostbursting model. In fact, similar to the present results, a previous study [2] indicated that an increase in I_s changes the dynamical state of the ghostbursting model, e.g., (1) from a quiescent state to a repetitive spiking state to a bursting state when κ takes a large value but (2) from a quiescent state to a bursting state when the value κ takes is small value. In addition, taking the previous [2] and the present results into account reveals that when we inject the current, whether I_s or I_d, into the ghostbursting model, the increase in κ induces an increase in the threshold to the injected current for the transition from a repetitive spiking state to a bursting state. Interestingly, however, the results from the previous [2] and present studies show not only quantitative but also qualitative differences between I_s and I_d in their effect on the ghostbursting model. The quantitative difference is that when κ is between 0.30 and 0.40, the I_d threshold for the transition from a quiescent state to a repetitive or bursting state is smaller than the I_s threshold for the same transition. The qualitative difference is that an increase in κ induces an *increase* in the I_d threshold for the transition from a quiescent state to a repetitive or bursting state, whereas it induces a *decrease* in the I_s threshold for that transition. In conclusion, the present study, in combination with the previous one, contributes to an in-depth understanding of the similarities and differences between the effects of I_s and I_d on the ghostbursting model.

κ	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6
0.40	×	×	×	×	×	×	○	○	○	○	○	○	○	○	●
0.38	×	×	×	×	×	○	○	○	○	○	○	●	●	●	●
0.36	×	×	×	×	○	○	○	○	●	●	●	●	●	●	●
0.34	×	×	×	×	○	○	●	●	●	●	●	●	●	●	●
0.32	×	×	×	●	●	●	●	●	●	●	●	●	●	●	●
0.30	×	×	×	●	●	●	●	●	●	●	●	●	●	●	●
0.28	×	×	●	●	●	●	●	●	●	●	●	●	●	●	●
0.26	×	×	●	●	●	●	●	●	●	●	●	●	●	●	●
0.24	×	●	●	●	●	●	●	●	●	●	●	●	●	●	●

I_d (μA/cm^2)

×: quiescence, ○: spiking, ●: bursting

Figure 1. The dependence of the dynamical state of the ghostbursting model on I_d and κ. × indicates a quiescent state, ○ indicates a repetitive spiking state, and ● indicates a bursting state.

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References

[1] B. Doiron, C. Laing, A. Longtin and L. Maler, Ghostbursting: a novel neuronal burst mechanism, *Journal of Computational Neuroscience*, **12** (2002), 5-25. https://doi.org/10.1023/a:1014921628797

[2] C. Laing, B. Doiron, A. Longtin and L. Maler, Ghostbursting: the effects of dendrites on spike patterns, *Neurocomputing*, **44-46** (2002), 127-132. https://doi.org/10.1016/s0925-2312(02)00373-9

[3] A. Kepecs and X.J. Wang, Analysis of complex bursting in cortical pyramidal neuron models, *Neurocomputing*, **32-33** (2000), 181-187. https://doi.org/10.1016/s0925-2312(00)00162-4

[4] A. Kepecs, X.J. Wang and J. Lisman, Bursting neurons signal input slope, *Journal of Neuroscience*, **22** (2002), 9053-9062.

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