Abstract

In this paper the statements of Classical Mechanics are given in terms of the measurability notion introduced in previous works of the author.

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1 Introduction

This article is the continuation of the previous works of the author [1]–[8], the first of which [1] was published in autumn, 2014. The main idea of these works is as follows. At the present time physics is using (not without success) the mathematical apparatus based on the infinitesimal space-time variations (increments)

\[ dt, dx_i, i = 1, ..., 3 \]  \hspace{1cm} (1)

This mathematical apparatus comes from mathematical analysis [9], calculus of variations [10] and classical mechanics [11],[12]. Continuous space-time forms the base thereof.
The article [6] shows that while going over to the quantum theory at natural assumptions mentioned in [8] Principle of Bounded Space-Time Variations (Increments) the notion of continuous space-time becomes empty. And this is related to the fact that measurement procedure and Heisenberg’s Uncertainty Principle (HUP) [13] play a fundamental role in the quantum theory.

If Principle of Bounded Space-Time Variations (Increments) is correct, minimal length $l_{\text{min}}$ and time $t_{\text{min}} = l_{\text{min}}/c$ appear in the nature, (where $c$ is light speed). Then, based on $l_{\text{min}}$ and $t_{\text{min}}$ definitions of measurability and measurable quantities may be correctly input in theory. Some examples show, although in this case it becomes discrete, but in low energies, $E$, far from Planck $E \ll E_P$, it is close to the initial theory in continuous space-time. Real discreteness of the theory is manifested only at high energies $E$ close to Planck $E \approx E_P$ [1],[6],[8].

The main objective (hypothesis) of the author is as follows [6],[2],[8]:

*It is possible to correctly construct the quantum theory and gravity as discrete theories in terms of measurable quantities.*

The word correctness in this case means the following:

**I1.** At low energies these theories must, to a high accuracy, represent the results of the corresponding continuous theories.

**I2.** This theories should not have the problems of transition from low to high energies and vice versa and, specifically, the ultraviolet (UV) and infrared (IR) divergences problem.

In this work a preliminary step is made on the way to the above-mentioned objective:

Based on measurable quantities the construction of Classical Mechanics is given.

As the mathematical apparatus based on the use of infinitesimal space-time variations (increments) (1) for Classical Mechanics is absolutely adequate, then the main objectives of this work are as follows:

**I3.** To show how in the natural passage to the limit measurable quantities transform into the infinitesimal space-time variations (1) and fundamental ingredients of Classical Mechanics.

**I4.** To improve methods, and to make more precise and generalize main definitions and formulae from [1]–[8] to solve the problems set up in **I1.** and **I2.**
2 Previous Information and Some Specializing and Generalization

This section gives the necessary preliminary information from [1]–[8]. Part of previous results is presented in detail [6],[8], as without this it’s not possible to understand improvement and generalization of the basic determinations: Principle of Bounded Space-Time Variations (Increments), Definition 1, Definition 2 and formulae (for example formula (38)).

2.1 Minimal Length and Definition of Primary and Generalized Measurability

The present study is based on the Definition I. (it is improvement of the Supposition I in [6],[8]) and on the Supposition II. from [6],[8]:

Definition I. Let’s call as primarily measurable variation any small variation (increment) $\Delta x_\mu$ of any spatial coordinate $x_\mu$ of the arbitrary point $x_\mu, \mu = 1, ..., 3$ in some space-time system $R$ if it may be realized in the form of the uncertainty (standard deviation) $\Delta x_\mu$ when this coordinate is measured within the scope of Heisenberg’s Uncertainty Principle (HUP) [13]

$$\Delta x_\mu = \Delta x_\mu, \Delta x_\mu \simeq \frac{\hbar}{\Delta p_\mu}, \mu = 1,2,3$$ (2)

for some $\Delta p_\mu \neq 0$. 

Similarly, for $\mu = 0$ for pair “time-energy” ($t, E$), let’s call any small variation (increment) by primarily measurable variation in the value of time $\Delta t_0 = \Delta t_0$ if it may be realized in the form of the uncertainty (standard deviation) $\Delta x_0 = \Delta t$ and then

$$\Delta t = \Delta t, \Delta t \simeq \frac{\hbar}{\Delta E}$$ (3)

for some $\Delta E \neq 0$. Here HUP is given for the nonrelativistic case. In the relativistic case HUP has the distinctive features [14] which, however, are of no significance for the general formulation of Definition I., being associated only with particular alterations in the right-hand side of the second relation Equation (3).

It is clear that at low energies $E \ll E_P$ (momenta $P \ll P_{\text{pl}}$) Definition I. sets a lower bound for the primarily measurable variation $\Delta x_\mu$ of any space-time coordinate $x_\mu$.

At high energies $E$ (momenta $P$) this is not the case if $E$ ($P$) have no upper limit. But, according to the modern knowledge, $E$ ($P$) are bounded by some maximal quantities $E_{\text{max}}$, ($P_{\text{max}}$)

$$E \leq E_{\text{max}}, P \leq P_{\text{max}},$$ (4)
where in general $E_{\text{max}}, P_{\text{max}}$ may be on the order of Planck quantities $E_{\text{max}} \propto E_P, P_{\text{max}} \propto P_P$ and also may be the trans-Planck’s quantities.

In any case the quantities $P_{\text{max}}$ and $E_{\text{max}}$ lead to the introduction of the minimal length $l_{\text{min}}$ and of the minimal time $t_{\text{min}}$.

**Supposition II.** There is the minimal length $l_{\text{min}}$ as a minimal measurement unit for all primarily measurable variations having the dimension of length, whereas the minimal time $t_{\text{min}} = l_{\text{min}}/c$ as a minimal measurement unit for all quantities or primarily measurable variations (increments) having the dimension of time, where $c$ is the speed of light.

$l_{\text{min}}$ and $t_{\text{min}}$ are naturally introduced as $\Delta x_{\mu}, \mu = 1, 2, 3$ and $\Delta t$ in Equations (2) and (3) for $\Delta p_\mu = P_{\text{max}}$ and $\Delta E = E_{\text{max}}$.

For definiteness, we consider that $E_{\text{max}}$ and $P_{\text{max}}$ are the quantities on the order of the Planck quantities, then $l_{\text{min}}$ and $t_{\text{min}}$ are also on the order of Planck quantities $l_{\text{min}} \propto l_P, t_{\text{min}} \propto t_P$.

**Definition I.** and **Supposition II.** are quite natural in the sense that there are no physical principles with which they are inconsistent.

The combination of **Definition I.** and **Supposition II.** will be called the Principle of Bounded Primarily Measurable Space-Time Variations (Increments) or for short Principle of Bounded Space-Time Variations (Increments) with abbreviation (PBSTV).

Then, since in fact PBSTV introduce the minimal length $l_{\text{min}}$, instead of HUP, we can consider its widely known high-energy generalization—the Generalized Uncertainty Principle (GUP) that naturally leads to the minimal length $l_{\text{min}}$ [15]–[26]:

$$\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \frac{l_P^2}{\hbar} \Delta p .$$

(5)

Here $\alpha'$ is the model-dependent dimensionless numerical factor and $l_P$ is the Planckian length. As Equation (5) is a quadratic inequality, then it naturally leads to the minimal length $l_{\text{min}} = \xi l_P = 2\sqrt{\alpha' l_P}$.

As the minimal unit of measurement $l_{\text{min}}$ is available for all the quantities $L$ having the dimensions of length, the “Integrality Condition” (IC) is the case

$$L = N_L l_{\text{min}},$$

(6)

where $N_L > 0$ is an integer number.

In a like manner the same “Integrality Condition” (IC) is the case for all the quantities $t$ having the dimensions of time. And similar to Equation (6), we get the for any time $t$:

$$t \equiv t(N_t) = N_t t_{\text{min}},$$

(7)

Due to (6), we have

$$\Delta x = N_{\Delta x} l_{\text{min}}.$$

(8)
Then the transition from high to low energies in GUP, i.e. \((GUP, \Delta p \to 0) = (HUP)\), is nothing else but

\[ (N_{\Delta x} \approx 1) \to (N_{\Delta x} \gg 1). \] (9)

Substituting (8) into (5) and making the necessary calculations, we can see that in the general case

\[ \Delta p \equiv \Delta p_{N_{\Delta x}} = \frac{\hbar}{(N_{\Delta x} - \frac{1}{4N_{\Delta x}})l_{min}}. \] (10)

Whereas at low energies \(E \ll E_P\)

\[ \Delta p \equiv \Delta p_{N_{\Delta x}} = \frac{\hbar}{N_{\Delta x}l_{min}}. \] (11)

At the same time, for the corresponding energy \(E\) we get

\[ \Delta E \equiv \Delta E(N_t) = \frac{\hbar}{(N_t - \frac{1}{4N_t})l_{min}} \] (12)

or for low energies

\[ \Delta E \equiv \Delta E(N_t) = \frac{\hbar}{N_tl_{min}}. \] (13)

In the relativistic case the formulae corresponding to (18),(12) have been derived in [2],[6].

Note that the above-mentioned formulae may be conveniently rewritten in terms of \(l_{min}\) with the use of the deformation parameter \(\alpha_a\) [6]. This parameter has been introduced earlier in the papers [27]–[34] as a deformation parameter (in terms of paper [35]) on going from the canonical quantum mechanics to the quantum mechanics at Planck’s scales (early Universe) that is considered to be the quantum mechanics with the minimal length (QMML):

\[ \alpha_a = l_{min}^2/a^2, \] (14)

where \(a\) is the measuring scale.

Actually, with the equality \((\Delta p\Delta x = \hbar)\) Equation 5 is of the form

\[ \Delta x = \frac{\hbar}{\Delta p} + \frac{\alpha_{\Delta x}}{4} \Delta x. \] (15)

In this case due to Equations (6), (9) and (15) takes the following form:

\[ N_{\Delta x}l_{min} = \frac{\hbar}{\Delta p} + \frac{1}{4N_{\Delta x}}l_{min} \] (16)
or
\[(N\Delta x - \frac{1}{4N\Delta x})l_{min} = \frac{\hbar}{\Delta p}.\]  
(17)

That is
\[\Delta p = \frac{\hbar}{(N\Delta x - \frac{1}{4N\Delta x})l_{min}}.\]  
(18)

From Equations (16)–(18) it is clear that HUP Equation (2) appears to a high accuracy in the limit \(N\Delta x \gg 1\) in conformity with Equation 9.

It is easily seen that the parameter \(\alpha_a\) from Equation (14) is discrete as it is nothing else but
\[\alpha_a = \frac{l^2_{min}}{a^2} = \frac{l^2_{min}}{N^2a^2_{min}} = \frac{1}{N^2}.\]  
(19)

At the same time, from Equation (19) it is evident that \(\alpha_a\) is irregularly discrete.

It is clear that from Equation (18) at low energies \(|N\Delta x| \gg 1\), up to a constant
\[\frac{\hbar^2}{l^2_{min}} = \frac{\hbar c^3}{4\alpha'G}\]  
(20)
we have
\[\alpha_{\Delta x} = (\Delta p)^2, (i.e.\alpha_{\Delta x} \propto (\Delta p)^2).\]  
(21)

**Definition 1 (Primary or Elementary Measurability.)**

(1) In accordance with the PBSTV let us define the quantity having the dimensions of length \(L\) or time \(t\) as **primarily (or elementarily) measurable**, when it satisfies the relation Equation (6) (and respectively Equation (7)).

(2) Let us define any physical quantity **primarily (or elementarily) measurable**, when its value is consistent with points (1) of this Definition.

However, physical quantities complying with **Definition 1** won’t be enough for the research of physical systems.

Indeed, such a variable as
\[\alpha_{Nl_{min}}(Nl_{min}) = l_{min}/N,\]  
(22)
(where \(\alpha_{Nl_{min}}\) is taken from formula (19) at \(a = Nl_{min}\)), is fully expressed in terms only **Primarily Measurable Quantities** of **Definition 1** and that’s why it may appear at any stage of calculations, but apparently doesn’t comply with **Definition 1**. That’s why it’s necessary to introduce the following definition generalizing **Definition 1**:

**Definition 2. Generalized Measurability**

We shall call any physical quantity as **generalized-measurable** or for simplicity **measurable** if any of its values may be obtained in terms of **Primarily**
Measurable Quantities of Definition 1.

In what follows for simplicity we will use the term Measurability instead of Generalized Measurability.

It’s evident that any primarily measurable quantity (PMQ) is measurable. Generally speaking, the contrary is not correct, as indicated by formula (22).

Naturally, of course that, a minimal possible primarily measurable change of length is $l_{\text{min}}$. It corresponds to some maximal value of the energy $E_{\text{max}}$ or momentum $P_{\text{max}}$. If $l_{\text{min}} \propto l_P$, then $E_{\text{max}} \propto E_P, P_{\text{max}} \propto P_P$, where $P_{\text{max}} \propto P_P$, where $P_P$ is where the Planck momentum. Then denoting in nonrelativistic case with $\Delta_p(w)$ a minimal measurable change every spatial coordinate $w$ corresponding to the energy $E$ we obtain

$$\Delta_{P_{\text{max}}}(w) = \Delta_{E_{\text{max}}}(w) = l_{\text{min}}. \quad (23)$$

Evidently, for lower energies (momenta) the corresponding values of $\Delta_p(w)$ are higher and, as the quantities having the dimensions of length are quantized Equation (6), for $p \equiv p(N_p) < p_{\text{max}}, \Delta_p(w)$ is transformed to

$$|\Delta_{p(N_p)}(w)| = |N_p|l_{\text{min}}. \quad (24)$$

where $|N_p| > 1$ is an integer number so that we have

$$|N_p - \frac{1}{4N_p}|l_{\text{min}} = \frac{\hbar}{|p(N_p)|}. \quad (25)$$

In the relativistic case the Equation (23) holds, whereas Equations (24) and (25) for $E \equiv E(N_E) < E_{\text{max}}$ are replaced by

$$|\Delta_{E(N_E)}(w)| = |N_E|l_{\text{min}}, \quad (26)$$

where $|N_E| > 1$ is an integer.

Next we assume that at high energies $E \propto E_P$ there is a possibility only for the nonrelativistic case or ultrarelativistic case.

Then for the ultrarelativistic case, formula (25) takes the form [6]:

$$|N_E - \frac{1}{4N_E}|l_{\text{min}} = \frac{\hbar c}{E(N_E)} = \frac{\hbar}{|p(N_p)|}, \quad (27)$$

where $N_E = N_p$.

In the relativistic case at low energies we have

$$E \ll E_{\text{max}} \propto E_P. \quad (28)$$
and formula (24) is of the form
\[ |\triangle E(N_E)(w)| = |N_E| |l_{\min} = \frac{\hbar c}{E(N_E)}, |N_E| \gg 1 - \text{integer}. \quad (29) \]

In the nonrelativistic case at low energies Equation (28) due to Equation (25) we get
\[ |\triangle p(N_p)(w)| = |N_p| |l_{\min} = \frac{\hbar}{|p(N_p)|}, |N_p| \gg 1 - \text{integer}. \quad (30) \]

In a similar way for the time coordinate \( t \), by virtue of Equations (7)–(13), at the same conditions we have similar Equations (23)–(25)
\[ \triangle_{E_{\max}}(t) = t_{\min}. \quad (31) \]

For \( E \equiv E(N_t) < E_{\max} \)
\[ |\triangle E(N_t)(t)| = |N_t| |t_{\min}, \quad (32) \]
where \( |N_E| > 1 \) is an integer, so that we obtain
\[ |N_t - \frac{1}{4N_t}| |t_{\min} = \frac{\hbar c}{E(N_t)}. \quad (33) \]

In the relativistic case at low energies
\[ E \ll E_{\max} \propto E_P, \quad (34) \]
equation (24) takes the form [6]:
\[ |\triangle E(N_t)(w)| = |N_t| |l_{\min} = \frac{\hbar c}{E(N_t)}, |N_t| \gg 1 - \text{integer}. \quad (35) \]

We shall make two important Commentaries:

Comment 2.1.
What’s the main difference between Definition 1 and Definition 2?

2.1.1. Definition 1 defines variables which may be obtained as a result of an immediate experiment.

2.1.2. Definition 2 defines the variables which may be calculated based on primarily measurable quantities, i.e. based on the data obtained in previous clause 2.1.1.
Comment 2.2.
It’s evident that HUP-derived (2) $\Delta p_i \doteq \Delta p_{i,HUP}; i = 1, ..., 3$ are primarily measurable quantities:

$$\Delta p_i \simeq \frac{\hbar}{\Delta x_i} = \frac{\hbar}{N_{\Delta x_i} l_{\text{min}}} \quad (36)$$

However, variables $\Delta p_i \doteq \Delta p_{i,GUP}$ obtained from GUP (5) and defined by formula (18) are already obviously not the same, but only measurable quantities.

From formulae (20) and (21) follows that in case of correctness of HUP (2) i.e. in low energies $E \ll E_{\text{max}} \propto E_P$, in notations of formulae (24)–(35)

$$\alpha_{N_p l_{\text{min}}} (HUP) \doteq \alpha_{\Delta x} = p(N_p)^2 l_{\text{min}}^2 \frac{\hbar^2}{\bar{\hbar}^2} = \frac{1}{N_p^2} \quad (37)$$

where $\Delta x = N_p l_{\text{min}}$ and $p(N_p)$ is calculated from formula (30).

However, in high energies $E \approx E_P$, HUP is replaced with GUP, primarily measurable quantity $p(N_p)$ from formula (30) is replaced with generalized measurable quantity $\Delta p_i \doteq \Delta p_{i,GUP}$ from formula (25).

Then $\alpha_{N_p l_{\text{min}}} (HUP)$ may be replaced with $\alpha_{N_p l_{\text{min}}} (GUP)$:

$$\alpha_{N_p l_{\text{min}}} (GUP) = p(N_p, GUP)^2 l_{\text{min}}^2 \frac{\hbar^2}{\bar{\hbar}^2} =$$

$$= \frac{l_{\text{min}}^2}{(N_p - \frac{1}{4N_p})^2} \frac{l_{\text{min}}^2}{(N_p - \frac{1}{4N_p})^2} = \frac{1}{(N_p - \frac{1}{4N_p})^2} \quad (38)$$

When going over from high energies $E \approx E_P$ to low energies $E \ll E_P$ we have:

$$\alpha_{N_p l_{\text{min}}} (GUP) \overset{|N_p| \approx 1}{\longrightarrow} (|N_p| \gg 1) \alpha_{N_p l_{\text{min}}} (HUP) \quad (39)$$

In what follows all the considerations are given in terms of measurable quantities in the sense of Definition 2 given in this Section. Of course, this apply and to variations of space-time coordinates.

2.2 Space-Time Lattice of Primarily Measurable Quantities and Dual Lattice

For convenience, we denote the minimal length $l_{\text{min}} \neq 0$ by $\ell$ and $t_{\text{min}} \neq 0$ by $\tau = \ell/c$.

So, provided the minimal length $\ell$ exists, two lattices are naturally arising.

I. Lattice of the space-time variation—$\text{Lat}_{S-T}$ representing, to within the known multiplicative constants, for sets of nonzero integers $N_w \neq 0$ and $N_t \neq 0$
in corresponding formulae from the set Equations 24 and (35) for each of the three space variables \( w = x; y; z \) and the time variable \( t \)

\[
\text{Lat}_{S-T} \doteq (N_w \ell, N_t \tau).
\]  

(40)

Which restrictions should be initially imposed on these sets of nonzero integers?

It is clear that in every such set all the elements \((N_w \ell, N_t \tau)\) should be sufficiently “close”, because otherwise, for one and the same space-time point, variations in the values of its different coordinates are associated with principally different values of the energy \( E \) which are “far” from each other.

Note that the words “close” and “far” will be elucidated further in this text.

Thus, at the admittedly low energies (Low Energies) \( E \ll E_{max} \propto E_P \) the low-energy part (sublattice) \( \text{Lat}_{S-T}[LE] \) of \( \text{Lat}_{S-T} \) is as follows:

\[
\text{Lat}_{S-T}[LE] = (N_w \ell, N_t \tau); |N_x| \gg 1, |N_y| \gg 1, |N_z| \gg 1, |N_t| \gg 1.
\]  

(41)

At high energies (High Energies) \( E \to E_{max} \propto E_P \) we, on the contrary, have the sublattice \( \text{Lat}_{S-T}[HE] \) of \( \text{Lat}_{S-T} \)

\[
\text{Lat}_{S-T}[HE] = (N_w \ell, N_t \tau); |N_x| \approx 1, |N_y| \approx 1, |N_z| \approx 1, |N_t| \approx 1.
\]  

(42)

We will call lattice \( \text{Lat}_{S-T} \) (40) as primary (or primitive) lattice of the space-time variation.

II. Next let us define the lattice momenta-energies variation \( \text{Lat}_{P-E} \) as a set to obtain \((p_x(N_x,p), p_y(N_y,p), p_z(N_z,p), E(N_t))\) in the nonrelativistic and ultra-relativistic cases for all energies, and as a set to obtain \((E_x(N_x,E), E_y(N_y,E), E_z(N_z,E), E(N_t))\) in the relativistic (but not ultrarelativistic) case for low energies \( E \ll E_P \), where all the components of the above sets conform to the space coordinates \((x, y, z)\) and time coordinate \( t \) and are given by corresponding formulae (23)–(35) from the previous Section.

Note that, because of the suggestion made after formula Equation 28 in the previous Section, we can state that the foregoing sets exhaust all the collections of momentums and energies possible for the lattice \( \text{Lat}_{S-T} \).

From this it is inferred that, in analogy with point I of this Section, within the known multiplicative constants, we have

\[
\text{Lat}_{P-E} \doteq \left( \frac{1}{N_w - \frac{1}{1/4N_w}}, \frac{1}{N_t - \frac{1}{1/4N_t}} \right),
\]  

(43)

where \( N_w \neq 0, N_t \neq 0 \) are integer numbers from Equation 40. Similar to Equation 41, we obtain the low-energy (Low Energy) part or the sublattice
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\[ \text{Lat}_{P-E}[LE] \text{ of } \text{Lat}_{P-E} \]

\[ \text{Lat}_{P-E}[LE] \approx \left( \frac{1}{N_w}, \frac{1}{N_t} \right), |N_w| \gg 1, |N_t| \gg 1. \quad (44) \]

In accordance with Equation 42, the high-energy (High Energy) part (sublattice) \( \text{Lat}_{P-E}[HE] \) of \( \text{Lat}_{P-E} \) takes the form

\[ \text{Lat}_{P-E}[HE] \approx \left( \frac{1}{N_w - \frac{1}{4N_w}}, \frac{1}{N_t - \frac{1}{4N_t}} \right), |N_w| \rightarrow 1, |N_t| \rightarrow 1. \quad (45) \]

It is important to note the following.

In the low-energy sublattice \( \text{Lat}_{P-E}[LE] \) all elements are varying very smoothly enabling the approximation of a continuous theory.

3 Classical Mechanics in “Measurable Format”

3.1 Preliminary Information

We will preserve the lattice \( \text{Lat}_{P-E} \), but primary lattice \( \text{Lat}_{S-T} \) will be replaced with “\( \alpha \) – lattice”, measurable space-time quantities, which will be denoted by \( \text{Lat}^\alpha_{S-T} \):

\[ \text{Lat}^\alpha_{S-T} \doteq (\alpha_{N_w \ell} N_w, \alpha_{N_t \tau} N_t) = \left( \frac{\ell}{N_w}, \frac{\tau}{N_t} \right). \quad (46) \]

In the last formula by the variable \( \alpha_{N_t \tau} \) we mean the parameter \( \alpha \) corresponding to the length \( (N_t \tau)c \):

\[ \alpha_{N_t \tau} \doteq \alpha_{(N_t \tau)c}. \quad (47) \]

As in this case low energies \( E \ll E_P \) are discussed, \( \alpha_{N_w \ell} \) in this formula is consistent with the corresponding parameter from formula (37):

\[ \alpha_{N_w \ell} = \alpha_{N_w \ell}(HUP) \quad (48) \]

As it was mentioned in the previous section, in the low-energy \( E \ll E_{max} \propto E_P \) all elements of sublattice \( \text{Lat}_{P-E}[LE] \) are varying very smoothly enabling the approximation of a continuous theory.

It is similar to the low-energy part of the \( \text{Lat}^\alpha_{S-T}[LE] \) of lattice \( \text{Lat}^\alpha_{S-T} \) will vary very smoothly:

\[ \text{Lat}^\alpha_{S-T}[LE] = \left( \frac{\ell}{N_w}, \frac{\tau}{N_t} \right); |N_x| \gg 1, |N_y| \gg 1, |N_z| \gg 1, |N_t| \gg 1. \quad (49) \]

In Section 5 of [6] three following cases were selected:
(a) “Quantum Consideration, Low Energies”:

\[ 1 \ll |N_w| \leq \tilde{N}; \]

(b) “Quantum Consideration, High Energies”:

\[ |N_w| \approx 1; \]

(c) “Classical Picture”:

\[ 1 \ll \tilde{N} \ll |N_w|. \]

Here \( \tilde{N} \) is a cutoff parameter, defined by the current task [6]. It is assumed that there is a correct transition to the infinite limit in “Classical Picture” (c)

\[ |N_w| \to \infty, |N_t| \to \infty \quad (50) \]

That’s why, if for three space coordinates \( x_i; i = 1, 2, 3 \) we introduce the following notation:

\[ \Delta(x_i) = \tilde{\Delta} = \Delta[\alpha N_{\Delta x_i}] = \alpha N_{\Delta x_i} \ell(N_{\Delta x_i} \ell) = \ell/N_{\Delta x_i}; \]

\[ \Delta[F(x_i)]/\Delta(x_i) = F(x_i + \Delta(x_i)) - F(x_i) \Delta(x_i), \]

then it’s evident that

\[ \lim_{|N_{\Delta x_i}| \to \infty} \Delta[F(x_i)]/\Delta(x_i) = \lim_{\Delta(x_i) \to 0} \Delta[F(x_i)]/\Delta(x_i) = \partial F/\partial x_i. \quad (51) \]

Respectively, for time \( x_0 = t \) we have:

\[ \Delta(t) = \tilde{\Delta} = \tilde{\Delta}[\alpha N_{\Delta t}] = \alpha N_{\Delta t} \tau(N_{\Delta t} \tau) = \tau/N_{\Delta t}; \]

\[ \Delta[F(t)]/\Delta(t) = F(t + \Delta(t)) - F(t) \Delta(t), \]

then

\[ \lim_{|N_{\Delta t}| \to \infty} \Delta[F(t)]/\Delta(t) = \lim_{\Delta(t) \to 0} \Delta[F(t)]/\Delta(t) = dF/dt. \quad (52) \]

We shall designate for momenta \( p_i; i = 1, 2, 3 \)

\[ \Delta p_i = \frac{\hbar}{N_{\Delta x_i} \ell}; \]

\[ \Delta p_i F(p_i) = \frac{F(p_i + \Delta p_i) - F(p_i)}{\Delta p_i} = \frac{F(p_i + \hbar/N_{\Delta x_i} \ell) - F(p_i)}{\hbar/N_{\Delta x_i} \ell}. \quad (53) \]
From where similarly (52) we get

\[
\lim_{|N_{\Delta x_i}| \to \infty} \frac{F(p_i + \Delta p_i) - F(p_i)}{\Delta p_i} = \lim_{|N_{\Delta x_i}| \to \infty} \frac{F(p_i + \frac{\hbar}{N_{\Delta x_i} \ell}) - F(p_i)}{\frac{\hbar}{N_{\Delta x_i} \ell}} = \\
= \lim_{\Delta p_i \to 0} \frac{F(p_i + \Delta p_i) - F(p_i)}{\Delta p_i} = \frac{\partial F}{\partial p_i}. \tag{56}
\]

Therefore, in low energies \(E \ll E_P\), i.e. at \(|N_{\Delta x_i}| \gg 1; i = 0, \ldots, 3\) in passages to the limit (52),(54),(56) it's possible to obtain known partial derivatives like in case of continuous space-time.

**Definition Cl1.**

Let some quantity \(\Xi\) depend on integers \(N_{\Delta x_i}, N_{\Delta t}\), at all values of \(N_{\Delta x_i}, N_{\Delta t}\) is measurable and formula (50) is correct, i.e. we have “Classical Picture” (c). Then, if there are passages to the limit

\[
\lim_{|N_{\Delta x_i}| \to \infty} \Xi(N_{\Delta x_i}) = \Xi_x; \lim_{|N_{\Delta t}| \to \infty} \Xi(N_{\Delta t}) = \Xi_t,
\]

then the respective limits \(\Xi_x, \Xi_t\) shall be also called measurable quantities.

Particularly, if \(F\) in formulae (51)–(56) is a measurable quantity, then from Definition 2 follows directly that the values

\[
\frac{\Delta F(x_i)}{\Delta (x_i)}, \frac{\Delta F(t)}{\Delta (t)}, \frac{\Delta F(p_i)}{\Delta p_i}
\]

are also measurable quantities. Then, according to this definition, the same are the quantities \(\frac{\partial F}{\partial x_i}, \frac{\partial F}{\partial p_i}, \frac{dF}{dt}\) in formulae (52),(54),(56).

**Commentary to Definition Cl1.**

By virtue of (50) it’s evident that Definition.Cl1 is applicable only to case (c) above (Classical Picture) and not applicable to cases (a) and (b),(Quantum Consideration, Low Energies) and (Quantum Consideration, High Energies) respectively.

We shall make two notes

**Remark 3.1**

There is a significant difference between formulae (52),(54) on the one hand and formula (56) on the other hand.

Limits in (52) and in (54) may be obtained also when going over to continuous space-time

\[
\ell \to 0; \tau \to 0; \lim_{\ell \to 0} \frac{\Delta F(x_i)}{\Delta (x_i)} = \frac{\partial F}{\partial x_i}; \lim_{\tau \to 0} \frac{\Delta F(t)}{\Delta (t)} = \frac{dF}{dt}. \tag{58}
\]
But in formula (56) passage to the limit at $\ell \to 0$ is not possible, as it leads to an infinitely great denominator.

**Remark 3.2** The above-mentioned calculations show that in this offered discrete approach by virtue of **Definition Cl1.** in low energies in classical consideration it’s possible to obtain all the main attributes of the continued theory, particularly, for any respective function $F$ the quantities $\frac{\partial F}{\partial x_i}$, $\frac{dF}{dt}$, $\frac{\partial F}{\partial p_i}$ are defined correctly.

Here it’s not necessary to observe the condition $\hbar \to 0$, i.e. $\hbar \neq 0$ remains also in the classical situation and is suppressed due to the passage to the limit $|N_{\Delta x_i}| \to \infty$.

### 3.2 Lagrangian Formalism and Principle of Least Action in Terms of Measurable Quantities

By virtue of **Definition Cl1.** and formulae (51)–(56), as well as some of their generalizations, it’s possible to show that all the main provisions of classical mechanics both in Lagrangian and Hamiltonian formalism remain correct in terms of measurable quantity, in the presence of quite natural additional assumptions.

Hereinafter we will use standard terminology of classical mechanics [11],[12]. Let there be a Lagrangian $L = L(q, \dot{q}, t)$, where $q$ are generalized coordinates; $\dot{q}$ are generalized speeds and $t$ is time. However, in the discussed case $t$ changes discretely, according to the formulae above.

**Definition Cl2.** We shall call $L$ as a measurable analogue and denote by $L_{\text{meas}}(q, \dot{q}, t)$, the quantity satisfying the following properties:

**Cl1.1.** Time $t$, and the generalized coordinate $q$ included into $L_{\text{meas}}(q, \dot{q}, t)$ are primarily measurable quantities in terms of **Definition 1**

**Cl1.2.** The quantity $\dot{q}$ is obtained from formulae (53),(54), (where $F(t) = q(t)$) and that’s why according to **Definition Cl1.**, it’s a measurable quantity.

**Cl1.3.** In case of fulfillment of conditions **Cl1.1.** and **Cl1.2.**

$$L_{\text{meas}}(q, \dot{q}, t) = L(q, \dot{q}, t)$$  \hfill (59)

Hereinafter we will assume that the Lagrangian $L(q, \dot{q}, t)$ is measurable, i.e.

$$L(q, \dot{q}, t) = L_{\text{meas}}(q, \dot{q}, t)$$  \hfill (60)

It’s necessary to make an important note:

**Remark 3.3.**
In formulae (51)–(56) Cartesian coordinates $x_i$ were used and respective pulses $p_i$ in terms of measurable quantities. However, it’s not difficult to obtain analogue (51), (52), (55), (56) for measurable generalized coordinates $q$ and speeds $\dot{q}$.

Indeed, let $\Delta q$ and $\Delta \dot{q}$ be measurable small increments $q$ and $\dot{q}$ respectively. We shall introduce the following notations for the measurable value of time $t_i$:

$$\Delta q(t_i) = \alpha_i \Delta q(t_i); \Delta \dot{q}(t_i) = \alpha_i \Delta \dot{q}(t_i),$$

(61)

where $\alpha_i = \alpha_{N_i \tau}$ from formula (47).

It’s clear that as $\Delta q$ and $\Delta \dot{q}$ are measurable small increments of $q$ and $\dot{q}$ respectively, then $\Delta q$ and $\Delta \dot{q}$ will be the same, and as we are discussing low energies and, consequently, for each $t_i$ from formula (61) $t_i = N_i \tau, |N_i| \gg 1$, then $|\Delta q(t_i)| \ll |\Delta t(t_i)|, |\Delta \dot{q}(t_i)| \ll |\Delta \dot{q}(t_i)|$.

In formula (61) measurable small increments are set with the help of the corresponding parameter $\alpha$ by actual generalization for the case $\Delta q, \Delta \dot{q}$ of “$\alpha$ – lattice” measurable space-time quantities $\text{Lat}_3^{S-T}$ (46).

However, it’s possible to act in a more simple way: as under the definition $q$ and $\dot{q}$ are measurable quantities, then $\Delta q(t_i) = \frac{1}{N_i} q(t_i) = (\frac{\ell}{\tau} p_{N_i}) q(t_i)$ as well as $\Delta \dot{q}(t_i) = \frac{1}{N_i} \dot{q}(t_i) = (\frac{\ell}{\tau} p_{N_i}) \dot{q}(t_i)$ are measurable small increments $q$ and $\dot{q}$ at $|N_i| \gg 1$, which go to zero, at $|N_i| \to \infty$.

Next, we shall define

$$\frac{\Delta F(q(t_i))}{\Delta q(t_i)} = \frac{F(q(t_i) + \Delta q(t_i)) - F(q(t_i))}{\Delta q(t_i)}$$

(62)

and, respectively,

$$\frac{\Delta F(\dot{q}(t_i))}{\Delta \dot{q}(t_i)} = \frac{F(\dot{q}(t_i) + \Delta \dot{q}(t_i)) - F(\dot{q}(t_i))}{\Delta \dot{q}(t_i)}$$

(63)

Then it’s evident that for the measurable function $F$ right parts (62) and (63) will also be measurable and according to Definition Cl1. it’s possible to obtain measurable limits:

$$\lim_{|N_i| \to \infty} \frac{\Delta F(q(t_i))}{\Delta q(t_i)} = \lim_{\Delta q(t_i) \to 0} \frac{\Delta F(q(t_i))}{\Delta q(t_i)} = \frac{\partial F}{\partial q};$$

$$\lim_{|N_i| \to \infty} \frac{\Delta F(\dot{q}(t_i))}{\Delta \dot{q}(t_i)} = \lim_{\Delta \dot{q}(t_i) \to 0} \frac{\Delta F(\dot{q}(t_i))}{\Delta \dot{q}(t_i)} = \frac{\partial F}{\partial \dot{q}}$$

(64)

As according to Definition Cl2. the time $t$ is a primarily measurable quantity we shall denote as follows

$$\bar{t} - \tilde{t} = \Delta t = N_{\Delta t \tau}$$

(65)
In this case it’s possible to define a measurable action as a sum:

\[
S_{\text{meas}, N\Delta t}(q, \dot{q}, t) = \sum_{1 \leq i \leq N, \hat{t} \leq t_i \leq \tilde{t}} L_{\text{meas}}(q(t_i), \dot{q}(t_i), t_i) \alpha_{N\Delta t}(N\Delta \tau) = \sum_{1 \leq i \leq N, \hat{t} \leq t_i \leq \tilde{t}} L_{\text{meas}}(q(t_i), \dot{q}(t_i), t_i) \frac{\tau}{N\Delta t},
\]

(66)

where \(L(q, \dot{q}, t)\) satisfies (60).

However, by virtue of Definition Cl1, in this particular case of classical mechanics the passage to the infinite limit is correct:

\[
S_{\text{meas}, N\Delta t}(q, \dot{q}, t) \bigg|_{N\Delta t \to \infty} \rightarrow S_{\text{meas}}(q, \dot{q}) = \int_{\hat{t}}^{\tilde{t}} L_{\text{meas}}(q, \dot{q}, t) \, dt
\]

(67)

Based on Definition Cl1, (67) may be rewritten as

\[
S_{\text{meas}, N\Delta t}(q, \dot{q}, t) \bigg|_{N\Delta t \to \infty} \rightarrow S_{\text{meas}}(q, \dot{q}) = \int_{\hat{t}}^{\tilde{t}} L_{\text{meas}}(q, \dot{q}, t) \, dt
\]

(68)

Next, quite a natural supposition will be taken:

**Supposition.Cl1.**

For each measurable quantity \(\kappa\) and quite large \(\Delta t\) (or the same for quite large \(|N\Delta t|\) and, naturally, for \(|N\Delta t| \to \infty\) there is a measurable variation of \(\delta \kappa\).

(Indeed, this is a very natural supposition. As \(q\) is a primary measurable, then \(q/N = \frac{P_N}{N}q\) is a measurable quantity and at quite large \(N\) it may be made arbitrary close to the measurable variation of \(\delta \kappa\).

Taking as \(\kappa\) a measurable quantity \(q\), according to Supposition.Cl1., we may obtain a measurable variation \(\delta q\). Considering Definition Cl1., we obtain

\[
\frac{d}{dt}(\delta q) = \delta \dot{q}
\]

(69)

is a measurable quantity as well.

Next, step by step we may obtain Principle of Least Action [11],[12] in terms of measurable quantities. For this we need to make sure that at each step of proof of this principle only measurable quantities appear.

Indeed, as \(S_{\text{meas}, N\Delta t}(q, \dot{q}, t)\) is a measurable quantity, then by virtue of measurability \(\delta q\) and \(\delta \dot{q}\), the sum \(S_{\text{meas}, N\Delta t}(q + \delta q, \dot{q} + \delta \dot{q}, t)\) will be also measurable. By virtue of Definition Cl1., using the passage to the limit (68), but already for \(S_{\text{meas}, N\Delta t}(q + \delta q, \dot{q} + \delta \dot{q}, t)\), we obtain measurable quantity
$S_{\text{meas}}(q + \delta q, \dot{q} + \delta \dot{q})$:

$$S_{\text{meas}, N\Delta t}(q + \delta q, \dot{q} + \delta \dot{q}, t) \xrightarrow{N\Delta t \to \infty} S_{\text{meas}}(q + \delta q, \dot{q} + \delta \dot{q}) = \int_{\tilde{t}}^{\hat{t}} L_{\text{meas}} \left( q + \delta q, \dot{q} + \delta \dot{q}, t \right) dt \quad (70)$$

From where it follows directly that the variation $\delta S_{\text{meas}}(q, \dot{q})$ is also measurable:

$$\delta S_{\text{meas}}(q, \dot{q}) = S_{\text{meas}}(q + \delta q, \dot{q} + \delta \dot{q}) - S_{\text{meas}}(q, \dot{q}) = \left[ \delta S(q, \dot{q}) \right]_{\text{meas}} \quad (71)$$

Equating the right part (71) to zero we obtain the equation in which all the components are measurable quantities:

$$\left[ \delta S(q, \dot{q}) \right]_{\text{meas}} = \delta \int_{\tilde{t}}^{\hat{t}} L_{\text{meas}} \left( q, \dot{q}, t \right) dt = \int_{\tilde{t}}^{\hat{t}} \left( \frac{\partial L_{\text{meas}}}{\partial q} \delta q + \frac{\partial L_{\text{meas}}}{\partial \dot{q}} \delta \dot{q} \right) dt = 0 \quad (72)$$

Indeed, $\partial L_{\text{meas}}/\partial q, \partial L_{\text{meas}}/\partial \dot{q}$ are measurable according to Remark 3.3. and respective formulae. $\delta q, \delta \dot{q}$ are measurable according to Supposition.Cl1., Definition Cl1. and formula (69).

That’s why using formula (69) and an integration by parts [11], which evidently does not destroy measurability we obtain the following from (72):

$$\delta S = \left[ \frac{\partial L_{\text{meas}}}{\partial \dot{q}} \delta \dot{q} \right]_{\tilde{t}}^{\hat{t}} + \int_{\tilde{t}}^{\hat{t}} \left( \frac{\partial L_{\text{meas}}}{\partial q} - \frac{d}{dt} \frac{\partial L_{\text{meas}}}{\partial \dot{q}} \right) \delta q dt = 0 \quad (73)$$

where as usually $q(\hat{t}) = q(\tilde{t}) = 0, \frac{d}{dt} \frac{\partial L_{\text{meas}}}{\partial \dot{q}}$ are measurable by virtue of Definition Cl1. and formula (54) at $F(t) = \frac{\partial L_{\text{meas}}}{\partial \dot{q}}$ and as it was already used in formulae (66), (67) in case of classical mechanics $dt$ is also a measurable quantity, as according to Definition Cl1. it appears within the limits for a measurable quantity

$$\tau/N\Delta t \xrightarrow{N\Delta t \to \infty} dt \quad (74)$$

From where the following representation follows Euler-Lagrange equations [11] in terms of only measurable quantities:

$$\frac{\partial L_{\text{meas}}}{\partial q} - \frac{d}{dt} \frac{\partial L_{\text{meas}}}{\partial \dot{q}} = 0,$$

$$\frac{d}{dt} \left( \frac{\partial L_{\text{meas}}}{\partial q_i} \right) - \frac{\partial L_{\text{meas}}}{\partial q_i} = 0, \quad (i = 1, 2, \ldots, s) \quad (75)$$
3.3 Hamiltonian Formalism and Measurability

Using the results of the previous Subsection it’s not difficult to obtain also Hamiltonian Formalism in terms of measurable quantities. As well as in the previous Subsection it’s necessary to make sure that at each step all members in respective formulae are measurable.

Indeed, using “measurable” Euler-Lagrange equations (75) it’s possible to introduce measurable generalized momenta and their time derivatives:

\[ p_{\text{meas}} = \frac{\partial L_{\text{meas}}}{\partial \dot{q}}; \quad \dot{p}_{\text{meas}} = \frac{\partial L_{\text{meas}}}{\partial q} \]

(76)

From where, using Legendre transformation [11], the following appears “measurable” Hamiltonian \( H_{\text{meas}} \):

\[ H_{\text{meas}}(q, p, t) = \sum_{i} \dot{q}_i (p_i)_{\text{meas}} - L_{\text{meas}}(q, \dot{q}, t) \]

(77)

Here we don’t put subscript meas for variables \( q, \dot{q} \), as they are measurable by virtue of Definition Cl2.

Total differential of left part (73) will be equal to:

\[ d[H_{\text{meas}}(q, p, t)] = \sum_{i} [\dot{q}_i d[(p_i)_{\text{meas}}] - (p_i)_{\text{meas}} dq_i] - \frac{\partial L_{\text{meas}}}{\partial t} dt \]

(78)

In right part (78) the member \( \frac{\partial L_{\text{meas}}}{\partial t} dt \) will be measurable by virtue of Definition Cl1. and formulae (54) and (74). \( q_i, \dot{q}_i \) are measurable by virtue of Definition Cl1., Definition Cl2., measurability \( (p_i)_{\text{meas}} \) and \( (p_i)_{\text{meas}} \) was obtained in (76). Finally, \( dq_i \) and \( d[(p_i)_{\text{meas}}] \) will be measurable according to Definition Cl1. and formulæ (61)–(64).

Therefore, right part (78) is a measurable quantity, that’s why also left part (78) is a measurable quantity. From where

\[ d[H_{\text{meas}}(q, p, t)] = d[H_{\text{meas}}(q, p, t)]_{\text{meas}} \]

(79)

From (79) and standard representation of total differential for \( H_{\text{meas}}(q, p, t) \), which will also evidently be a measurable quantity Canonical Hamilton’s Equations in terms of measurable quantities follow immediately:

\[ \frac{\partial H_{\text{meas}}}{\partial q_i} = -(p_i)_{\text{meas}}, \quad \frac{\partial H_{\text{meas}}}{\partial (p_i)_{\text{meas}}} = \dot{q}_i, \quad \frac{\partial H_{\text{meas}}}{\partial t} = -\frac{\partial L_{\text{meas}}}{\partial t}. \]

(80)

Next, it’s tacitly supposed that \( p = p_{\text{meas}} \). Then any function of canonical variables \( G(q, p, t) \) will be a measurable quantity, in that sense that any of its meanings may be obtained in terms of measurable set of variables \( G(q, p, t) \).
By virtue of the results obtained above, *Poisson bracket* \([,]_{PB}\) of two *measurable* functions \(G(q,p,t)\) and \(\Phi(q,p,t)\) \([11]\) will also be a *measurable* function:

\[
[\Phi, G]_{PB} = \sum_j \left( \frac{\partial \Phi}{\partial p_j} \frac{\partial G}{\partial q_j} - \frac{\partial \Phi}{\partial q_j} \frac{\partial G}{\partial p_j} \right)
\]  

(81)

Particularly, if \(\Phi(q,p,t) = H(q,p,t) = H_{meas}(q,p,t)\), we come to the basic equation of the Hamiltonian mechanics \([11]\), obtained in terms of *measurable* quantities:

\[
\frac{dG}{dt} = \frac{\partial G}{\partial t} + [H, G]_{PB}
\]

(82)

**Note.**
It’s evident that in this formalism *Canonical Hamilton’s Equations* in terms of *measurable* quantities (80) may be obtained from *Principle of Least Action*, if in **Definition Cl2.** we make a replacement \(L \to H, L_{meas} \to H_{meas}\) and add *measurable* generalized momentum \(p\).

### 4 Final Commentaries, Explanations and Conclusion

**F1.** Primary *measurable* the generalized coordinates \(q\) and *measurable* the generalized velocities \(\dot{q}\) from **Definition Cl2.** are standard quantities of classical mechanics \([11],[12]\), on which only one limitation is imposed: *Changes of all parameters, (naturally including time \(t\)), on which \(q\) and \(\dot{q}\) depend satisfy Definition 1 and Definition 2 respectively.*

The exception is the procedure to obtain \(\dot{q}\) by \(q\), as here \(q\) cannot be considered as a *primary measurable* quantity, but only a *measurable* quantity. This is discussed in details in clause **F4.**

**F2.** If the theory supposes the transition to the infinite limit (50), then this theory may be considered a *Classical Theory* and then **Definition Cl1.** is absolutely correct, as for limits \(\Xi_{x_i}\) and \(\Xi_t\) from (57) at quite large \(|N_{\Delta x_i}|, |N_{\Delta t}|\) it’s always possible to find *measurable* quantities \(\Xi(N_{\Delta x_i}), \Xi(N_{\Delta t})\) arbitrary close to \(\Xi_{x_i}\) and \(\Xi_t\). I.e. with high precision \(\Xi_{x_i}\) and \(\Xi_t\) may be replaced with primary *measurable* quantities \(\Xi(N_{\Delta x_i})\) and \(\Xi(N_{\Delta t})\).

**F3.** It may seem that “\(\alpha - lattice\)” \(\text{Lat}^{\alpha}_{S_{-T}}\) (formula (46)) is introduced in this work artificially. But in reality this is not true. It appears, but with “factor” 1/4 from equation (15) written in the form

\[
\Delta x = \frac{\hbar}{\Delta p} = \frac{1}{4} \alpha_{\Delta x}\Delta x.
\]

(83)
It’s evident that factor 1/4 in right part (83) is not significant in this case.

**F4.** Despite the fact that the generalized coordinate \( q \) from **Definition Cl2.** is initially a **primarily measurable quantity** in terms of **Definition 1**, “the speed of its variation in time”, i.e. \( \dot{q} \) already cannot be the same and is just a **measurable quantity** in terms of **Definition 2**. Moreover, for its definition, according to formulae (53),(54), (at \( F(t) = q(t) \)) the generalized coordinate \( q \) and time \( t \) should be also considered as **measurable quantities**. There is no contradiction here. If during definition of \( \dot{q} \) we considered \( q \) as a **primarily measurable quantity**, then in formula (53) at larger \( |\Delta t| \) and \( F(t) = q(t) \) we would obtain generally a discrete divergent row of values

\[
\frac{\Delta(t)}{\Delta(t)} = N_{\Delta(t)} \tau;
\]

where \( N_{\Delta t} = N_{\Delta(t)} \).

And then the limit (54), i.e. \( \dot{q} \) would not even exist.

**F5.** It’s clear that the transition to the limit (74) from a **measurable quantity** \( \tau/\Delta t \) to infinitesimal quantity \( dt \), which in case of **Classical Mechanics** by virtue of **Definition Cl1.** will also be a **measurable quantity**, may be generalized as space variables and written as follows:

\[
\left( \frac{\tau}{N_{\Delta t}} = p_{N_{\Delta t}} \frac{\ell^2}{\hbar} \right)_{N_{\Delta t} \to \infty} dt,
\]

\[
\left( \frac{\ell}{N_{\Delta x}} = p_{N_{\Delta x}} \frac{\ell^2}{\hbar} \right)_{N_{\Delta x} \to \infty} dx,
\]

\[
\left( \frac{\ell}{N_{\Delta y}} = p_{N_{\Delta y}} \frac{\ell^2}{\hbar} \right)_{N_{\Delta y} \to \infty} dy,
\]

\[
\left( \frac{\ell}{N_{\Delta z}} = p_{N_{\Delta z}} \frac{\ell^2}{\hbar} \right)_{N_{\Delta z} \to \infty} dz.
\]

Left parts of all four limits given in formula (85), are **measurable quantities**, which depend on available energies. They will be necessary in the construction of the **Quantum Theory and Gravity** in terms of **measurable quantities**.

**F6. Remarks 3.1,3.2** and formula (56) show that in this formalism, as distinct from the standard case of continuous space-time it will be possible to keep \( \hbar \neq 0 \) during the passage from **Quantum Picture** to **Classical Picture**.

Therefore, summing up it should be stated that at some natural suppositions
Classical Mechanics may be correctly formulated in terms of measurable quantities of the [1]–[8] and present paper. The author will show in the following papers (material in progress) how to obtain in terms of measurable quantities the correct quantum field theory and measurable analogue (measurable pre-image) of metric in gravity. The results of [6], [8] and of the present paper will contribute to solution of this task.

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