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Induced Gravity and Topological Quantum Field Theory

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Abstract

We construct an induced gravity (pregeometry) where both the Newton constant and the cosmological constant appear as integration constants in solving field equations. By adding the kinetic terms of ghosts and antighosts, an action of the induced gravity is transformed to a topological field theory. Moreover, by solving field equations of the topological field theory in the FRW universe, we find an inflation solution. The present study might shed some light on a close relationship between the induced gravity and the topological quantum field theory.

1 Introduction

Induced gravity, or pregeometry, has a long history whose origin traces back to a very short paper by Sakharov in 1968 [1]. The basic idea is that gravity is not fundamental but might be induced by quantum fluctuations of matter fields. In particular, the Newton constant could be induced at the one-loop level although it is vanishing at the tree level.

In the framework of the induced gravity, the existence of the Lorentzian manifold is assumed *a priori*, but the dynamics of the geometry is not assumed and determined by radiative corrections of matter. Thus, the geometry is

regarded as a classical background and is not quantized unlike matter fields. Phrased in an equation, integrating over matter fluctuations at the one-loop level turns out to lead to an effective action

$$S_{IG} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(-2\Lambda + R + \cdots\right), \tag{1}$$

where dots denote the higher-derivative terms such as R^2. Even if there are some difficulties and subtleties in the induced gravity, the basic idea is extremely of interest in the sense that quantum gravity is now believed to be somehow an emergent phenomenon like the induced gravity [2].

There are a number of versions of the induced gravity thus far [3]-[10], [1] but the folklore of the induced gravity is to derive general relativity or its generalizations via quantum corrections of matter. However, there might be a logical possibility such that without relying on quantum fluctuations gravity could be generated at the classical level by solving field equations of a new type of induced gravity. For instance, in the pregeometrical approach of string field theory [12, 13], starting with only a purely cubic action, the dynamics of the geometry or the kinetic term involving a background metric, is indeed induced by solving field equations and picking up a classical solution where quantum corrections play no role. One of motivations in this article is to construct such an induced gravity within the framework of a local field theory.

In a recent work [14], we have constructed a new induced gravity where the Newton constant is derived by fixing a part of diffeomorphisms by which the full diffeomorphisms are broken down to the transverse diffeomorphisms (TDiff). However, it was difficult to derive the cosmological constant at the same time since there remain no available diffeomorphisms for introducing a gauge condition corresponding to the cosmological constant without violating the general covariance. In this paper, we will resolve this issue and derive the Einstein equations with the cosmological constant by beginning with a new induced gravity action.

It turns out that adding a sector of ghosts, we can transform the new induced gravity to a topological quantum field theory. Its moduli space is composed by constraints such that the curvature density and the volume element are determined in terms of a divergence of vector densities. One of these constraints is reminiscent of *unimodular gravity* where the determinant of the metric tensor is taken to be -1, that is, $\det g_{\mu\nu} = -1$ [15]-[27].

Moreover, beginning with the topological theory, we derive the whole set of field equations and find an inflation solution in the framework of the Friedmann-Robertson-Walker (FRW) universe with spacially flat metric.

[1] See a recent review on induced gravity [11].

The structure of this article is the following: In Section 2, we present a simple model of the induced gravity and adding the ghost sector we construct a topological field theory. In Section 3, we examine the cosmological implications by finding the classical solution to the field equations stemming from the topological model. We conclude in Section 4.

2 New model of induced gravity

Let us start by presenting an action of our new induced gravity: [2]

$$S = \int d^4x\sqrt{-g}\left[\gamma\left(R - \varepsilon_0\nabla_\mu\tau^\mu\right) + \lambda\left(1 - v_0\nabla_\mu\omega^\mu\right)\right], \tag{2}$$

where $\gamma(x)$ and $\lambda(x)$ are the Lagrange multiplier fields enforcing the constraints $R = \varepsilon_0\nabla_\mu\tau^\mu$ and $1 = v_0\nabla_\mu\omega^\mu$, respectively. And v_0 and ε_0 are some constants, and τ^μ and ω^μ are vector fields. The first term in this action was introduced to make the curvature density constraint $\sqrt{-g}R = 1$ be invariant under not the transverse diffeomorphisms (TDiff) but the full diffeomorphisms [14]. The second term was also introduced to keep the unimodular constraint $\sqrt{-g} = 1$ be invariant under diffeomorphisms [18].

Now it is easy to show that the action (2) induces the Einstein-Hilbert action with the cosmological constant by solving its field equations as follows: Variation with respect to the vector fields reads

$$\begin{aligned} \frac{\delta S}{\delta\tau^\mu} &= \sqrt{-g}\varepsilon_0\nabla_\mu\gamma = 0, \\ \frac{\delta S}{\delta\omega^\mu} &= \sqrt{-g}v_0\nabla_\mu\lambda = 0, \end{aligned} \tag{3}$$

from which the classical solution is given by

$$\gamma(x) = \bar{\gamma}, \quad \lambda(x) = \bar{\lambda}, \tag{4}$$

where $\bar{\gamma}$ and $\bar{\lambda}$ are certain constants. Then, provided that we set

$$\bar{\gamma} = \frac{1}{16\pi G}, \quad \bar{\lambda} = -\frac{2\Lambda}{16\pi G}, \tag{5}$$

we obtain

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R - 2\Lambda\right), \tag{6}$$

[2] We follow notation and conventions by Misner et al.'s textbook [28], for instance, the flat Minkowski metric $\eta_{\mu\nu} = diag(-,+,+,+)$, the Riemann curvature tensor $R^\mu{}_{\nu\alpha\beta} = \partial_\alpha\Gamma^\mu_{\nu\beta} - \partial_\beta\Gamma^\mu_{\nu\alpha} + \Gamma^\mu_{\sigma\alpha}\Gamma^\sigma_{\nu\beta} - \Gamma^\mu_{\sigma\beta}\Gamma^\sigma_{\nu\alpha}$, and the Ricci tensor $R_{\mu\nu} = R^\alpha{}_{\mu\alpha\nu}$. The reduced Planck mass is defined as $M_p = \sqrt{\frac{c\hbar}{8\pi G}} = 2.4\times10^{18}GeV$ where G is the Newton constant.

which is nothing but the Einstein-Hilbert action with the cosmological constant Λ. In this way, we can arrive at the action (6) of general relativity by starting with the action (2) of the induced gravity simply by solving field equations at the classical level.

A peculiar feature of this new induced gravity is that the Newton constant (G) and the cosmological constant (Λ) appear as integration constants which are not related to any parameters in the original induced gravity action. Let us recall that in unimodular gravity [15]-[27], the similar phenomenon emerges for the cosmological constant and is expected to play an important role in understanding the well-known cosmological constant problem.

We are now ready to show that the induced gravity action (2) can be deformed into a topological quantum field theory by adding ghosts and antighosts as shown shortly.

First of all, let us consider two types of the BRST transformations. One type of the BRST transformation δ_B is related to the first term in the action (2) and is given by

$$\delta_B \gamma = \delta_B c = 0, \quad \delta_B \tau^\mu = \frac{1}{\varepsilon_0} \nabla^\mu c, \quad \delta_B b = \gamma, \tag{7}$$

where c and b are respectively a ghost with the ghost number $+1$ and an antighost with the ghost number -1. The Lagrange mutiplier field γ is identified with the Nakanishi-Lautrup auxiliary field in this BRST transformation. The other BRST transformation $\hat{\delta}_B$ operates on the second term in the action (2) and takes the form

$$\hat{\delta}_B \lambda = \hat{\delta}_B \hat{c} = 0, \quad \hat{\delta}_B \omega^\mu = \frac{1}{v_0} \nabla^\mu \hat{c}, \quad \hat{\delta}_B \hat{b} = \lambda, \tag{8}$$

where $\hat{c}$ and $\hat{b}$ are respectively a ghost with the ghost number $+1$ and an antighost with the ghost number -1. The Lagrange mutiplier field λ corresponds to the Nakanishi-Lautrup auxiliary field as well. Note that the two types of the BRST transformations are nilpotent, $\delta_B^2 = \hat{\delta}_B^2 = 0$ and anticommute to each other, $\{\delta_B, \hat{\delta}_B\} = 0$, so we can define the physical state, $|phys\rangle$ by the physical state conditions, $Q_B|phys\rangle = \hat{Q}_B|phys\rangle = 0$ where Q_B and $\hat{Q}_B$ are the BRST charges [29].

Next, we add the kinetic terms for the ghosts to the action (2). As a result, the total action S_T is given by

$$\begin{aligned} S_T &= S + \int d^4x \sqrt{-g} \left(-\nabla_\mu b \nabla^\mu c - \nabla_\mu \hat{b} \nabla^\mu \hat{c} \right) \\ &= \int d^4x \sqrt{-g} \left[\gamma \left(R - \varepsilon_0 \nabla_\mu \tau^\mu \right) - \nabla_\mu b \nabla^\mu c + \lambda \left(1 - v_0 \nabla_\mu \omega^\mu \right) - \nabla_\mu \hat{b} \nabla^\mu \hat{c} \right] \\ &= \int d^4x \sqrt{-g} \left\{ \delta_B \left[b \left(R - \varepsilon_0 \nabla_\mu \tau^\mu \right) \right] + \hat{\delta}_B \left[\hat{b} \left(1 - v_0 \nabla_\mu \omega^\mu \right) \right] \right\}. \end{aligned} \tag{9}$$

The last expression clearly implies that S_T is an action of the topological quantum field theory where its moduli space is defined by two equations $R = \varepsilon_0 \nabla_\mu \tau^\mu$ and $1 = v_0 \nabla_\mu \omega^\mu$.

About twenty five years ago, we have constructed a model of topological pregeomery where a classical action has been taken to be trivially zero and quantum fluctuations of matter fields have generated the Einstein-Hilbert action with the cosmological constant at the one-loop level via the cutoff which violates the topological symmetry [8, 9].

On the other hand, in this paper, we have started with a non-trivial classical action (2) of the induced gravity, which reduces to the Einstein-Hilbert action with the cosmological constant at the classical level, and then added the kinetic term of the ghosts, thereby transforming the induced gravity to a topological field theory. The above two models of the induced gravity are obviously different, but the close relationship between the induced gravity and the topological field theory seems to suggest that the induced gravity and a topological quantum field theory might be tantalizingly equivalent.

3 Cosmological solutions

In this section, we work with the action (9) of a topological field theory to derive the inflation universe as a classical solution in the framework of the Friedmann-Robertson-Walker (FRW) universe with spacially flat metric. This derivation not only shows indirectly that the topological action (9) is equivalent to that of general relativity but also suggests that it could be applied to cosmology, in particular, the inflation universe.

The Einstein equations, which stem from variation with respect to the metric tensor $g^{\mu\nu}$, read

$$\begin{aligned} &2\gamma G_{\mu\nu} - 2\left(\nabla_\mu \nabla_\nu \gamma - g_{\mu\nu} \nabla^2 \gamma\right) + 2\left[\varepsilon_0 \tau_{(\mu} \nabla_{\nu)} \gamma - \nabla_{(\mu} b \nabla_{\nu)} c + v_0 \omega_{(\mu} \nabla_{\nu)} \lambda - \nabla_{(\mu} \hat{b} \nabla_{\nu)} \hat{c}\right] \\ &- g_{\mu\nu}\left[\lambda + \varepsilon_0 \tau^\rho \nabla_\rho \gamma - \nabla_\rho b \nabla^\rho c + v_0 \omega^\rho \nabla_\rho \lambda - \nabla_\rho \hat{b} \nabla^\rho \hat{c}\right] = 0, \end{aligned} \tag{10}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is the Einstein tensor and the round bracket indicates the symmetrization of indices of weight $\frac{1}{2}$ such as $A_{(\mu} B_{\nu)} = \frac{1}{2}\left(A_\mu B_\nu + A_\nu B_\mu\right)$. In deriving this equation, by performing the integration by parts, it is convenient to rewrite the action (9) as

$$S_T = \int d^4x \sqrt{-g}\left[\gamma R + \varepsilon_0 \tau^\mu \nabla_\mu \gamma - \nabla_\mu b \nabla^\mu c + \lambda + v_0 \omega^\mu \nabla_\mu \lambda - \nabla_\mu \hat{b} \nabla^\mu \hat{c}\right]. \tag{11}$$

Taking variation of the vector fields τ^μ, ω^μ gives rise to Eq. (3) whose solution is given by Eq. (4). The field equations for the Lagrange multiplier

fields γ read

$$R = \varepsilon_0 \nabla_\mu \tau^\mu, \tag{12}$$

which can be rewritten as

$$\sqrt{-g}R = \varepsilon_0 \partial_\mu \left(\sqrt{-g}\tau^\mu \right). \tag{13}$$

Likewise, for the λ variation, we have

$$1 = v_0 \nabla_\mu \omega^\mu, \tag{14}$$

which can be recast to the form

$$\sqrt{-g} = v_0 \partial_\mu \left(\sqrt{-g}\omega^\mu \right). \tag{15}$$

Eqs. (13) and (15) imply that the scalar curvature density ($\sqrt{-g}R$) and the volume element ($\sqrt{-g}$) are determined by a divergence of the vector densities $\sqrt{-g}\tau^\mu$ and $\sqrt{-g}\omega^\mu$, respectively.

Finally, field equations for the ghosts and the antighosts satisfy the same form of equation $\nabla_\mu \nabla^\mu \Phi = 0$ where $\Phi = \{c, b, \hat{c}, \hat{b}\}$. Since the ghosts and the antighosts carry non-zero ghost numbers, we simply take $c = b = \hat{c} = \hat{b} = 0$ as the classical solution.

With the solution Eq. (4) and the vanishing ghosts and antighosts, the Einstein equations (10) takes the simpler form

$$G_{\mu\nu} - \frac{\bar{\lambda}}{2\bar{\gamma}} g_{\mu\nu} = 0, \tag{16}$$

which is equivalent to the standard Einstein equations with the cosmological constant $\Lambda = -\frac{\bar{\lambda}}{2\bar{\gamma}}$.

At this stage, having the cosmological application of the model at hand in mind, let us work with the FRW metric with spacially flat metric ($k = 0$)

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t)^2 (dx^2 + dy^2 + dz^2), \tag{17}$$

with $a(t)$ being the scale factor. Since the equations (16) are usual Einstein equations with the cosmological constant, the Hubble parameter $H = \frac{\dot{a}}{a}$ takes a constant value $H = \bar{H}$ where $\bar{H}$ is a constant. Then, the scale factor is of form $a(t) = a(0)\mathrm{e}^{\bar{H}t}$ so that we have the inflation universe.

The remaining field equations (12) and (14) must be solved. In order to solve these equations in an analytical way, we assume that

$$\begin{aligned} \tau_0 &= \tau_0(t), \quad \dot{\tau_0} = \tau_i = 0, \\ \omega_0 &= \omega_0(t), \quad \dot{\omega_0} = \omega_i = 0, \end{aligned} \tag{18}$$

where the overdot means the differentiation with respect to the time variable and the index $i = 1, 2, 3$ denotes the space component. Then, Eq. (12) yields the equation

$$\bar{H} = -\frac{1}{4}\varepsilon_0\tau_0, \tag{19}$$

and Eq. (14) is solved to be

$$\bar{H} = -\frac{1}{3}\frac{1}{v_0\omega_0}. \tag{20}$$

Hence, provided that we make use of Eq. (5) and set $\varepsilon_0 = v_0 = 1$, the time components τ_0, ω_0 of the vector fields are described in terms of the cosmological constant Λ

$$\tau_0 = -4\sqrt{\frac{\Lambda}{3}}, \quad \omega_0 = -\frac{1}{\sqrt{3\Lambda}}. \tag{21}$$

4 Conclusion

In this article, motivated with unimodular gravity, we have constructed a new type of induced gravity. In unimodular gravity, the cosmological constant emerges as an integration constant whereas in our induced gravity, both the Newton constant and the cosmological constant appear as integration constants. This physically intriguing feature is utilized to derive general relativity from the induced gravity at the classical level. We think that our induced gravity is an analog of the pregeometrical approach of string field theory in local field theory.

Furthermore, we have pursued our original idea that the induced gravity is closely connected with the topological field theory and shown that indeed the induced gravity considered in this paper can be deformed into a topological field theory by adding the kinetic terms of ghosts.

Here we wish to comment on one of the particular motivations for considering our particular gravitational action (2). Recently, the absence of radiative corrections to the cosmological constant in the framework of unimodular gravity, which is parametrized by Weyl-transverse gravity, has been extensively discussed in Refs. [30, 31]. This absence of radiative corrections allows us to avoid the stability problem of the cosmological constant problems [32], and is closely related to the topological nature of some of the terms in the action. We therefore expect similar results to be valid for the theory under consideration, but this time the Newton constant would be unchanged by radiative corrections. This problem is now under active investigation.

We have also found the inflation solution by solving analytically the field equations of the topological field theory. Of course, adding additional matter fields in our model, it is possible to derive various types of cosmological solutions as in general relativity. Indeed, it is straightforward to couple various matter fields to the present theory. For instance, in case of a scalar field, the action (2) can be extended to

$$S = \int d^4x\sqrt{-g}\left\{\gamma\left(R + \nabla_\mu \tau^\mu\right) + \lambda\left(1 + \nabla_\mu \omega^\mu\right) + \beta\left[-\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi) + \nabla_\mu\rho^\mu\right]\right\}, \quad (22)$$

where for simplicity we have set $\varepsilon_0 = v_0 = -1$. This possibility of coupling the matter fields to our theory guarantees that the present theory would be part of a more realistic, physical setup.

In this article, we have treated with the metric tensor field as a classical background field along the spirit of the induced gravity, but it might be interesting to regard the metric tensor as a quantized field as well and investigate its physical implications [3] Moreover, even within the present framework, it is significant to clarify how the new fields introduced in our theory would affect physical behavior in a quantum field setup.

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