Advanced Studies in Theoretical Physics
Vol. 10, 2016, no. 8, 421 - 431
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/astp.2016.6930

The Hubble Constant and the Deceleration Parameter in Anisotropic Cosmological Spaces of Petrov type D

R. Alvarado

CINESPA, Escuela de Fisica
Universidad de Costa Rica, Costa Rica

Abstract

In this paper the Hubble parameter and the deceleration parameter are analyzed for a group of anisotropic homogeneous solutions of the Petrov type-D. It is obtained that for the said set of solutions can be constructed a representative average value of the Hubble constant and the deceleration parameter; both matching with their analogues obtained for the FRWL Flat model or the Kasner's solution depending on the time values; however, the parameters depend on time, so their values or tendency, evolve significantly regarding time, in most cases compared to the ones of FRWL's or Kasner's. The average value for those parameters does not depend on whether the expansion is greater on one axis than on the perpendicular plane to this, or otherwise. The deceleration parameter q, for models where $\lambda < -1/3$ change signs, when time augments, so it is presented a process of initial deceleration that trough the augmentation changes to another one of acceleration.

Keywords: cosmology, Hubble, deceleration, parameter, exact, solution, einstein

1 Introduction

The data of the microwave background radiation obtained by the satellites COBE, WMAP and PLANCK and the discovery of the acceleration of the Universe [1, 2], lead to series of problems which have been discussed recently

(cf. [3]).
The anisotropic acceleration of the type Ia supernovae and the gamma-ray burst data (towards the Vulpecula constellation) with a level of anisotropy of the deceleration parameter $\Delta q_{0,max}/q_0 = 0.79^{-0.28}_{+0.27}$ [4],[5], the evidence, trough the study of the Ia supernovae, that the Universe went through a stage of deceleration before commencing the acceleration stage [6] [7], [8], an asymmetry value of the Hubble parameter of $\Delta H/H < 0.038$ [9] as well as a type of anisotropy of redshift in studies of Quasares and Seyfert Galaxies [10].
This motivates to find and formulate new ideas or models that allow, at least in part, giving an explanation to the cosmological observation; those ideas are of experimental and theoretical interest and may or may not break with standards or principals that are taken into cosmology, such as the Universe's isotropy and homogeneity.
In the present study, the averaged parameters of deceleration and the averaged parameters of Hubble in anisotropic solutions of the Petrov type-D (Bianchi-I, con $g_{00} = g_{00}(t)$) and their respective changes in tendency trough time.

2 The Hubble Parameter

The Hubble parameter H is defined for the FRWL solution as $H = \dot{R}/R$, where R is the scale factor. For the FRWL flat model,

$$ds^2 = dt^2 - R(t)^2(dx^2 + dy^2 + dz^2), \tag{1}$$

and considering Einstein's equations [11]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}, \tag{2}$$

where $T_{\mu\nu} = (\mu + P)u_\mu u_\nu - g_{\mu\nu}P$ is the energy-momentum tensor of the perfect fluid model, and when considering that $P = \lambda\mu$ it follows that $R(t) \sim t^{2/3\,(\lambda+1)^{-1}}$; therefore, Hubble's parameter is

$$H = \frac{2}{3\,(\lambda+1)\,t}. \tag{3}$$

For the Bianchi-I symmetry,

$$ds^2 = dt'^2 - a^2dx^2 - b^2dy^2 - c^2dz^2, \tag{4}$$

Hubble's parameter is defined as [12]

$$H = \frac{\left((abc)^{1/3}\right)'}{abc} = \frac{1}{3}\left(H_x + H_y + H_z\right) = \frac{1}{3}\left(\frac{a'}{a} + \frac{b'}{b} + \frac{c'}{c}\right), \tag{5}$$

where ($'$) represents derivatives for t'; a, b, and c are scale factors dependent on t', and the coordinates x, y and z, respectively.
The above definition can be generalized for a symmetry in the shape of

$$ds^2 = e^2 dt^2 - A^2 dx^2 - B^2 dy^2 - C^2 dz^2, \tag{6}$$

where e, A, B, and C are functions of t, with only noticing that when writing $dt' = edt$, (6) it transform into a metric type (4). Therefore, (5) can be defined as follows:

$$H = \frac{\left((ABC)^{1/3}\right)^{\cdot}}{(ABC)^{1/3}\, e} = \frac{1}{3}\left(H_x + H_y + H_z\right) = \frac{1}{3}\left(\frac{\dot{A}}{Ae} + \frac{\dot{B}}{Be} + \frac{\dot{C}}{Ce}\right), \tag{7}$$

the point represents the t derivative.

3 The Deceleration Parameter q

The deceleration parameter q is defined for the FRWL solution as

$$q = -\ddot{R}R/\dot{R}^2.$$

For the flat model of FRWL ($R(t) \sim t^{2/3\,(\lambda+1)^{-1}}$), so that

$$q = 1/2 + 3/2\,\lambda. \tag{8}$$

For the Bianchi-I symmetry, it is defined in an analogous manner, but changing $R \longrightarrow (abc)^{1/3}$ or using the symmetry (4) and of (5). It can be written as

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right) = -\left(1 + \frac{1}{3}(q_x + q_y + q_z)\right), \tag{9}$$

where $q_x = \frac{1}{H^2}\left(\frac{\dot{a}}{a}\right)^{\cdot}$, $q_y = \frac{1}{H^2}\left(\frac{\dot{b}}{b}\right)^{\cdot}$, $q_z = \frac{1}{H^2}\left(\frac{\dot{c}}{c}\right)^{\cdot}$.
For a symmetry type (6), considering the Hubble parameter given in (7), the deceleration parameter can be written as

$$q = -\left(1 + \frac{\dot{H}}{H^2 e}\right) = -\left(1 + \frac{1}{3}(Q_x + Q_y + Q_z)\right), \tag{10}$$

where $Q_x = \frac{1}{eH^2}\left(\frac{\dot{A}}{eA}\right)^{\cdot}$, $Q_y = \frac{1}{eH^2}\left(\frac{\dot{B}}{eB}\right)^{\cdot}$, $Q_z = \frac{1}{eH^2}\left(\frac{\dot{C}}{eC}\right)^{\cdot}$.
A particular case is the Kasner vacumm solution (external solution) whose form is [13]

$$\begin{gathered} ds^2 = dt^2 - t^{2a_1}dx^2 - t^{2a_2}dy^2 - t^{2a_3}dz^2, \\ a_1 + a_2 + a_3 = 1 \quad \textit{and} \quad a_1^2 + a_2^2 + a_3^2 = 1, \end{gathered} \tag{11}$$

from which it is obtained that the value of the deceleration parameters (9) and Hubble's (5), respectively are

$$q = 2, \quad H = 1/(3t), \tag{12}$$

those do not depend on the supremacy of any of the axis, which is explained by being average parameters of H_i and q_i (not to be confused with relativistic invariants).

4 Petrov Type-D Anisotropic Cosmological Solutions

In [3], it was obtained a set of cosmological solutions for symmetry given by

$$ds^2 = F dt^2 - t^{2/3} K(dx^2 + dy^2) - \frac{t^{2/3}}{K^2} dz^2, \tag{13}$$

where F and K are functions of t of the following form

$$F = \frac{1}{1 + \alpha t^{1-\lambda}}, \quad K_\pm = \left(\frac{4(\sqrt{1 + \alpha\, t^{1-\lambda}} - 1)}{\alpha(\sqrt{1 + \alpha\, t^{1-\lambda}} + 1)} \right)^{\pm 2/3\,(1-\lambda)^{-1}} \tag{14}$$

for $\lambda \neq 1$ and

$$F = 1, \quad K_{Z\pm} = t^{\pm 2/3\,\sqrt{1-3M}} \tag{15}$$

when $\lambda = 1$. In (14) and (15) α (different for each λ) and M are considered positive constants (by requiring that $\mu \geq 0$), and $0 \leq M \leq 1/3$ (by requiring that $g_{\mu\nu} \in \mathbb{R}$). The density and the pressure, regardless of the chosen sign for K, are

$$\mu = \frac{\alpha}{3t^{1+\lambda}}, \quad P = \frac{\lambda\,\alpha}{3t^{1+\lambda}}, \tag{16}$$

when $\lambda \neq 1$ and when $\lambda = 1$, $\alpha/3$ should be changed for M.
For those solutions the parameters (7) and (10) have the following form:

$$H = \frac{\sqrt{1 + \alpha\, t^{1-\lambda}}}{3t}, \quad q = \frac{\alpha\, t(1 + 3\lambda) + 4\, t^\lambda}{2(t^\lambda + \alpha\, t)} \tag{17}$$

and for (15)

$$H = \frac{1}{3t}, \quad q = 2. \tag{18}$$

5 Analysis of Parameters H and q in Regard to Possible Cosmological Models

5.1 Phantom Type Model

The Phantom model has as a state equation $\lambda < -1$ and its only soltuion (14) can be written in the following form [3]:

$$F = \frac{1}{1+\alpha t^{1+|\lambda|}}, \quad K = \left(\frac{4(\sqrt{1+\alpha\, t^{1+|\lambda|}}-1)}{\alpha(\sqrt{1+\alpha\, t^{1+|\lambda|}}+1)} \right)^{\pm 2/3\,(1+|\lambda|)^{-1}}. \tag{19}$$

$t = (-1/2\,\eta\,\sqrt{\alpha}\ (|\lambda|-1))^{-2\,(|\lambda|-1)^{-1}}$ $(t \longrightarrow \infty,\ \eta \longrightarrow 0,\ \eta < 0)$, so that

$$ds^2 \longrightarrow d\eta^2 - \left(-1/2\,\eta\,\sqrt{\alpha}\ (|\lambda|-1)\right)^{-4/3\,(|\lambda|-1)^{-1}} \left(dx'^2 + dy'^2 + dz'^2\right), \tag{20}$$

where $x' = (4/\alpha)^{\pm 1/(3(1+|\lambda|))} x$, analogous y, $z' = (4/\alpha)^{\mp 2/(3(1+|\lambda|))} z$. Parameters (17), with the new coordinates (η, x', y', z'), have the following form:

$$H \longrightarrow -2/3\,\frac{1}{\eta\ (|\lambda|-1)}, \qquad q \longrightarrow 1/2 - 3/2\ |\lambda|\,, \tag{21}$$

therefore, for all the values of $\lambda < -1$, the deceleration parameter is negative when $t \longrightarrow \infty$, which indicates that for models of type Phantom under the analyzed symmetry, the Universe has an accelerated expansion in this limit; however, if $t \longrightarrow 0$, the solution tends to $F \longrightarrow 1$ and $K \longrightarrow t^{\pm 2/3}$, so the metric (13) tends to

$$ds^2 \longrightarrow dt^2 - t^{2/3 \pm 2/3}(dx^2 + dy^2) - t^{2/3 \mp 4/3} dz^2, \tag{22}$$

because of their form, they are the Kasner vacuum with $a_1 = a_2 = -2a_3 = 2/3$ or $a_1 = a_2 = 0$, $a_3 = 1$, thus they are consistent with (12), so the parameter $q \longrightarrow 2$, hence it is positive and indicates a decelerated expansion (similar to the one obtained for the Zeldovich type model, $\lambda = 1$ in FRWL), i.e. in the Phantom type models, the Universe, for $t \longrightarrow 0$, expands decelerating and as time increases, the acceleration increases. Meanwhile, Hubble's parameter $H \sim 1/\eta$ $(\lambda \neq -1)$, if $t \longrightarrow \infty$, $\eta \longrightarrow 0$; therefore, in the models Phantom type, the Hubble's parameter tends to infinity for those values, allowing what is known as Big Rip and for values of $t \longrightarrow 0$ the parameter H, of (17), also tends to $H \longrightarrow \infty$. From the foregoing, both for very small values of t, as well as for very large values, $H \longrightarrow \infty$, and goes through a minimum value of $H_{min} = 1/3\sqrt{1 + 2\ (|\lambda|-1)^{-1}}\,(1/2\,\alpha\ (|\lambda|-1))^{(1+|\lambda|)^{-1}}$, when $t_{min} = (1/2\,\alpha\ (|\lambda|-1))^{-(1+|\lambda|)^{-1}}$.

5.2 Dark Energy Model

The Dark Energy model has as a state equation the relation $\mu = -P \sim \Lambda$ (where Λ is the cosmological constant), or what is the same $\lambda = -1$. The solution obtained gives as a result, if $\lambda = -1$, that $\mu = -P = -\alpha/3$ or if it is considered the usual relation that is given to the cosmological constant Λ with the dark energy $\alpha = 3c^2\Lambda/(8\pi G)$.
Analogous to what was done in the prior section, it would be done the following change of temporal coordinate $t = \frac{\sinh\left(\eta\sqrt{\alpha}\right)}{\sqrt{\alpha}}$. In this case the metric can be written in the form [3]

$$ds^2 = d\eta^2 - \left(\sinh\left(\sqrt{\alpha}\eta\right)\right)^{2/3} N_0^{\pm 1/3}\alpha^{-1/3}\left(dx^2 + dy^2 + N_0^{\mp 1} dz^2\right). \qquad (23)$$

where $N_0 = 4\left(\cosh\left(\sqrt{\alpha}\eta\right) - 1\right)/\left(\alpha\left(\cosh\left(\sqrt{\alpha}\eta\right) + 1\right)\right)$.
For such solution the parameter H y q de (17), when $t \longrightarrow \infty$, tend to

$$H \longrightarrow 1/3\sqrt{\alpha}, \qquad q \longrightarrow -1, \qquad (24)$$

and when $t \longrightarrow 0$, tend to

$$H \longrightarrow \infty, \qquad q \longrightarrow 2, \qquad (25)$$

Therefore, according to the parameter q, as in the previous case, indicates an expansion of the Universe initially decelerated until a time $t = \sqrt{2/\alpha}$, from which starts a stage of acceleration and as time augments, asymptotically approaching to -1. Regarding the Hubble parameter, this has very large values when $t \longrightarrow 0$, but asymptotically becomes in a constant as time augments.

5.3 Quintessence Model

Fort the Quintessence model, the state equation has values of $-1 < \lambda < 0$. The solution of said model can be seen at [3] and has similarities to the solution of the Phantom model for values of $t \longrightarrow 0$. The parameters H y q, if $t \longrightarrow 0$, H and q tend to

$$H \longrightarrow \infty, \qquad q \longrightarrow 2. \qquad (26)$$

For values when $t \longrightarrow \infty$, the metric solution (14) for the Quintessence model tends to an isotropic regimen of the form [3]

$$ds^2 \approx d\eta^2 - \xi^{4/3\,(1-|\lambda|)^{-1}}\left(dx'^2 + dy'^2 + dz'^2\right), \qquad (27)$$

where $\xi = 1/2\,\eta\sqrt{\alpha}$, so in (7)

$$A \approx B \approx C \approx (1/2\,\eta\sqrt{\alpha})^{2/3\,(1-|\lambda|)^{-1}}, \qquad e \approx 1.$$

It is evident that the definitions used in FRWL modes, for the parameters H and q, have the same validity in such limit and has the form

$$H \longrightarrow \frac{2}{3\eta(1-|\lambda|)}, \qquad q \longrightarrow \frac{1-3|\lambda|}{2}, \tag{28}$$

Thus representing a similar situation to the dark energy model for values of $-1/3 > \lambda$; for the value of $\lambda = -1/3$, the parameter of deceleration tends to zero, implying that the space begins in a deceleration process until entering in a regimen when ther is no acceleration or deceleration, and if $-1/3 < \lambda < 0$, it is positive; therefore, for the last set of values the possible scenarios of Universes decelerate with the augmentation of t, though the q value decreases with the augmentation of t.

5.4 Models Ordinary, Relativistic under Pressure, Ultrarelativistic and Hard Universe

It is reffered to the Dust models ($\lambda = 0$), Ordinary Relativistic at Pressure ($0 < \lambda < 1/3$), Ultrarelativistic ($\lambda = 1/3$) and Hard Universe ($1/3 < \lambda < 1$), for which the equations have values of $0 \leq \lambda < 1$. For these, the solutions of (14) have been analyzed in their different limits for which these can be determined as synchronous ($g_{00} = 1$, $g_{i0} = 0$) [3]and it was found that in the proximities with $t = 0$, the solutions can be written as

$$ds^2 \longrightarrow dt^2 - t^{2/3\pm 2/3}\left(dx^2 + dy^2\right) - t^{-4/3\pm 2/3} dz^2, \tag{29}$$

and for values of $t \longrightarrow \infty$, they become isotropic and can be written as

$$ds^2 \approx d\eta^2 - \xi^{4/3\,(1+\lambda)^{-1}}\left(dx'^2 + dy'^2 + dz'^2\right), \tag{30}$$

where

$$\xi = 1/2\,\eta\,\sqrt{\alpha}\,(1+\lambda) \quad y \quad t = \xi^{2\,(1+\lambda)^{-1}}, \tag{31}$$

Therefore, the parameters H and q can be defined using the same definitions for the solutions in symmetry of FRWL. For these it is obtained that

$$H \longrightarrow \frac{2}{3(1+\lambda)\eta} \longrightarrow 0, \qquad q = \frac{1+3\lambda}{2}, \tag{32}$$

they do not differ significantly with previous cases.
If $t \longrightarrow 0$, the parameters H and q, tend to

$$H \longrightarrow \frac{1}{3t} \longrightarrow \infty, \qquad q = 2, \tag{33}$$

From the above, the previous models give common scenario, for all those the possible Universes in their beginnings have a Hubble's parameter that tend to infinity and decay towards zero with the augmentation of time; meanwhile the deceleration parameter decreases its value over time from $q_{t\longrightarrow 0} = 2 > 0$ to $q_{t\longrightarrow\infty} = (1+3\lambda)/2 > 0$, but not its sign. Therefore, it is presented a process of deceleration for all the t values; however, since the q parameter decreases it can be interpreted that the space has accelerated with the augmentation of t in relation to the one that had at the beginning. A similar situation is presented for the Quintessence models with values of $\lambda > -1/3$.

5.5 Zeldovich Type Model

The Zeldovich type model, has as state equation $\lambda = 1$. For this model, the solution is [3]

$$ds^2 = dt^2 - t^{2/3(1\pm\sqrt{1-3\,M})}\left(dx^2 + dy^2\right) - t^{2/3(1\mp 2\sqrt{1-3\,M})} dz^2. \tag{34}$$

where M is a constant with values that may be between $0.48\dot{1}0^{-40} \leq M \leq 1/3$ and does not tend to be isotropic. The solution (34) is synchronic. The Zeldovich type Model is the only case of anisotropic models of Petrov type D, for which the parameters H and q do not change their form depending on the value of t, and has the following form:

$$H = \frac{1}{3t}, \qquad q = 2. \tag{35}$$

From (35) it is concluded that the Hubble's parameter tends to infinity when $t \longrightarrow 0$ and tends to zero when $t \longrightarrow \infty$, and it is consistent accurately with the same parameter obtained in the case of the FRWL model. In relation to the deceleration parameter q, it happens the same as with H regarding that no difference is found with the same parameter in FRWL and represents an expanding Universe that decelerates constantly

5.6 Ekpirotic Modelo (ekpyrotic matter)

The Ekpirotic Model has a state equation where $\lambda > 1$. For values of $t \longrightarrow 0$, the solution is a synchronous system, making a change of temporary variable $(\alpha \neq 0)$ of the form $t = \xi^{2\,(\lambda+1)^{-1}}$, where $\xi = 1/2\,\sqrt{\alpha}\,(\lambda+1)\,\eta$, has the form

$$ds \approx d\eta^2 - \xi^{4/3\,(\lambda+1)^{-1}}\left(dx'^2 + dy'^2 + dz'^2\right) \tag{36}$$

$$\tag{37}$$

where $x' = x\ (\alpha/4)^{\pm 2(3(\lambda-1))^{-1}}$, analogous y and $z' = z(\alpha/4)^{\mp 4(3(\lambda-1))^{-1}}$. For values of $t \longrightarrow \infty$, the solution has the form

$$ds^2 \approx dt^2 - t^{\pm 2/3+2/3}\left(dx^2 + dy^2\right) - t^{\mp 4/3+2/3} dz^2, \tag{38}$$

For those limits it is obtained that

$$\lim_{\eta \longrightarrow 0} H \longrightarrow \frac{2}{3(1+\lambda)\eta} \longrightarrow \infty, \qquad \lim_{\eta \longrightarrow 0} q = \frac{1+3\lambda}{2}, \tag{39}$$

and if $t \longrightarrow \infty$, the parameters H and q tend to

$$H \longrightarrow \frac{1}{3t} \longrightarrow 0, \qquad q = 2. \tag{40}$$

Of the foregoing, in the ekpirotic model the behavior of the H parameter is similar to the above cases, since it tends to zero (when $t \longrightarrow \infty$) or to infinity (when $t \longrightarrow 0$), in the same way as in the previous models, but it differs in that the speed with which tends to zero is constant for any value of $\lambda > 1$, and when it tends the same does not happen. In the opposite way to models with $\lambda < 1$, the q parameter behaves oppositely to the rule represented for the same parameter, but for models with $\lambda < 1$.

6 Conclusions

For Petrov type D anisotropic models, both the Hubble parameter as the deceleration parameter depend on time, which marks a fundamental difference in relation to the same parameters obtained for the case of the flat universe of FRWL or the Kasner vacuum solution, where the deceleration parameter is constant for each type of model or for all of them, in the Kasner's case. The parameters depend on time, so in certain limits they behave as analogues obtained in the Kasner solution and for other limits they behave as analogues of the FRWL flat model solution; a transitional path exists between both of them through time. For the Phantom model, a unique situation related to the Hubble's constant is presented, for both the anisotropic models of the Petrov type D, as well as for the equivalents in FRWL and Kasner. For this case there is a value $H_{min} = 1/3\sqrt{1 + 2\,(|\lambda| - 1)^{-1}}\,(1/2\,\alpha\,(|\lambda| - 1))^{(1+|\lambda|)^{-1}}$, en un tiempo $t_{min} = (1/2\,\alpha\,(|\lambda| - 1))^{-(1+|\lambda|)^{-1}}$, before which $H \longrightarrow \infty$ cuando $t \longrightarrow 0$ and after which $H \longrightarrow \infty$ cuando $t \longrightarrow \infty$. For the dark energy models and some quintessence models with $\lambda < -1/3$ occurs a situation that also takes place and is common in the Phantom model, when $t \longrightarrow 0$ the Hubble parameter tends to infinity, and the deceleration one $q \longrightarrow 2$, but with the augmentation of t time; the H parameter tends to be constant in the case of the dark energy, or tends to zero in the case of the quintessence model, and the q parameter changes its value and sign meaning that there is a period of deceleration for small values of t and then changes for a process of acceleration as time augments which agrees with some observations mentioned before [6, 7, 8].

For the quintessence model with $\lambda = -1/3$, the q parameter tends toz ero with the augmentation of time, and for the rest of the values $< -1/3\lambda < 0$, there is no sing change by the q parameter, but there is a significant decrease in its value: a similar situation to the quintessence models with $-1/3 < \lambda < 0$ is presented for the models type Dust, relativistic ordinary under pressure, Ultrarelativistic and Hard Universe. For the Zeldovich model the situation is different, since for this model the parameters $H = 1/(3t)$ and $q = 2$ do not change over time in the dynamic way in which parameters change for other models. In the case of the Zeldovich model, there is no difference regarding the form in which parameters change, or not, in relation to their analogue when using the FRWL symmetry or the Kasner vacuum solution. In the case of the ekpirotic model, for values of time $t \longrightarrow 0$, the Hubble parameter tends to infinity and the deceleration parameter is $q > 2$, and as time augments they tend to behave as if it was a Kasner vacuum solution, that is $H \longrightarrow 1/(3t)$ and $q \longrightarrow 2$, so they go through a process of high to low deceleration.

References

[1] A. G. Riess, A. V. Filippenko, P. Challis et al., Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant, *Astron. Journal*, **116** (1998), no. 3, 1009-1038. https://doi.org/10.1086/300499

[2] S. Perlmutter, G. Aldering, G. Goldhaber et al., Measurements of Ω and Λ from 42 High-Redshift Supernovae, *The Astrophysical Journal*, **517** (1999), no. 2, 565-586. https://doi.org/10.1086/307221

[3] R. Alvarado, Cosmological Exact Solutions Set of a Perfect Fluid in an Anisotropic Space-Time in Petrov Type D, *Advanced Studies in Theoretical Physics*, **10** (2016), 267-295. https://doi.org/10.12988/astp.2016.6311

[4] Rong-Gen Cai, Zhong-Liang Tuo, Direction Dependence of the Deceleration Parameter, *Journal of Cosmology and Astroparticle Physics*, **2012** (2012), no. 2. https://doi.org/10.1088/1475-7516/2012/02/004

[5] Rong-Gen Cai, Yin-Zhe Ma, Bo Tang, Zhong-Liang Tuo, Constraining the Anisotropic Expansion of the Universe, *Phys. Rev. D*, **87** (2013). https://doi.org/10.1103/physrevd.87.123522

[6] A.G. Riess, L.G. Strolger, J. Tonry et al., Type la Supernova Discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution, *The Astrophysical Journal*, **607** (2004), no. 2, 665-687. https://doi.org/10.1086/383612

[7] J. A. S. Lima, R. F. L. Holanda, J. V. Cunha, Are Galaxy Clusters Suggesting an Accelerating Universe?, *AIP Conf. Proc*, **1241** (2010). https://doi.org/10.1063/1.3462638

[8] Omer Farooq, Bharat Ratra, Hubble parameter measurement constraints on the cosmological deceleration-acceleration transition redshift, *The Astrophysical Journal Letters*, **766** (2013), no. 1. https://doi.org/10.1088/2041-8205/766/1/L7

[9] B. Kalus, D. J. Schwarz, M. Seikel, A. Wiegand, Constraints on anisotropic cosmic expansion from supernovae, *Astron. and Astrophys.*, **553** (2013), A56. https://doi.org/10.1051/0004-6361/201220928

[10] A.V. Glushkov, The Big Bang Problems: Anisotropy of $z \leq 6$ Redshifts, (2005), arXiv:astro-ph/0512276.

[11] L. Landau, E. Lifshitz, *The Classical Theory of Fields*, Addison-Wesley, Reading (Massachusetts), 1951.

[12] S. Kurma and C.P. Singh, Anisotropic Bianchi type-I models with constant deceleration parameter in general relativity, *Astrophys. Space Science*, **312** (2007), 57-62. https://doi.org/10.1007/s10509-007-9623-4

[13] E. Kasner, Geometrical Theorem on Einstein's Cosmological Equations, *American Journal of Mathematics*, **43** (1921), no. 2, 217-221. https://doi.org/10.2307/2370192

Received: October 7, 2016; Published: November 21, 2016